Core Degeneracy on the RGB

For low and intermediate-mass stars on the red giant branch, the density in the helium core becomes sufficiently high (relative to the temperature) that the electron degeneracy is important. This is illustrated in Fig. 1, which shows the path followed by a *MESA* solar model in the $\log \rho_c$–$\log T_c$ plane, as it evolves from the pre-main sequence to the tip of the RGB. During the subgiant branch, the center of the star passes over the threshold $\psi = 4$ that (approximately) divides the classical and degenerate regimes[1].

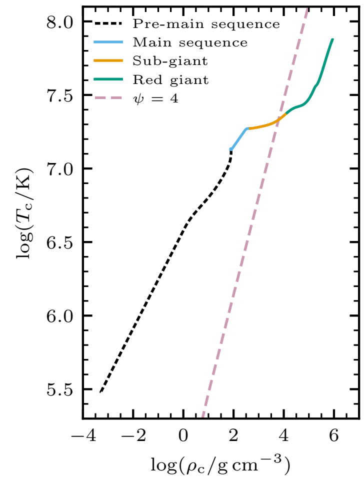

Figure 1: Path followed in the $\log \rho_c$–$\log T_c$ plane for a *MESA* model of the Sun, as it evolves from the pre-main sequence to the tip of the RGB. The diagonal dashed line divides the plane approximately into classical (left) and degenerate (right) regimes.

[1] Recall from *Handout* XVI that the classical limit corresponds to a large and negative degeneracy parameter ψ, and that the completely degenerate limit corresponds to a large and positive ψ. The value $\psi = 4$ is a (somewhat-arbitrary) choice between these two extremes.

The onset of electron degeneracy has an important consequence for how the radius R_c of the core changes as its mass M_c grows. Hydrostatic equilibrium and the mass equation require that the central pressure P_c and density ρ_c follow the scalings

$$P_c \sim \frac{M_c^2}{R_c^2}, \qquad \rho_c \sim \frac{M_c}{R_c^3}, \tag{1}$$

respectively. Combining these with the degenerate equation-of-state $P \propto \rho^{5/3}$, we find a mass-radius relation for degenerate cores:

$$R_c \sim M_c^{-1/3} \tag{2}$$

This scaling is very approximate, but as Fig. 2 shows, it doesn't do to bad of a job.

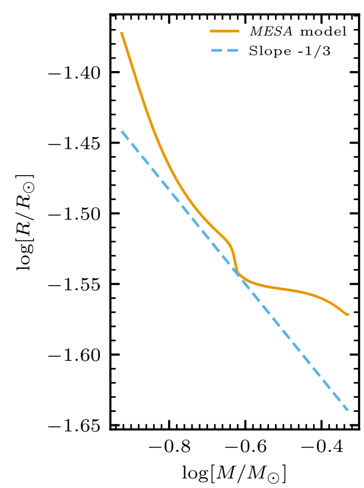

Figure 2: The path followed in the $\log M_c$–$\log R_c$ for a *MESA* model of the Sun, as it evolves up the RGB. The dashed line has a slope of $-1/3$, corresponding to the power-law mass-radius relation for degenerate cores (see eqn. 2).

The Approach to Helium Ignition

In *Handout* XXII, we showed that the temperature at the bottom of the hydrogen-burning shell of an RGB star scales as $T_b \sim M_c/R_c$. With the core mass-radius relation found above, we can expect the shell temperature to grow as $\sim M_c^{4/3}$ as the star ascends the RGB. During this evolutionary phase, the high thermal conductivity of the helium coremaintains its temperature close to that of the shell; as the shell gets hotter, so does the core. Eventually, the core reaches the temperatures necessary for helium burning via the *triple alpha* (3-α) reaction:

$$\underbrace{3(^4\mathrm{He} + 2e^-)}_{\text{3 helium-4 atoms}} \longrightarrow \underbrace{^{12}\mathrm{C} + 6e^-}_{\text{carbon-12 atom}} + \underbrace{N\gamma}_{\text{energy}} \tag{3}$$

The ignition of helium doesn't take place precisely at the center of the star, because non-nuclear neutrino production (see *Handout* X) causes the gradual removal of energy from the core, cooling it (see Fig.3). Instead, the ignition occurs off-center (but still within the core) at the point which first reaches the ignition temperature $T \approx 10^8$ K.

