Star Formation in Brief

Stars form from the gravitational collapse of giant molecular clouds (GMCs) — huge ($\lesssim$ 100 pc), massive ($\lesssim 10^5\,M_\odot$), cold ($\approx$ 30 K) agglomerations of dust[1] and gas. This collapse ultimately leads to the formation of one or more *protostars* — large, luminous, spherical orbs of gas surrounded by accretion disks. Eventually, the material in a protostar's accretion disk either falls onto the star, or is driven away in a jet outflow. From this point onward, the mass M of the star is fixed, and we label it a *pre-main sequence* (PMS) star.

[1] Interstellar dust isn't the same stuff as household dust; it's the generic name given to small ($\lesssim$ 0.1 μm) solid grains composed primarily of carbon, silicon and oxygen compounds.

Kelvin-Helmholtz Contraction

PMS stars differ from regular main-sequence stars through their lack of significant internal energy production by nuclear reactions. Nevertheless, because they are hot compared to their surroundings[2], they lose energy via electromagnetic radiation from their surface layers. Conservation of energy requires that

[2] Empty space has the same temperature $\approx$ 2.73 K as the cosmic microwave background (the afterglow of the Big Bang).

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -L, \tag{1}$$

where as usual E and L are as the star's total energy and surface luminosity, respectively. Using the virial theorem (eqn. 11 of *Handout* VII) and the expression for the star's gravitational potential energy (eqn. 8 of *Handout* IV), we can recast this as

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{4-3\langle\gamma\rangle}{3-3\langle\gamma\rangle} f_U \frac{GM^2}{R}\right) = L. \tag{2}$$

Let's assume that the average ratio of specific heats remains fixed at the canonical value $\langle\gamma\rangle = 5/3$, and that the shape factor f_U doesn't change with time. Since M is also fixed (see above), we can rearrange this equation to find the rate-of-change of the star's radius:

$$\frac{\mathrm{d}R}{\mathrm{d}t} = -\frac{2R^2L}{f_U GM^2}. \tag{3}$$

Because $\mathrm{d}R/\,\mathrm{d}t$ is negative, the star shrinks as it loses energy — a process known[3] as *Kelvin-Helmholtz contraction*. The characteristic timescale for the contraction is

[3] After the physicists William Thompson (Lord Kelvin) and Hermann von Helmholtz, who first considered the process in the late 19th century.

$$\tau_{\mathrm{KH}} \equiv \frac{GM^2}{RL}, \tag{4}$$

which is known as the *Kelvin-Helmholtz timescale* of the star. During the Sun's PMS phase, when its radius was $10\,\mathrm{R}_\odot$, its luminosity was $\approx 63\,\mathrm{L}_\odot$, leading to $\tau_{\mathrm{KH}} \approx 50\,000$ yr. This is small compared to the Sun's main-sequence lifetime $\approx 10^{10}$ yr, indicating that the PMS phase is typically brief.

Pre-Main Sequence Tracks

Although eqn. (3) tells us that PMS stars must contract, it doesn't indicate how they move in the Hertzsprung-Russell diagram. Fig. 1 reveals what happens, plotting evolutionary tracks for PMS stars with a range of masses. The tracks start in the top-right (low-temperature, high-luminosity, large-radius) of the HR diagram, and evolve toward the *zero-age main sequence* (ZAMS) — the line marking where the main-sequence phase begins. For most of the stars, the evolution occurs in two distinct stages:

- the *Hayashi track*, where energy is transported within the star by convection, and the star evoles vertically downward in the HR diagram (approximately constant $T_{\rm eff}$, decreasing L and R);
- the *the Henyey track*, where energy is transported within the star primarily by radiation, and the star evolves horizontally leftward in the HR diagram (approximately constant L, increasing $T_{\rm eff}$, decreasing R).

The switch from the Hayashi track to the Henyey track, corresponding to the transition from convective energy transport to radiative energy transport, occurs toward hotter effective temperatures and higher luminosities as the stellar mass increases. For stars with masses $M \lesssim 0.4\,{\rm M}_{\odot}$, this switch never takes place and they remain on the Hayashi track all the way to the ZAMS.

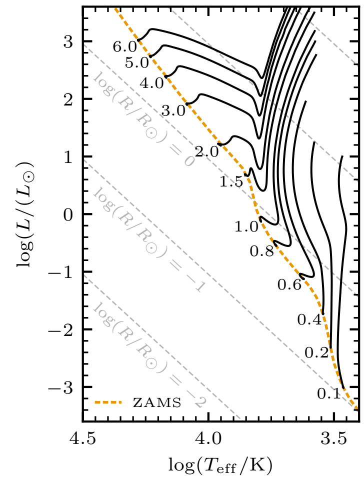

Figure 1: Evolutionary tracks in the Hertzsprung-Russell diagram for stars in the pre-main sequence phase, calculated using *MESA*. Each track is labeled at the zero-age main sequence (ZAMS) with its initial mass in $M_{\odot}$. The dashed lines mark contours of constant stellar radius.

A Simple Hayashi Track Model

With $L = 4\pi R^2 \sigma T_{\rm eff}^4$ (cf. eqn. 8 of *Handout* II), we can write eqn. (3) as

$$\frac{{\rm d}R}{{\rm d}t} = -\frac{8\pi R^4 \sigma T_{\rm eff}^4}{f_U G M^2}. \tag{5}$$

To develop a simple model for radius evolution on the Hayashi track, let's assume that $T_{\rm eff}$ does not vary at all. Then, we solve this equation to find

$$\frac{1}{R^3} = \frac{1}{R_0^3} + \frac{24\pi\sigma T_{\rm eff}^4}{f_U G M^2}\, t, \tag{6}$$

where t is the time elapsed since the radius equaled the (arbitrary) value R_0. This equation tells us that R^{-3} should increase linearly with time for a PMS star on the Hayashi track; Fig. 2 confirms this approximate behavior for the Sun.

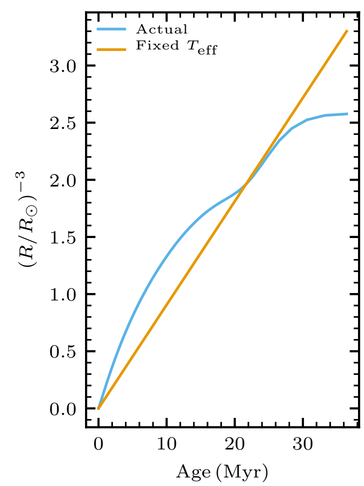

Figure 2: The quantity $(R/R_{\odot})^{-3}$, plotted as a function of age for a *MESA* model of the Sun during the pre-main sequence phase. The straight line shows the behavior predicted by eqn. (6), assuming $f_U = 1.5$ and a fixed effective temperature $T_{\rm eff} = 5300\,{\rm K}$ on the Hayashi track.

Further Reading

Ostlie & Carroll, §12.3; *Prialnik*, §9.1.