Binary Stars and Optical Doubles

When we observe a pair of stars that are very close together on the sky — a so-called *double star* — it's natural to ask whether there's any kind of relationship between the stars. In some cases, the answer is 'no'; the stars' proximity is just a chance alignment, and in fact they're nowhere near one another in space. These are known as *optical doubles*.

In other cases, however, the answer is 'yes'; the stars are bound together by their own gravitational attraction, and orbit around their common center of mass. Such cases, known as *binary systems*, are particularly important tools for studying the structure and evolution of stars, as they offer unique opportunities to directly measure quantities like stellar mass. Moreover, if the stars in a binary system are sufficiently close, they can exchange mass during their lives, often with profound impacts on their final fate.

Types of Binary Systems

We divide binary systems into a number of different types, based on their observational characteristics. If we're able to resolve the two stars in a binary system (either with the naked eye, or with the use of a telescope) we classify the system as a *visual binary*. Otherwise, if we can only see one star in the system, or if both stars are so close together that they appear as a single point of light on the sky, we classify it as a *non-visual binary*.

Clearly, there has to be some property of a non-visual binary that tells us it comprises two stars. Again, there are a number of subtypes based on observational characteristics:

- *Spectrum binaries* show a spectrum that combines absorption lines from stars at different temperatures — for instance, lines due to singly ionized helium (only seen in stars with $T_{\rm eff} \gtrsim 25\,000\,{\rm K}$) together with lines due to titanium dioxide (only seen in stars with $T_{\rm eff} \lesssim 4000\,{\rm K}$).
- *Spectroscopic binaries* show a spectrum with absorption lines that undergo periodic changes in wavelength. These changes are due to the Doppler shifts arising as the stars move toward/away from the Earth. If the lines from only one star are seen[1], the binary is classified as a *single-lined* spectroscopic binary; if both sets are seen, it is a *double-lined* spectroscopy binary.
- *Astrometric binaries* show periodic changes in the position of one star on the sky, as it orbits around the system center of mass; the other star is too dim to detect.

[1] This can happen, for example, if the other star is so dim that it contributes negligibly to the binary spectrum.

- *Eclipsing binaries* show periodic changes in the overall brightness of the system, occurring as one star passes in front of the other and blocks out its light. For such eclipses to occur, the binary must be viewed from within the plane of the stars' orbits.

Often, a binary system may fall into multiple classifications; for instance, the famous eclipsing binary Algol[2](β Per) is also a spectroscopic binary. Also, binaries often occur in hierarchical systems; for instance, the 'star' Mizar[3]is a visual binary, with each component itself being a spectroscopic binary (for a total of four stars).

[2] From the Arabic *al ghūl*, literally 'the ghoul'.

[3] Mizar is one of the stars in the handle of the Big Dipper; it forms an optical double with Alcor (some argue that two systems are not just a chance alignment, but are in fact gravitationally bound).

Kepler's Laws

Stars in a binary system orbit around their common center of mass for the same reason that planets orbit around the Sun: mutual gravitational attraction. The motion of the planets was first (mostly) correctly described by Johannes Kepler, as an improvement on the earlier heliocentric model proposed by Nicolaus Copernicus. Kepler described this motion in the form of three laws:

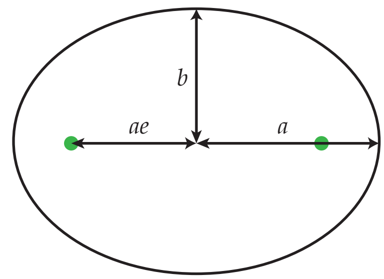

Figure 1: Schematic of an ellipse. The *semi-major axis a* and *semi-minor axis b* of the ellipse are the maximal and minimal distances of the boundary of the ellipse from its center. In terms of the eccentricity e of the ellipse, $b = a\sqrt{1-e^2}$. For any point on the boundary, the sum of the distances from that point to the two loci (shown by the green circles) is always $2a$.

1. The orbit of a planet is an ellipse with the Sun at one of the two foci (see Fig. 1 for a reminder of the properties of ellipses).
2. A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time (see Fig. 2 for a depiction of this equal-area law).
3. The square of the orbital period P of a planet is directly proportional to the cube of the semi-major axis a of its orbit. When the period is expressed in years, and the semi-major axis in astronomical units, the constant of proportionality is unity:

$$\left(\frac{P}{1\,\mathrm{yr}}\right)^2 = \left(\frac{a}{1\,\mathrm{au}}\right)^3 \tag{1}$$

Kepler's first law tells us about the shape of a planet's orbit (an ellipse); the second law tells us about the relative speed of the planet at different points on its orbit (fastest when closest to the Sun, slowest when furthest away); and the third law tells us about how the average speed changes from one planet to the next (dependent on the size of the orbit, but not on the mass of the planet). As we shall see, all three laws can be derived from Newton's laws of motion and gravitation.

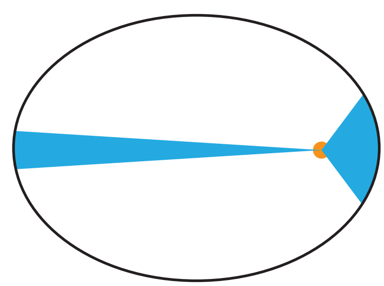

Figure 2: Kepler's second law illustrated graphically. The Sun is shown as the yellow circle at one focus of the elliptical orbit followed by a planet (there is nothing at the other focus). The areas of the shaded regions are the same, indicating that the planet takes equal times to sweep out these areas; therefore, it must be moving fastest when closest to the Sun (*perihelion*) and slowest when furthest from the Sun (*aphelion*).

Further Reading

Ostlie & Carroll, §§2.1,7.1; *Prialnik*, §11.1.