Assignment 2 — due Thursday February 17$^{\rm th}$ [*Revision* : 1.1]

Question 1

By measuring the apparent magnitude and effective temperature of stars within a young cluster, an observer tells you that stars on the main sequence obey the approximate luminosity-radius relation $L \propto R^3$.

Armed with this knowledge, you — a budding stellar theoretician — set out to figure out how the internal density of the main-sequence stars varies as a function of stellar radius/luminosity. Are the bigger/brighter stars more or less dense than the smaller/dimmer stars?

This sounds like a tricky question, given that you don't know anything about the stars' *masses* (these are all single stars, so you can't take advantage of binary orbits to make mass measurements). However, you soon realize that invoking hydrostatic and thermal equilibrium should give you enough constraints on stellar properties to reach a definitive answer.

To keep things simple, assume that the stars have uniform density (see §1.4 of *Stellar Interiors* for a review of the constant-density model). Your task is then to find a density-radius relation of the form $\rho \propto R^\chi$, and — by considering the sign of the exponent χ — thence determine the answer to the question.

In addition to the empirical luminosity-radius relation and the constant-density approximation, you should assume the following:

- An ideal gas law with constant mean molecular weight;
- A pressure that goes to zero at the stellar surface;
- Hydrogen burning via the PP chain, with an energy release rate $\epsilon \propto \rho T^4$ (cf. Table 1.1 of *Stellar Interiors*).

You will also need to apply the stellar structure equations that embody the conservation laws for mass, momentum and energy.

This is not a trivial question — it involves all you have learned to date. Points will be awarded not only for the correct answer, but also for the narrative that explains how this answer is arrived at. In this respect, a long sequence of equations, with no written explanation of the physical principles being applied, will not score very highly!

If you find yourself going around in circles, then pause to figure out (i) what your unknowns are, and (ii) what relationships (i.e., equations) exist between these unknowns. If there aren't the same number of unknowns as equations, then you're missing a bit of information — see if you can take a step back, re-read the question, and find out what it is.

A final hint to simplify the algebra: you do *not* need to calculate the constant of proportionality in the equation $\rho \propto R^\chi$. Thus, don't worry about carrying along constants such as G and k in your math — just focus on the overall scaling of the relations between various quantities.