The Proton-Proton Chain

In *Handout* ix we wrote down a reaction for the fusion of four hydrogen nuclei into one helium nucleus (see eqn. 5, *ibid.*). In this reaction there are a sequence of intermediate steps that we're now going to examine in detail.

The simplest sequence, known as the *proton-proton (pp) chain*, begins with the interaction of a proton with another proton. During this interaction, one of the protons transforms into a neutron[1], forming a deuterium nucleus and releasing a positron and an electron neutrino. In balanced form (with zero net charge on each side),

$$\underbrace{2({}^{1}\mathrm{H} + e^{-})}_{\text{2 hydrogen atoms}} \longrightarrow \underbrace{{}^{2}\mathrm{D} + 1e^{-}}_{\text{deuterium atom}} + \underbrace{e^{-} + e^{+}}_{\text{annihilate}} + \nu_e. \tag{1}$$

The net energy release, in the form of the neutrino, two gamma rays[2] and the kinetic energy of the nuclei, is given by the mass-energy difference between a deuterium atom and two hydrogen atoms: $\Delta\mathcal{E} = (2\,m_{\mathrm{H}} - m_{\mathrm{D}})c^2 = 1.44\,\mathrm{MeV}$ (see Table 1 for atomic mass data).

The next step in the pp chain is a reaction between the deuterium nucleus and another proton. Again in balanced form,

$$\underbrace{{}^{2}\mathrm{D} + 1e^{-}}_{\text{deuterium atom}} + \underbrace{{}^{1}\mathrm{H} + 1e^{-}}_{\text{hydrogen atom}} \longrightarrow \underbrace{{}^{3}\mathrm{He} + 2e^{-}}_{\text{helium-3 atom}} + \gamma \tag{2}$$

The energy release from this reaction is $\Delta\mathcal{E} = 5.49\,\mathrm{MeV}$. The final step involves two ^{3}He nuclei combining to form a ^{4}He nucleus and two protons:

$$\underbrace{2({}^{3}\mathrm{He} + 2e^{-})}_{\text{2 helium-3 atoms}} \longrightarrow \underbrace{{}^{4}\mathrm{He} + 2e^{-}}_{\text{helium-4 atom}} + \underbrace{{}^{2}\mathrm{H} + 1e^{-}}_{\text{2 hydrogen atoms}} \tag{3}$$

The energy release from this reaction is $\Delta\mathcal{E} = 12.86\,\mathrm{MeV}$. Taken together, these three reactions — with the first two occurring twice, to provide the two ^{3}He nuclei required for the third — release a total energy of $\Delta\mathcal{E} = 26.72\,\mathrm{MeV}$. This energy is released in the form of two neutrinos, six gamma rays and kinetic energy.

The above description of pp-chain chain reactions is complete, but rather verbose. An abbreviated notation writes a reaction between a target nucleus 'a' and an incoming particle 'b', to produce a final nucleus 'd' and an outgoing particle 'c', as a(b, c)d. Written in this form, the pp chain becomes

$$p\,(p, e^{+})\,d\,(p, \gamma)\,{}^{3}\mathrm{He}\,({}^{3}\mathrm{He}, 2p)\,\alpha, \tag{4}$$

where p, d and α are shorthands for a proton, a deuterium nucleus and an α particle (a ^{4}He nucleus). Note that neutrinos are not explicitly listed here, but are implied by lepton number conservation.

[1] This transformation is a highly unlikely process, because first the two protons must overcome their mutual Coulomb repulsion to get close enough for nuclear forces to become important; and then a weak interaction must take place. An average proton at the center of a star like the Sun waits billions of years before a successful transformation occurs.

[2] These gamma rays are released when the positron annihilates with the electron.

Isotope	Atomic Mass (u)
^{1}H	1.007 825
^{2}D	2.014 102
^{3}He	3.016 029
^{4}He	4.002 603
^{7}Li	7.016 004
^{7}Be	7.016 929
^{8}Be	8.005 305
^{8}B	8.024 607
^{12}C	12.000 000
^{13}C	13.003 355
^{13}N	13.005 739
^{14}N	14.003 074
^{15}N	15.000 109
^{15}O	15.003 066
^{16}O	15.994 915

Table 1: Atomic masses (in atomic mass units, $1\,\mathrm{u} = 1.6605 \times 10^{-24}\,\mathrm{g} = 931.5\,\mathrm{MeV}$) for selected isotopes. From Table D of Audi & Wapstra (1993, *Nucl. Phys A.*, **565**, 1).

