Planetary Nebulae

With the onset of a superwind (see *Handout* XXVI), an AGB star rapidly ejects most of its hydrogen-rich envelope. This exposes the star's deeper, hotter layers, causing it to evolve rapidly toward the blue in the Hertzsprung-Russell diagram (see Fig. 1). Once the star's effective temperature increases above about 3×10^4 K, it emits sufficient radiation in the UV to ionize the ejecta. The ionized material appears as a glowing shell — or sometimes, a more-complex structure — around the star (see Fig. 2), and is known as a *planetary nebula*[1].

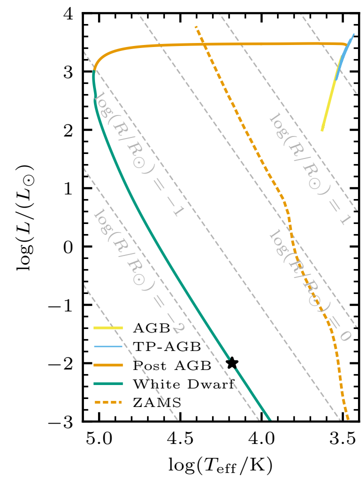

Figure 1: Evolutionary track in the Hertzsprung-Russell diagram for a *MESA* model of the Sun, spanning the red clump phase to the white dwarf phase. The asterisk marks the case plotted in Fig.3.

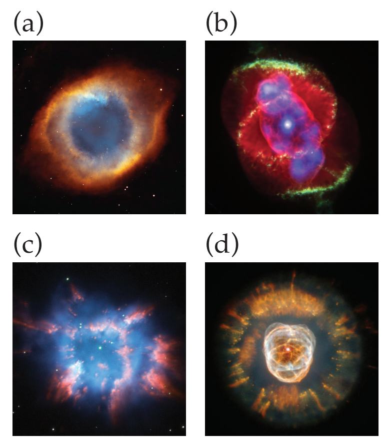

Figure 2: Images of four different planetary nebulae: (a) the Helix Nebula, (b) the Cat's Eye Nebula, (c) the Eskimo Nebula, and (d) NGC 6326. Image credits: *NASA*.

White Dwarfs

When the effective temperature of a post-AGB star reaches around 10^5 K, it turns a corner in the HR diagram: its luminosity drops rapidly, and then it begins to evolve diagonally down and to the red. The star has become a *white dwarf*, the final evolutionary state for all stars with initial masses $\lesssim 9\,M_\odot$.

White dwarfs (WDs) are compact objects, with radii $\sim 10^{-2}\,R_\odot$ comparable to the Earth. Their central densities exceed $10^6\,{\rm g\,cm^{-3}}$, while central temperatures are $\lesssim 10^7$ K; therefore, the free electrons at the center of white dwarfs are extremely degenerate[2] . In fact, degeneracy holds throughout almost all of WD interiors; in only a very thin surface layer, comprising around a billionth of the star by mass, does the material behave like an ideal gas.

Fig. 3 plots the composition profile of a $0.54\,M_\odot$ WD, the end state of the *MESA* model of the Sun we've been examining in previous handouts. The inner $\sim$ 95% by mass is composed of carbon and oxygen; most of the remaining $\sim$ 5% is composed of helium; and a tiny fraction $\sim$ 0.01% is composed of hydrogen, the remnant of the hydrogen-rich layers that were ejected to form a planetary nebula.

White Dwarf Cooling

WDs cool down via the loss of thermal energy from the ions in their interior[3]. To explore this process, let's create simple scaling relations for the luminosity of a WD based on the following assumptions:

(i) the star can be treated as a completely degenerate, isothermal core with radius R and mass M, surrounded by a thin ideal-gas atmosphere with radius Δr and negligible mass;

(ii) the boundary between core and atmosphere occurs where the ideal and degenerate pressures match;

[1] A name coined in the 18th century by the astronomy William Herschel. In reality, planetary nebulae have nothing to do with planets.

[2] To see this, locate the $(\log \rho_c, \log T_c) = (6, 7)$ point in Fig. 1 of *Handout* XXIII.

[3] Even though the electrons are completely degenerate, the ions still behave as an ideal gas.

(iii) the temperature, density and pressure at the top of the atmosphere are negligible compared to their values at the bottom of the atmosphere;

(iv) the atmosphere is radiative, with a Kramers opacity law.

