Thermal Pulses

As stars approach the top of the asymptotic giant branch, the helium-burning shell[1]exhibits a series of unstable events that are reminiscent of the core helium flash at the tip of the RGB. We'll delve into the cause of these *shell flashes* below, but first let's examine their consequences. During a shell flash, the luminosity produced by triple-alpha reaction spikes up by a factor $\sim 10^3$. This excess energy cannot be radiated by the star over short timescales, and is instead absorbed by the helium layer between the burning shells, causing it to expand. This expansion cools the hydrogen-burning shell atop the layer, to such an extent that its nuclear reactions almost cease. The overall effect is to significantly reduce the star's surface luminosity, in a manner reminiscent of the luminosity drop that occurs after the helium flash.

A shell flash cannot be sustained for long; having consumed most of the helium layer, the helium-burning shell runs out of fuel. After some time, the hydrogen-burning shell recovers and begins to rebuild the helium layer. Eventually, the layer becomes thick enough for another shell flash to occur, and the whole cycle — known as a *thermal pulse* — repeats. Fig. 1 illustrates a single thermal pulse for a *MESA* model of the Sun.

The phase on the AGB when thermal pulses arise is known as the *thermally pulsing AGB* (*TP-AGB*). During this phase, a star moves up and down the asymptotic giant branch in the Hertzsprung-Russell diagram (see Fig. 2), with maximal surface luminosity corresponding to minimal helium-burning luminosity, and vice versa (this correlation can clearly be see in Fig. 1).

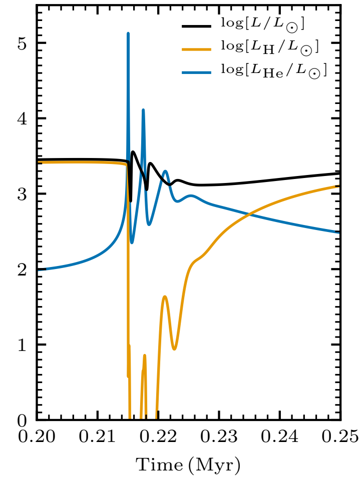

Figure 1: The surface luminosity L, hydrogen-burning luminosity $L_{\rm H}$ and helium-burning luminosity $L_{\rm He}$, plotted as a function of time for a *MESA*model of the Sun during the TP-AGB phase. The time zero-point is at an (arbitrary) age 1.358×10^{10} yr after the ZAMS. The plot reveals the interplay between L, $L_{\rm H}$ and $L_{\rm He}$ during a single thermal pulse cycle.

[1] Recall that AGB stars are powered by two shells: an outer hydrogen-burning shell and an inner helium-burning shell, with a helium layer sandwiched between then. See *Handout* xxv for more details.

The Thin-Shell Instability

While the core helium flash is a consequence of electron degeneracy, shell helium flashes arise even in non-degenerate conditions. To understand this, let's consider a thin shell which spans radial coordinates $[r_{\rm b}, r_{\rm t}]$ and mass coordinates $[m_{\rm b}, m_{\rm t}]$[2]. The density in the shell can be approximated as its mass $\Delta m \equiv m_{\rm t} - m_{\rm b}$ divided by its volume,

$$\rho \approx \frac{\Delta m}{4\pi r_{\rm b}^2 \Delta r}, \tag{1}$$

where $\Delta r \equiv r_{\rm t} - r_{\rm b}$ is the shell's radial thickness. Likewise, the pressure in the shell can be approximated by integrating the equation of hydrostatic equilibrium from the surface downward to the mass

[2] As in *Handout* xxii, the subscripts 'b' and 't' refer to the bottom and top of the shell, respectively.