Other pp Chains

The pp chain described above is one possible variant of the reaction sequence that begins with a proton-proton reaction. It is known as pp-I, to distinguish it from other possibilities. The pp-II chain is

$$p\,(p, e^+)\,d\,(p,\gamma)\,^3\mathrm{He}\,(\alpha,\gamma)\,^7\mathrm{Be}\,(e^-,\nu_e)\,^7\mathrm{Li}\,(p,\alpha)\,\alpha, \tag{5}$$

and the pp-III chain is

$$p\,(p, e^+)\,d\,(p,\gamma)\,^3\mathrm{He}\,(\alpha,\gamma)\,^7\mathrm{Be}\,(p,\gamma)\,^8\mathrm{B}\,(\cdot, e^+)\,^8\mathrm{Be}\,(\cdot,\alpha)\,\alpha. \tag{6}$$

The latter chain involves a couple of spontaneous decays, marked by the dot ($\cdot$) in front of the comma to indicate that they do not require an incoming particle. Like pp-I, both pp-II and pp-III release 26.72 MeV per ^{4}He nucleus produced; however, the distribution of this energy between neutrinos, gamma rays and kinetic energy differs between the three chains.

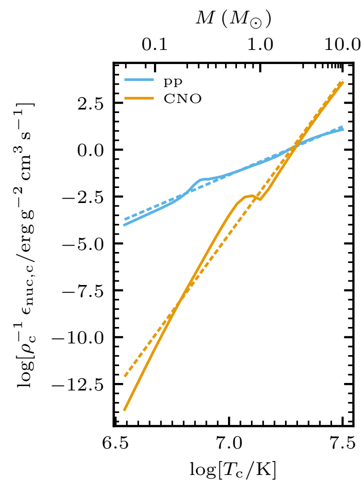

Figure 1: The logarithm of the central nuclear energy generation rate ϵ_c divided by the central density ρ_c squared, plotted as a function of the logarithm of the central temperature T_c for zero-age main sequence (ZAMS) stars spanning the mass range $0.1\,M_\odot \lesssim M \lesssim 10\,M_\odot$ (indicated along the top axis). Separate curves are shown for energy generation by the pp chains and the CNO cycle. The dotted lines are linear best fits to the data, and have slopes $\mathrm{d}\log(\rho_c^{-1}\,\epsilon_{\rm nuc,pp})/\,\mathrm{d}\log T_c \approx 5$ and $\mathrm{d}\log(\rho_c^{-1}\,\epsilon_{\rm nuc,CNO})/\,\mathrm{d}\log T_c \approx 17$.

The CNO Cycle

While the pp chains are responsible for hydrogen burning in the Sun, stars with hotter cores burn hydrogen by a different path: the *CNO (carbon-nitrogen-oxygen) cycle*. In shorthand form, the cycle is

$$^{12}\mathrm{C}\,(p,\gamma)\,^{13}\mathrm{N}\,(\cdot,e^+)\,^{13}\mathrm{C}\,(p,\gamma)\,^{14}\mathrm{N}\,(p,\gamma)\,^{15}\mathrm{O}\,(\cdot,e^+)\,^{15}\mathrm{N}\,(p,\alpha)\,^{12}\mathrm{C} \tag{7}$$

Importantly, the C, N and O involved in this reaction are *catalytic*[3]. As with the pp chains, the total energy release is 26.72 MeV per ^{4}He nucleus produced.

[3] I.e., they facilitate the reaction, but are not consumed.

The pp-CNO Switchover

The switchover between pp chains and the CNO cycle occurs because the latter is far more temperature sensitive than the former[4]. The energy generation rates for the two reaction networks follow the approximate scalings

[4] Ultimately, this is a consequence of the steeper Coulomb barrier of heavier nuclei such as carbon-12, compared to a proton.

$$\epsilon_{\rm nuc,pp} \approx \epsilon_{0,pp}\,\rho\,T^{6.6}, \qquad \epsilon_{\rm nuc,CNO} \approx \epsilon_{0,CNO}\,\rho\,T^{17.9}, \tag{8}$$

respectively, where $\epsilon_{0,pp}$ and $\epsilon_{0,CNO}$ are constants (see Fig. 1). Because $\epsilon_{0,pp} \gg \epsilon_{0,CNO}$, the pp chains dominate at temperatures $\log[T/\mathrm{K}] \lesssim 7.3$; but at higher temperatures, as encountered in more-massive stars ($M \gtrsim 1.4\,M_\odot$) the CNO cycle is more important.

Further Reading

Kippenhahn, Weigert & Weiss, §18.5.1; *Ostlie & Carroll*, §10.3; *Prialnik*, §§4.3,4.4.