The Inclination Problem

When we observe a binary system, we face the difficulty that we typically don't see the orbits face-on or edge-on, but rather from an arbitrary and (initially) unknown orientation. Conventionally, we characterize this orientation via the *inclination* i, defined as the angle between the orbital plane and the plane of the sky (onto which the orbit is projected; see Fig. 1); this is equivalent to the angle between the line-of-sight and the normal to the orbital plane.

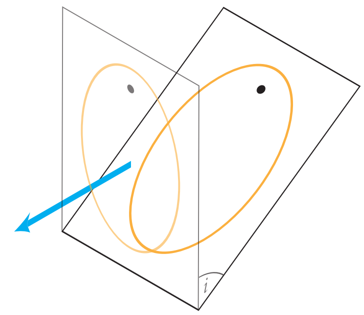

Figure 1: An elliptical orbit, with the center of mass at one focus marked by a black dot. The projection of the orbit on the sky plane is shown in faint, and the blue arrow indicates the line-of-sight. The angle between the orbit and sky planes defines the inclination i.

Observing Visual Binaries

For visual binaries, the effect of an inclination $i > 0°$ is to change the apparent shape of the orbits. Certain choices of i can make eccentric orbits appear circular, and vice versa. Fortunately, we can spot cases like this by finding the focus of the apparent orbit of each star (see Fig. 2). We know that these foci should both coincide with the center-of-mass of the system (see *Handout* XXXI). If they do, then we can be confident that we're seeing the system face-on ($i = 0°$).

We can determine the mass ratio of the stars in a visual binary by measuring the angular size α of the semi-major axis[1] for each star. Then, the ratio of semi-major axes is

$$\frac{a_1}{a_2} = \frac{\alpha_1}{\alpha_2}, \tag{1}$$

and from note 8 of *Handout* XXXI, it follows that the mass ratio is

$$\frac{M_2}{M_1} = \frac{a_1}{a_2} = \frac{\alpha_1}{\alpha_2}. \tag{2}$$

If the system happens to be face-on, and if we know its distance d, then we can calculate a_1 and a_2 from α_1 and α_2 using the small-angle formula

$$a_1/1\,\mathrm{au} = (\alpha_1/1'')\,(d/1\,\mathrm{pc}), \qquad a_2/1\,\mathrm{au} = (\alpha_2/1'')\,(d/1\,\mathrm{pc}). \tag{3}$$

The semi-major axis of the one-body problem is then $a = a_1 + a_2$. With a and the measured period P, we apply the generalized form of Kepler's third law (eqn. 13 of *Handout* XXXI) to find the combined mass $M_1 + M_2$ of the system. Together with the mass ratio M_2/M_1, we then have sufficient information to solve for the individual masses of the stars.

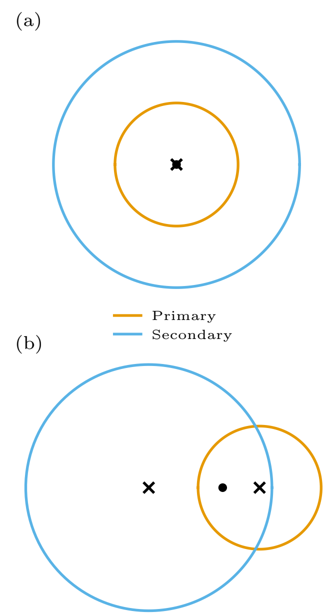

Figure 2: The apparent orbits of two binary systems, both having mass ratio $M_2/M_1 = 0.5$. (a) shows a circular ($e = 0$) system seen face-on; (b) shows an eccentric ($e = 0.6$) system seen at an inclination $i = 37°$. In both cases, the apparent orbit of each star is circular; however, only in case (a) do the apparent foci of the orbits (shown by the black crosses) coincide with the center-of-mass of the system (shown by the black dot).

[1] For circular systems, this corresponds to the angular radius of the orbit.

Observing Spectroscopic Binaries

For spectroscopic binaries, we can use the Doppler shifts of spectral lines to measure the *radial velocity* of each component — that is, the instantaneous orbital velocity projected along the line of sight. A *radial velocity curve* is a plot of these measurements as a function of time or orbital phase. The shape of the radial velocity curve provides information about the eccentricity: a sinusoidal curve indicates a circular orbit ($e = 0$; see Fig. 3), while non-sinusoidal curves are the hallmarks of eccentric orbits ($e > 0$).

We can determine the mass ratio of the stars in a spectroscopic binary from the semi-amplitude[2] v_r of the radial velocity curves:

$$\frac{M_2}{M_1} = \frac{v_{r,1}}{v_{r,2}}. \tag{4}$$

If the system is circular, then the velocity semi-amplitudes are related to the semi-major axes, inclination and period via

$$2\pi a_1 = \frac{v_{r,1}}{\sin i} P, \qquad 2\pi a_2 = \frac{v_{r,2}}{\sin i} P. \tag{5}$$

Combining this with Kepler's third law, we arrive at an expression for the combined mass of the system:

$$M_1 + M_2 = \frac{(v_{r,1} + v_{r,2})^3 P}{2\pi G \sin^3 i}. \tag{6}$$

If we don't know the inclination, this expression gives us only a lower bound on the combined mass (since $\sin i \leq 1$). However, if the system is also an eclipsing binary, the orbits must be close to edge-on ($i \approx 90°$); then, we can find the combined mass and solve for the individual masses of the stars.

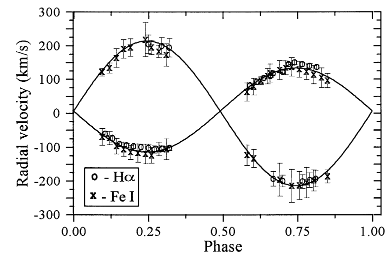

Figure 3: Radial velocity curves for the two components of SV Cam, a circular spectroscopic and eclipsing binary with $M_1 = 1.47\,M_\odot$ and $M_2 = 0.87\,M_\odot$. From Kjurkchieva et al. (2002, *A&A*, **386**, 548.

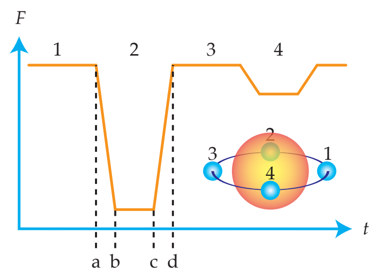

Figure 4: Schematic light curve for an eclipsing binary comprising a hot, smaller primary and a cool, larger secondary. The inset figure shows the relative positions of the two stars at four orbital phases.

[2] I.e, half the peak-to-peak amplitude.

Observing Eclipsing Binaries

Eclipsing binaries are especially useful systems; not only do we know their inclination, we can also place constraints on the size of each star[3]. Fig. 4 demonstrates the *light curve* of a typical eclipsing binary, plotting the flux F as a function of time t. If we measure the time difference between point a (when the primary begins to be eclipsed) and point b (when the primary completely disappears), and likewise between point a and point c (when the primary begins to reappear), then we can determine the radius of the two stars from

$$R_1 = v_t \frac{t_b - t_a}{2}, \qquad R_2 = v_t \frac{t_c - t_a}{2}. \tag{7}$$

Here, v_t is the *transverse velocity* of the stars relative to one another (for a circular eclipsing binary, $v_t = v_{r,1} + v_{r,2}$).

[3] In fact, we can also determine their effective temperatures; but for brevity we won't go into that here.

Further Reading

Ostlie & Carroll, §§7.2,7.3