The Approach to the Main Sequence

As a pre-main sequence (PMS) star approaches the main sequence, the ignition of hydrogen fusion deep within its core slows and eventually halts its Kelvin-Helmholtz contraction. To see why, we re-write the stellar energy conservation equation (eqn. 1 of *Handout* VIII) as

$$\frac{\mathrm{d}E}{\mathrm{d}t} = L_{\mathrm{nuc}} - L, \tag{1}$$

where the new term L_{nuc} represents the total energy produced per second by nuclear reactions within the star — its *nuclear luminosity*. Early during the PMS phase L_{nuc} is negligible, and so the star necessarily loses energy with time ($\mathrm{d}E/\,\mathrm{d}t < 0$) and contracts. However, near the main sequence L_{nuc} ramps up rapidly, reducing the rate-of-loss of energy and slowing the rate of contraction. Eventually, L_{nuc} is large enough to match the surface luminosity L, such that the star is generating energy as fast as it is losing it. This point formally defines the zero-age main sequence (ZAMS), and with no further energy loss from the star, the Kelvin-Helmholtz contraction ceases. Fig. 1 demonstrates the approach to the ZAMS for a model of the Sun.

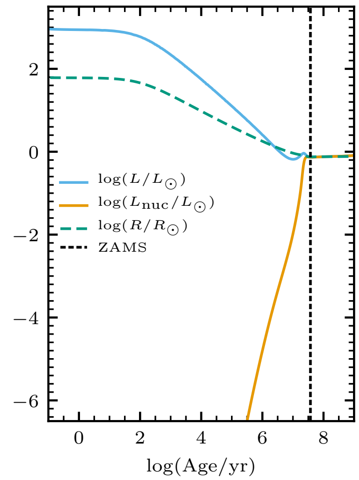

Figure 1: The surface luminosity L, nuclear luminosity L_{nuc} and stellar radius R, plotted as a function of age for a *MESA* model of the Sun. The age is measured relative to the arbitrary point on pre-main sequence when $L = 10^3\,\mathrm{L}_\odot$. The vertical dashed line shows the zero-age main sequence (ZAMS), where L_{nuc} first matches L; note how the radius stops changing from the ZAMS onward.

Evolution in the Density-Temperature Plane

To understand *why* hydrogen fusion ramps up near the main sequence, let's first note that this fusion involves *thermo*nuclear reactions — ones primarily driven by high temperature. Once the central temperature T_{c} of a star reaches a threshold $T_{\mathrm{c}} \approx 10^7\,\mathrm{K}$, depending on the density), the reaction rate rapidly climbs.

A useful way to visualize the approach to this ignition threshold is to plot the path followed by the star in the $\log \rho_{\mathrm{c}}$–$\log T_{\mathrm{c}}$ plane. Fig. 2 shows a number of these paths, for stars in the mass range $0.1\,M_\odot \leq M \leq 10\,M_\odot$[1]. The paths all take a similar form: starting at low central density and temperature, they evolve toward higher density due to KH contraction, and this evolution is accompanied by an steady increase in temperature. Eventually, T_{c} reaches the threshold for hydrogen ignition.

[1] We will discuss the $0.03\,M_\odot$ case separately, below

The paths followed by the stars are well approximated by parallel straight lines. To understand this behavior, let's estimate the central pressure of a star as[2]

$$P_{\mathrm{c}} \approx \frac{GM^2}{R^4}. \tag{2}$$

[2] This expression is based on the lower limit derived in eqn. (8) of *Handout* V, with the factor 8π dropped from the denominator.

Using the ideal-gas EOS (eqn. 4 of *Handout* VI), we express the central pressure in terms of the central density and temperature, so that

$$\frac{\rho_{\mathrm{c}} k_{\mathrm{B}} T_{\mathrm{c}}}{\mu \mathrm{m_H}} \approx \frac{GM^2}{R^4}. \tag{3}$$

Then, with the order-of-magnitude estimate $R \approx (M/\rho_c)^{1/3}$, we can eliminate the radius to find a relationship between $\log T_c$ and $\log \rho_c$:

$$\log T_c \approx \frac{1}{3} \log \rho_c + \frac{2}{3} \log M + C \qquad (4)$$

where C is a constant depending on μ and other quantities. This equation explains the behavior seen in Fig. 2: a contracting star follows a straight-line path in the $\log \rho_c$-$\log T_c$ plane, with slope $\mathrm{d}\log T_c / \mathrm{d}\log \rho_c \approx 1/3$. More-massive stars follow parallel paths displaced toward high temperatures and/or lower densities.