Then, hydrostatic equilibrium, radiative diffusion and the constitutive relations (see *Handout* XVIII) give then scalings

$$\frac{P_{\rm b}}{\Delta r} \sim \frac{M}{R^2}\rho_{\rm b}, \quad \ell_{\rm t} \sim \frac{R^2 T_{\rm b}^4}{\kappa \rho_{\rm b}}, \quad \kappa \sim \rho_{\rm b} T_{\rm b}^{7/2}, \qquad (1)$$
$$P_{\rm b} \sim \rho_{\rm b}^{5/3}, \quad P_{\rm b} \sim \rho_{\rm b} T_{\rm b},$$

where the subscripts 'b' and 't' refer to the bottom and top of the atmosphere, respectively. Solving for $\ell_{\rm t}$, we obtain the WD luminosity scaling as

$$L = \ell_{\rm t} \sim \frac{T_{\rm b}^{7/2}}{M}. \qquad (2)$$

The thermal energy of the ions in the interior scales[4]as $H \sim M T_{\rm b}$; with $L = -\,{\rm d}H/\,{\rm d}t$, we therefore arrive at a differential equation for the luminosity:

$$\frac{{\rm d}L}{{\rm d}t} \sim -M^{5/7} L^{12/7}. \qquad (3)$$

Solutions to this equation follow the scaling $L \sim t^{-7/5}$, a result confirmed empirically in Fig.4. Thus, the lumiunosity of a WD drops rapidly at first, but then decreasese more and and more gradually.

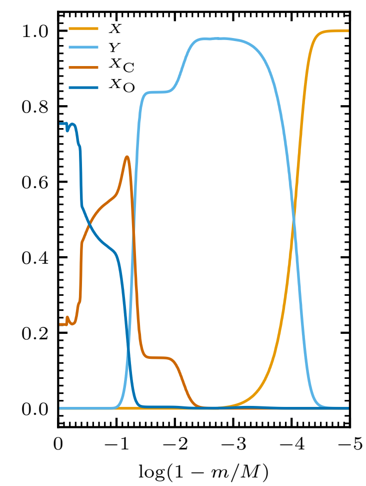

Figure 3: Composition profile of a *MESA* WD model ($M = 0.54\,M_\odot$; $L = 0.010\,L_\odot$; $R = 0.014\,R_\odot$), plotting mass fractions for hydrogen, helium, carbon and oxygen as a function of the logarithm of the fractional exterior mass $1 - m/M$.

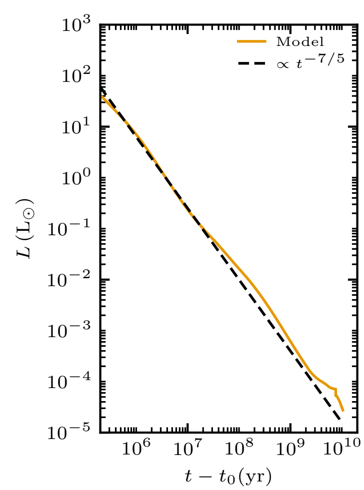

Figure 4: The luminosity L of the *MESA* WD model of the Sun (Fig. 1), plotted as a function of time t since the beginning of the WD phase. The dashed line shows the scaling $L \sim t^{-5/7}$ derived in the text. In the lower-right corner of the plot, the effect of *crystallization* can be seen: the release of latent heat during the crystallization process causes a pauses in the star's cooling.

Other White Dwarf Varieties

The WD model plotted in Fig. 3 is classified as a DA-type star — the 'D' stands for dwarf, and the 'A' indicates that it shows strong hydrogen absorption lines in its spectrum (see *Handout* III). Around 80% of WDs are DA type. Most of the remaining 20% are DB-type stars; these lack the thin outer layer of hydrogen shown in Fig. 3, and so show helium absorption lines instead of hydrogen in their spectra.

In addition to these atmospheric differences, it's also possible for WDs to have different interior compositions than the carbon-oxygen case considered here. Extremely low mass (ELM) WDs, with masses $M \sim 0.1 - 0.2\,M_\odot$ have helium interiors, and are thought to arise from binary evolution. Likewise, massive WDs ($M \gtrsim 1\,M_\odot$) have oxygen-neon interiors, and arise from the evolution of stars with initial masses $M \sim 6 - 9\,M_\odot$.

Further Reading

Kippenhahn, Weigert & Weiss, §§16.4,34.9,37.3; *Ostlie & Carroll*, §16.2,16.5; *Prialnik*, §§9.7,9.8.

[4] This is because the interior is isothermal; see also eqn. 10 of *Handout* VI.