Circularization and Synchronization

Observations of binary systems in clusters[1]reveal a lack of eccentric systems ($e > 0$) at short orbital periods (see Fig. 1). This is a consequence of *stellar tides*, which gradually transfer energy and angular momentum from the orbits to the individual stars. These tides, which are strongest for close (and hence short-period) binaries, ultimately leads both to the *circularization* of the orbits ($e \to 0$), and to the *synchronization* of the stars' rotation (such that the rotation period of each equals the orbital period P).

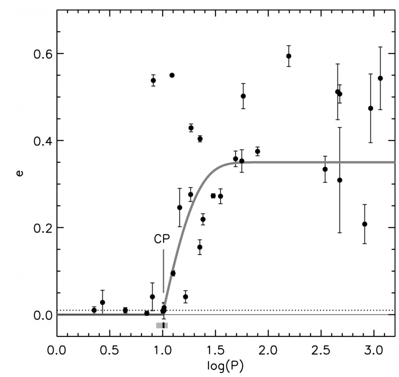

Figure 1: Scatter plot of eccentricity e versus orbital period P (in days) for binary systems in the open cluster M35. Note how systems with $P \lesssim 10\,\mathrm{d}$ are close to circular. From Meibom & Mathieu (2005, *ApJ*, **620**, 970).

[1] Remember that stars in clusters all have the same age.

The Roche Model

To determine the shapes of stars in circularized and synchronized binary system, we can use an approach developed originally by the astronomer Édouard Roche. In this so-called *Roche model*, we approximate the stars' gravitational potentials as arising from a pair of point masses. In the frame of reference co-rotating with the stars, the acceleration of an at-rest test particle can then be written as $\mathbf{a} = -\nabla\Phi_{\rm eff}$, where

$$\Phi_{\rm eff}(\mathbf{r}) = -\frac{GM_1}{|\mathbf{r} - \mathbf{r}_1|} - \frac{GM_2}{|\mathbf{r} - \mathbf{r}_2|} - \frac{1}{2}|\mathbf{r} \times \boldsymbol{\Omega}|^2 \qquad (1)$$

is the *effective potential* at position coordinate $\mathbf{r}$. Here $\mathbf{r}_1$ and $\mathbf{r}_2$ are the position coordinates of the primary and secondary stars, respectively, while $\boldsymbol{\Omega}$ is the angular velocity vector of the system[2]. On the right-hand side of this expression, the first two terms represent the gravitational potentials of the two stars, while the third term is a centrifugal potential.

The surfaces of constant effective potential define the level (horizontal) surfaces of the binary system, and surface of each star lies on one of these equipotentials. As shown in Fig. 2, the equipotentials are spherical close to the center of each star, but become distorted further away. Each star has a special equipotential known as a *Roche* lobe, which touches the L_1 Lagrange point[3] of the system.

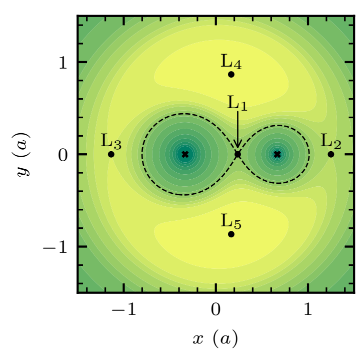

Figure 2: Contour plot of the effective potential $\Phi_{\rm eff}$ in the orbital plane, for a circularized and synchronized binary system with a mass ratio $M_2/M_1 = 0.5$. The black crosses mark the centers of each star (the more-massive star is on the left), and the black dots indicate the Lagrange points. The Roche lobe of each star, shown by the dashed black contour, intersects the L_1 point.

[2] This vector points along the rotation axis, and has magnitude $|\boldsymbol{\Omega}| = 2\pi/P$.

[3] This is the point on the line between the two stars (but not necessarily at the center of mass) where effective potential shows a saddle point, and the acceleration $\mathbf{a}$ vanishes. The L_1 point is marked in Fig. 2, together with the four other Lagrange points where $\mathbf{a}$ vanishes.

Roche Lobe Overflow

If a star expands to fill its Roche lobe, it will begin to spill mass onto its companion through the L_1 point. This *Roche lobe overflow* can have a significant impact on the evolution of the stars and their orbits. When the overflow occurs during the main-sequence phase of the overflowing star, we refer to the mass transfer as *case A*; during the RGB phase it is *case B*; and during the AGB it is *case C*.

Conservative Mass Transfer

Let's consider the special case of conservative mass transfer, meaning that all of the mass and angular momentum lost by the donor star is gained by the recipient star. With the further assumption that the orbits remain circular, the rate-of-change of semi-major axis is found[4] as

$$\frac{\dot{a}}{a} = -2\frac{\dot{M}_2}{M_2}\left(1 - \frac{M_2}{M_1}\right). \tag{2}$$

The corresponding rate-of-change of the period is, from Kepler's third law,

$$2\frac{\dot{P}}{P} = 3\frac{\dot{a}}{a}. \tag{3}$$

Let's assume[5] that the primary star is the donor ($\dot{M}_1 < 0$) and the secondary star is the recipient ($\dot{M}_2 = -\dot{M}_1 > 0$). Then, the above expressions show that the orbits will shrink and the period get shorter, if $M_2 < M_1$ — and vice versa. Put differently, mass transfer that tends to make the two stars closer in mass will also make them physically closer.

As the separation of the stars changes, the size of their Roche lobes will also change. The average radius[6] of the primary (donor) Roche lobe can be well approximated by the formula

$$R_{\mathrm{L},1} \approx 0.49a\left(\frac{M_1}{M_1 + M_2}\right)^{1/3}. \tag{4}$$

Taking the time derivative, we find after some algebra that

$$\frac{\dot{R}_{\mathrm{L},1}}{R_{\mathrm{L},1}} = -2\frac{\dot{M}_2}{M_2}\left(1 - \frac{5}{6}\frac{M_2}{M_1}\right). \tag{5}$$

With our assumption that $\dot{M}_2 > 0$, we can see that the primary Roche lobe will shrink with time ($\dot{R}_{\mathrm{L},1} < 0$) when $M_2/M_1 < 6/5$. The shrinkage leads to a positive feedback loop: as $R_{\mathrm{L},1}$ gets smaller, the mass loss from the donor star grows[7], driving up $\dot{M}_2$ and accelerating $\dot{R}_{\mathrm{L},1}$.

Unless halted by M_2/M_1 rising above 6/5, this *unstable mass transfer* ultimately deposits so much material on the recipient star that it, too, overflows its Roche lobe. The binary system then consists of a pair of stellar cores orbiting inside a *common envelope*. Friction between the stars and the envelope causes the stars to spiral in toward each other. The release of energy by this inspiral eventually drives off the envelope, leaving behind a very close binary system comprising the stripped donor and its companion.

[4] To derive this result, take the time derivative of eqn. (10) of *Handout* XXXI, with $e = 0$, $\dot{M}_1 = -\dot{M}_2$ and $\dot{L} = 0$.

[5] Our analysis doesn't rely on this assumption, since the labels 'primary' and 'secondary' are arbitrary; but it's easier to interpret results if we fix the signs of $\dot{M}_1$ and $\dot{M}_2$.

[6] Defined in an equal-volume sense.

[7] Compare what happens to a balloon filled with water, as the balloon starts to shrink.

Further Reading

Ostlie & Carroll, §18.1.