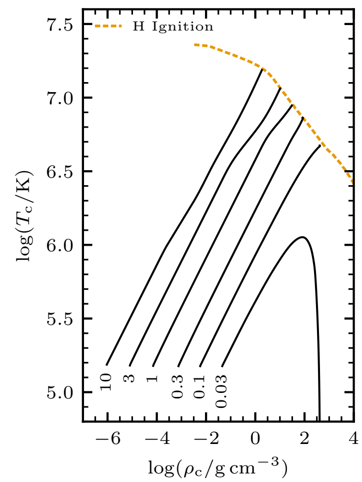

Figure 2: Paths followed by pre-main sequence stars in the $\log \rho_c$–$\log T_c$ plane. Each path is labeled at its start by the stellar mass in $M_\odot$. The dashed line shows the locus in the plane where hydrogen fusion ignites. Note that the path for $M = 0.03\,M_\odot$, which doesn't reach the ignition line, is for a brown dwarf rather than a star.

Let's now briefly consider the $M = 0.03\,M_\odot$ path in the figure, which describes the evolution of a *brown dwarf* — a sub-stellar object that never reaches a temperature sufficiently high for hydrogen ignition. As the brown dwarf contracts, the core temperature rises to a maximum $T_c \approx 10^6\,\mathrm{K}$ before decreasing again. Such behavior, which departs from the scaling given in eqn. (4), occurs because the electrons become degenerate[3] in the core of the brown dwarf. A similar fate is shared by all objects with masses $M \lesssim 0.08\,M_\odot$.

[3] Meaning that they depart from the ideal-gas EOS. The departure originates from the Pauli exclusion principle of quantum mechanics, which prohibits two fermions from being packed into the same state. We'll discuss degeneracy in further detail in a later lecture.

A First Look at Hydrogen Fusion

Without going into the specific details, we can write the reaction for the fusion of hydrogen into helium as

$$\underbrace{4\,{}^{1}\mathrm{H} + 4\,\mathrm{e}^-}_{\text{4 hydrogen atoms}} \longrightarrow \underbrace{{}^{4}\mathrm{He} + 2\,\mathrm{e}^-}_{\text{helium atom}} + \underbrace{2\,\nu_e + N\,\gamma}_{\text{energy}} \qquad (5)$$

where ν_e denotes an electron neutrino, and $N\,\gamma$ indicates some number of photons. There are typically many ways to write this reaction; however, the form given here is special in that not only does it conserve baryon number, lepton number and charge (as all valid versions of the reaction must), but each side is set up to have zero net charge.

This makes it possible to group together particles into discrete atoms, and so the energy release from the reaction, $\Delta\mathcal{E}$, can be expressed in terms of *atomic* masses (see Tab. 1):

$$\Delta\mathcal{E} = [4m_\mathrm{H} - m_\mathrm{He}]\,c^2 = 0.028\,698\,\mathrm{u}c^2 = 26.73\,\mathrm{MeV}. \qquad (6)$$

Isotope	Atomic Mass (u)
^{1}H	1.007 825
^{4}He	4.002 603
^{12}C	12.000 000
^{14}N	14.003 074
^{16}O	15.994 915

Table 1: Atomic masses (in atomic mass units, $1\,\mathrm{u} = 1.6605 \times 10^{-24}\,\mathrm{g} = 931.5\,\mathrm{MeV}$) for selected isotopes. From Table D of Audi & Wapstra (1993, *Nucl. Phys A.*, **565**, 1).

Some fraction[4] of this energy goes into the two neutrinos, which escape from the star without further interaction, while the remainder (typically denoted Q) is released locally into the star as photons.

[4] The precise fraction depends on the details of how the hydrogen fusion occurs. We'll discuss these details in a later lecture; for now, note that the process is more complicated than four protons magically coming together.

Further Reading

Kippenhahn, Weigert & Weiss, §§18.1,18.5.3; *Ostlie & Carroll*, §10.3; *Prialnik*, §§4.1,4.3.