The Helium Flash

When helium ignition first occurs, the release of energy by the 3-α reaction causes the temperature to rise. In ordinary matter, a temperature rise is accompanies by a pressure increase, which drives expansion and cooling. However, in degenerate matter this negative feedback loop doesn't work, because the pressure is decoupled from the temperature. Instead, the temperature continues to increase, raising the reaction rate and releasing even more energy. The runaway only ends when the temperature becomes so high that the electrons are no longer degenerate; then, their pressure finally increases, expanding and cooling the burning region.

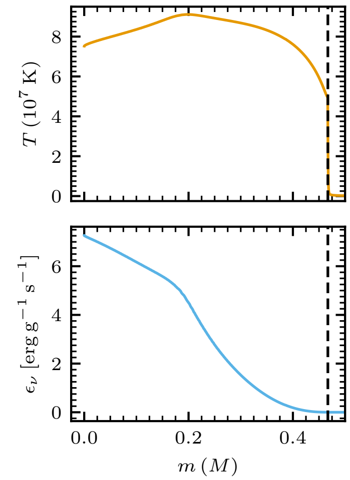

Figure 3: Temperature T (upper panel) and non-nuclear neutrino energy loss rate ϵ_ν (lower panel), plotted as a function of interior mass m in the core of a *MESA* model of the Sun just prior to helium ignition. Neutrino cooling at the center means that the temperature maximum, where the helium ignition will take place, is situated off-center at $m \approx 0.2\,M$. The vertical dashed line in each panel marks the position of the core boundary.

During this *helium flash* process, the luminosity generated by the 3-α reaction spikes up to values $\sim 10^9\,\mathrm{L}_\odot$ comparable to a whole galaxy's worth of stars. After a few days, however, it drops back down to small values $\lesssim \mathrm{L}_\odot$. A series of after-flashes then occur at locations progressively closer to the center of the star. The recurrence timescale between these flashes is on the order of a few hundred thousand years, and they are weaker but longer-lasting than the original. Eventually, helium ignites at the center of the star, which by now is non-degenerate, and the star settles into a state of steady helium burning. Fig. 4 illustrates this sequence of events for a *MESA* model of the Sun.

Although the luminosity generated by helium burning can reach immense values during flashes, none of this energy makes its way to the surface of the star. Instead, it is absorbed by the material in the outer parts of the core, driving the expansion of the core. Nevertheless, there *is* an observable consequence to a star passing through the helium flash(es) phase. With the scaling

$$\ell_{\mathrm{t}} \sim M_{\mathrm{c}}^{7} R_{\mathrm{c}}^{-25/3} \tag{4}$$

derived in *Handout* XXII, the luminosity ℓ_{t} at the top of the hydrogen-burning shell is significantly reduced when the core expands in response to helium ignition. This behavior can clearly be seen in Fig. 4, immediately after the initial flash and then after the subsequent flashes. Because ℓ_{t} largely determines the stellar luminosity L, the observable consequence of the helium flash is — perhaps counter-intuitively — that a star's luminosity decreases by over an order of magnitude.

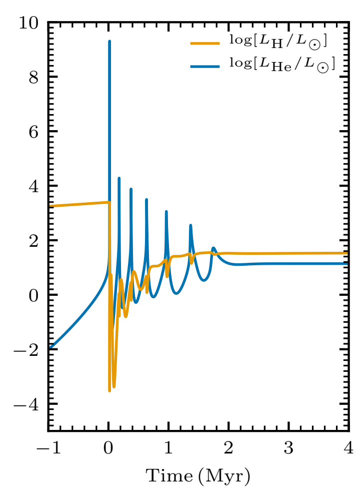

Figure 4: The hydrogen-burning luminosity L_{H} and helium-burning luminosity L_{He}, plotted as a function of time since the beginning of the helium flash for a *MESA* model of the Sun.

Further Reading

Kippenhahn, Weigert & Weiss, §§33.4,33.5.