The Triple Alpha Reaction

Helium in stars is fused to carbon via the triple alpha reaction. We've already encountered this reaction in schematic form (see eqn. 3 of *Handout* xxiii), but let's now examine it in detail. Because three-particle nuclear interactions are vanishingly improbable, the reaction proceeds via a pair of two-particle steps. In the first step, two alpha particles fuse to form a beryllium-8 nucleus:

$$\underbrace{2(^{4}\mathrm{He} + 2e^{-})}_{\text{2 helium-4 atoms}} \xrightarrow[+92\,\mathrm{keV}]{} \underbrace{^{8}\mathrm{Be} + 4e^{-}}_{\text{beryllium-8 atom}} \qquad (1)$$

Importantly, this is an *endothermic* reaction: it requires 92 keV of energy to occur, which is supplied through the kinetic energy of the alpha particles. The resulting beryllium-8 nucleus is highly unstable, and almost always fissions back into the alpha particles after $\sim 10^{-16}$ s. However, if a third alpha particle hits the beryllium-8 before this occurs, then something else can happen:

$$\underbrace{^{8}\mathrm{Be} + 4e^{-}}_{\text{beryllium-8 atom}} + \underbrace{^{4}\mathrm{He} + 2e^{-}}_{\text{helium-4 atom}} \xrightarrow[+287\,\mathrm{keV}]{} \underbrace{^{12}\mathrm{C}^{*} + 6e^{-}}_{\text{Hoyle-state carbon-12 atom}} \qquad (2)$$

Again, this is an endothermic reaction, requiring 287 keV of kinetic energy to occur. The asterisk on the carbon-12 nucleus on the right-hand side indicates that this is an excited state of the nucleus — the so-called *Hoyle state*[1], at an energy 7.654 MeV above the ground state. Most of the time, the Hoyle state fissions back into a beryllium-8 nucleus and an alpha particle. However, it can also undergo a two-photon radiative decay to the carbon-12 ground state:

$$\underbrace{^{12}\mathrm{C}^{*} + 6e^{-}}_{\text{Hoyle-state carbon-12 atom}} \longrightarrow \underbrace{^{12}\mathrm{C} + 6e^{-}}_{\text{carbon-12 atom}} + \underbrace{2\gamma}_{\text{energy}}\,. \qquad (3)$$

The net energy released by the triple alpha reaction is $\Delta\mathcal{E} = (4m_{\mathrm{He}} - m_{\mathrm{C}}) = 7.275\,\mathrm{MeV}$; but we could also have calculated this value by differencing the energy inputs (as kinetic energy) and outputs (as gamma rays): $\Delta\mathcal{E} = (7.654 - 0.92 - 0.287)\,\mathrm{MeV} = 7.275\,\mathrm{MeV}$.

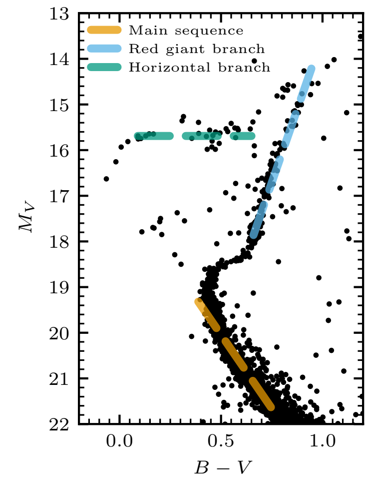

Figure 1: A *color-magnitude diagram* (CMD) for the globular cluster Messier 3, based on measurements by Rey et al. (2001, *AJ*, **122**, 3219). CMDs are an observational type of HR diagram, with color shown on the horizontal axis and absolute magnitude on the vertical axis. The colored lines highlight three evolutionary stages visible in the distribution of the cluster's stars.

[1] Named after Fred Hoyle, who hypothesized that this excited state must exist, otherwise there would be very little carbon formed in the universe — and hence no life as we know it. This is perhaps the most famous application of the *anthropic principle*. The existence of the Hoyle state was confirmed experimentally by Willie Fowler, who received the Nobel prize for his work.

The Horizontal Branch & Red Clump

Stars that are steadily burning helium in their cores[2]are known as *horizontal branch* (HB) stars. This terms originates from observations: the Hertzsprung-Russell diagrams of star clusters often show an over-density of stars lying along a horizontal line of approximately constant luminosity, extending from the RGB over to the the blue (see Fig. 1). A simple theoretical explanation for the position and orientation of the horizontal branch is not straightforward to formulate, but

[2] i.e., after the helium flash phase has completed.

numerical calculations confirm that stars on the horizontal branch must be burning helium in their cores.

