Assignment 5 — due Friday April 8$^{\text{th}}$ [$Revision$: 1.1]

Question 1

Use *EZ-Web* to construct structure files for $1\,M_\odot$ and $10\,M_\odot$ models approximately half-way through their main-sequence lifetimes (as determined from a core hydrogen mass fraction $X_{\rm c} \approx 0.35$; this quantity is reported in the summary file). For each model, create a graph showing ∇, $\nabla_{\rm ad}$ and $\nabla_{\rm rad}$ plotted together as a function of fractional radius r/R. On each graph, mark the regions where energy is (i) transported by radiation alone, (ii) transported by radiation and efficient convection, and (iii) transported by radiation and inefficient convection.

As a reminder, the *EZ-Web* page has some useful IDL procedures for reading in *EZ-Web* output.

Question 2

Write the diffusion equation for the radiative luminosity $L_{r,\rm rad}$ in terms of the gradient of the radiation pressure $P_{\rm rad} = aT^4/3$. Combine this with the equation of hydrostatic equilibrium, to find an expression for the spatial gradient of the gas pressure $P_{\rm gas}$.

Show that the gas pressure gradient becomes positive (i.e., the star experiences a pressure inversion) when

$$\Gamma \equiv \frac{\kappa L_{r,\rm rad}}{4\pi G M_r c} > 1. \tag{1}$$

This inequality represents the so-called *Eddington Limit* — an upper limit on the radiative luminosity, set by the threshold at which radiation pressure is able to overcome self-gravity.

Question 3

Express the Eddington limit in terms of the radiative temperature gradient $\nabla_{\rm rad}$ and the pressure ratio $\beta \equiv P_{\rm gas}/P = 1 - P_{\rm rad}/P$.

Question 4

For an ideal monatomic gas plus radiation pressure, write down an expression for the adiabatic temperature gradient $\nabla_{\rm ad}$ in terms of the pressure ratio β. (Use *Stellar Interiors* if you want to avoid spending more than one minute on this question).

Question 5

Combining the answers to the preceding questions, show that convection will always commence *before* the Eddington limit is reached.

Narrative aside: this is how a star avoids breaching the Eddington limit in its interior. Convection kicks in, reducing the radiative luminosity so that the star can remain below the limit.

The situation at the stellar surface is different, however — low densities mean that convection is too inefficient to transport any appreciable amount of energy (i.e., $L_{r,\rm rad}$ is unaffected when convection turns on). When the Eddington limit is breached at the surface, the result is a radiation-driven wind (as seen, e.g., in massive stars).