midpoint $m_\mathrm{m} \equiv (m_\mathrm{b} + m_\mathrm{t})/2$ of the shell:

$$P \approx \int_{m_\mathrm{m}}^{M} \frac{Gm}{4\pi r^4}\,\mathrm{d}m. \tag{2}$$

Now let's consider what happens to the shell pressure and density as r_t varies while keeping r_b fixed[3]. The change in density is

$$\delta\rho = \frac{\partial\rho}{\partial r_\mathrm{t}}\delta r_\mathrm{t} = -\rho\,\frac{\delta r_\mathrm{t}}{\Delta r}, \tag{3}$$

where δr_t is the change in r_t. To evaluate the change in pressure, we assume that the r term in the denominator of the integrand in eqn. (2) scales proportionately to r_t, leading to

$$\delta P = \frac{\partial P}{\partial r_\mathrm{t}}\delta r_\mathrm{t} = -4P\frac{\delta r_\mathrm{t}}{r_\mathrm{t}}. \tag{4}$$

Assuming the material in the shell behaves as an ideal gas, then the change δT in its temperature can be evaluated[4] from $\delta\rho$ and δP as

$$\frac{\delta T}{T} = \frac{\delta P}{P} - \frac{\delta\rho}{\rho} = \left(1 - 4\frac{\Delta r}{r_\mathrm{t}}\right)\frac{\delta r_\mathrm{t}}{\Delta r}. \tag{5}$$

For thin shells ($\Delta r \ll r_\mathrm{t}$) the term in parentheses is positive, indicating that an expansion of the shell is accompanied by a rise in its temperature. This *thin-shell instability* drives the helium shell flashes during the TP-AGB: an expansion of the shell raises its temperature, increasing the energy release rate of the triple alpha reaction, in turn causing the shell to expand further, and so on.

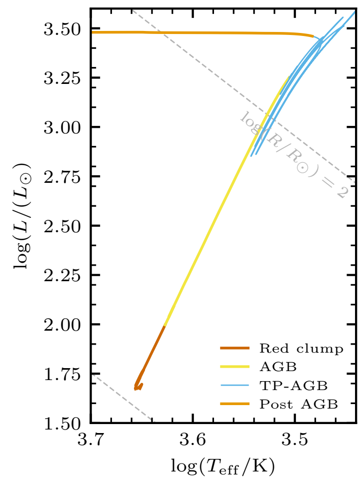

Figure 2: Evolutionary track in the Hertzsprung-Russell diagram for a *MESA* model of the Sun, spanning the red clump phase to beyond the TP-AGB phase.

[3] Since the shell always contains the same material, m_b and m_t also remain fixed.

[4] By applying eqn. (3) of *Handout* XIII.

Mass Loss on the Giant Branches

The TP-AGB phase is ultimately brought to an end by mass loss. The large luminosities of both RGB and AGB stars drive significant *stellar winds*, with mass-loss rates given approximately by[5]

$$\begin{aligned}
\dot{M}_\mathrm{RGB} &\approx -4\times10^{-13}\left(\frac{L}{L_\odot}\frac{R}{R_\odot}\frac{M_\odot}{M}\right)\mathrm{M_\odot/yr},\\
\dot{M}_\mathrm{AGB} &\approx 5\times10^{-9}\left(\frac{M}{M_\odot}\right)^{-2.1}\left(\frac{L}{L_\odot}\right)^{2.7}\dot{M}_\mathrm{RGB},
\end{aligned} \tag{6}$$

respectively. At the highest luminosities during the TP-AGB phase, $\dot{M}_\mathrm{AGB}$ can reach values $10^{-5}\,\mathrm{M_\odot/yr}$. This so-called *superwind* rapidly strips most of the remaining hydrogen envelope from the star, terminating the TP-AGB phase and launching the star toward the blue in the HR diagram (see Fig. 2).

[5] The first expression is from an empirical fit to observations of RGB stars by Reimers (1975, *Problems in Stellar Atmospheres and Envelopes*); the second from numerical simulations of AGB stars by Blöcker (1995, *A&A*, **297**, 727).

Further Reading

Kippenhahn, Weigert & Weiss, §§34.3,34.6; *Ostlie & Carroll*, §13.2; *Prialnik*, §§9.6,9.7.