The position of a given star on the HB depends on two important factors: the mass $M_{\rm env}$ of the hydrogen-rich envelope around the star's helium core, and the metallicity Z of this envelope. After the helium flash, all stars move onto the HB with approximately the same core mass $M_{\rm c} \approx 0.47\ M_{\odot}$; this is because the flash occurs when the core temperature reaches $T_{\rm c} \approx 10^8\,{\rm K}$, and — as we showed in *Handout* xxiii — the core temperature is set by $M_{\rm c}$.

However, the envelope mass depends on the total mass of the star via $M_{\rm env} = M - M_{\rm c}$. Stars with M only a little above the initial $M_{\rm c} \approx 0.47\,{\rm M}_{\odot}$ have a very thin envelope, and appear toward the blue end of the HB; whereas stars with an appreciably larger M have a correspondingly thicker envelope, and appear toward the red end of the HB. These trends are illustrated in Fig. 2, which plots the horizontal branches calculated by *MESA* for three different choices of metallicity. Each HB spans a range of stellar masses, with the low-mass end appearing on the left and the high-mass end on the right.

In addition to shifting the HB toward slightly lower luminosities, increasing the envelope metallicity Z reduces the relative number of stars at the blue end (see Fig. 2). For metallicities $Z \gtrsim 0.002$ (i.e., around one tenth of the Sun upward), the concentration of stars toward the red end becomes so pronounced that the horizontal branch no longer appears as a line in the HR diagram, but rather as a compact feature known as the *red clump*.

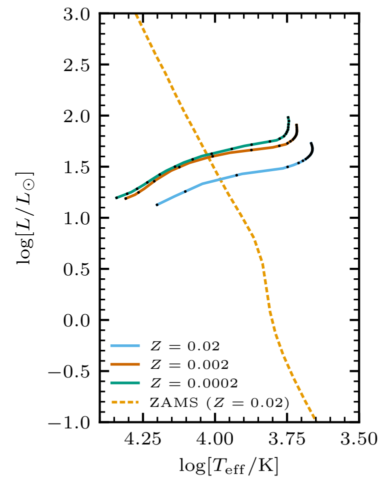

Figure 2: Horizontal branches in the Hertzsprung-Russell diagram calculated using *MESA* for three different choices of the metallicity Z. Each HB spans the mass range $0.49\,{\rm M}_{\odot} \leq M \leq 1.2\,{\rm M}_{\odot}$; the black dots show the positions of 20 stars in this range whose masses follow the distribution function $f(M) \propto M^{-2.35}$ (this is the so-called *Salpeter initial mass function*, which was derived from measurements of how many stars are born at each mass). Note how increasing the metallicity reduces the relative number of stars at the blue end of the HB.

Energy Production on the Horizontal Branch

Even though much of the foregoing discussion focuses on core helium burning, this isn't the only way horizontal branch (and red clump) stars generate energy. In fact, the hydrogen-burning shell that was responsible for energy generation during the red giant phase remains operative, just outside the helium core. This is demonstrated in Fig. 3 for a *MESA* model of the Sun on the HB (compare against Fig. 3 of *Handout* xxii). Note how the luminosity generated by the shell, at a mass coordinate $m \approx 0.47\,M_{\odot}$, exceeds the luminosity generated at the center. This indicates that more of the star's total luminosity is generated by hydrogen burning than by helium burning — even though we would typically label the star as helium burning.

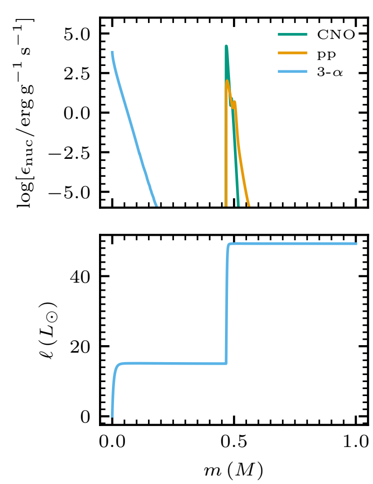

Figure 3: The logarithm of the nuclear energy generation rate $\epsilon_{\rm nuc}$ from the pp chains, CNO cycle and triple alpha reaction (upper panel), and the interior luminosity ℓ (lower panel), plotted as a function of interior mass m for a *MESA* model of the Sun soon after it arrives on the horizontal branch.

Further Reading

Kippenhahn, Weigert & Weiss, §33.6; *Ostlie & Carroll*, §13.2; *Prialnik*, §9.5.