Colors of Stars

Stars span a range of colors (see Fig. 1). These colors are due not to differences in composition[1], but to differences in surface temperature.

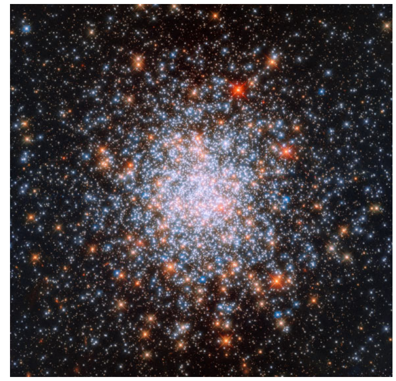

Figure 1: The globular cluster NGC 1866 — a densely packed, spherical collection of hundreds of thousands of stars. Credit: NASA/ESA/*Hubble*.

[1] Most stars have a composition similar to the Sun: around 70% (by mass) hydrogen, 28% helium, and the remainder a mixture of heavier elements.

Black-body Radiation

If we heat an opaque enclosure to a finite temperature, the interior cavity will fill with electromagnetic radiation. Through a continual process of absorption and re-emission by the walls of the enclosure, this radiation will eventually approach an equilibrium state that depends only on the temperature T of the walls. This state is known as *black-body radiation*, and to a reasonable level of approximation, stars can be modeled as emitters of black-body radiation.

If we open a small hole in the enclosure, allowing the black-body radiation to gradually escape, then the energy flux in the (narrow) wavelength interval $[\lambda, \lambda + \mathrm{d}\lambda]$ is given by *Planck's law*,

$$F_\lambda \, \mathrm{d}\lambda = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda k_\mathrm{B} T} - 1} \, \mathrm{d}\lambda \tag{1}$$

Here, k_B is Boltzmann's constant, h is Planck's constant, and c is the speed of light in a vacuum[2] . The quantity F_λ is the *flux density*, with units of flux per unit wavelength interval. Fig. 2 plots the flux density for three different temperatures. With rising temperature, F_λ increases at every wavelength; however, the increase is more pronounced at smaller λ, causing the F_λ peak to shift to shorter wavelengths. The result, when perceived in the visible part of the electromagnetic spectrum, is that hotter black-body radiation appears bluer, while cooler radiation appears redder.

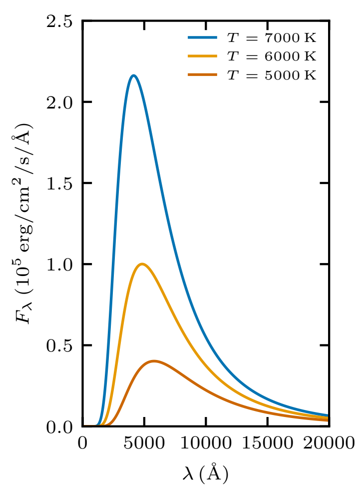

Figure 2: The flux density predicted by Planck's law (1), plotted against wavelength λ for three choices of the temperature T. The wavelength is measured in Ångstroms, another Astronomy-specific unit; $1\,\text{Å} = 0.1\,\text{nm} = 1 \times 10^{-8}\,\text{cm}$.

[2] In the cgs units preferred by Astronomers, these quantities have the values $k_\mathrm{B} = 1.380\,649 \times 10^{-16}\,\mathrm{erg\,K^{-1}}$, $h = 6.626\,069 \times 10^{-27}\,\mathrm{erg\,s}$ and $c = 2.997\,925 \times 10^{10}\,\mathrm{cm\,s^{-1}}$.

Wien's Law

This shift of the flux-density peak is described by *Wien's law*, which relates the wavelength λ_max of the peak to the temperature via

$$\lambda_\mathrm{max} = \frac{2.898 \times 10^7\,\text{Å}\,\mathrm{K}}{T} \tag{2}$$

This allows us to determine the temperature of a black-body emitter simply by measuring λ_max from its flux density.

The Stefan-Boltzmann Law

The *Stefan-Boltzmann law* is complementary to Wien's law, describing how the total flux

$$F \equiv \int_0^\infty F_\lambda \, \mathrm{d}\lambda \tag{3}$$

increases as the temperature rises. Substituting eqn. (1) into this expression, it can be shown [3] that

$$F = \sigma T^4, \tag{4}$$

[3] If you want to try the derivation for yourself, begin with the substitution $u = hc/\lambda k_B T$.

where the Stefan-Boltzmann constant is

$$\sigma \equiv \frac{2\pi^5 k_B^4}{15 h^3 c^2} = 5.6704 \times 10^{-5}\,\mathrm{erg\,cm^{-2}\,s^{-1}\,K^{-4}} \tag{5}$$

So, as with Wien's law, we can in principle use the Stefan-Boltzmann law to determine the temperature of a black-body emitter by measuring its total flux F. However, a big caveat here is that we have to measure F right at the surface of the emitter — we can't use the measured flux her on Earth, unless we correct for the effects of the inverse-square law.

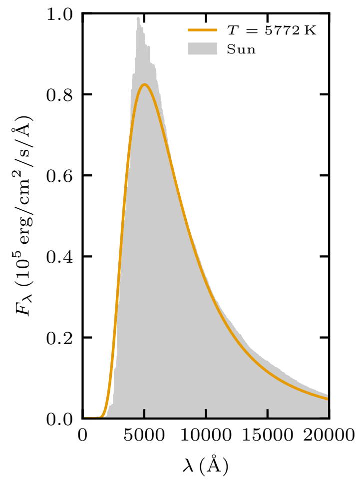

Figure 3: The measured flux density of the Sun (shaded), plotted together with the black-body curve at the Sun's effective temperature $T_{\mathrm{eff}} \simeq 5772$ K. Due to the way the effective temperature is defined, the area under the curves (i.e., the total flux F; cf. eqn. 3) is identical.

Application to Stars

Real stars aren't black-body emitters; their flux density doesn't follow Planck's law (1) exactly. However, as Fig. 3 illustrates, they come reasonably close. This motivates us to define the *effective temperature* of a star to be the temperature of a hypothetical black-body emitter that has the same surface flux of the star:

$$T_{\mathrm{eff}} \equiv \sqrt[4]{\frac{F}{\sigma}}. \tag{6}$$

The surface flux is itself determined by the star's luminosity L and radius R via

$$F = \frac{L}{4\pi R^2}; \tag{7}$$

putting these expressions together, we arrive at the Stefan-Boltzmann law as applied to stars,

$$L = 4\pi R^2 \sigma T_{\mathrm{eff}}^4. \tag{8}$$

This is one of the most important equations of stellar astrophysics, and we will be making extensive use of it.

In the case of the Sun, the effective temperature is $T_{\mathrm{eff}} \simeq 5772$ K. Fig. 3 shows that Wien's law doesn't hold exactly for the Sun — the flux-density peak is slightly bluer than predicted. Nevertheless, the discrepancy isn't large, and so in situations where we lack reliable measurements of L and/or R, we can estimate[4] T_{eff} by applying Wien's law.

[4] A more-accurate approach to estimating stellar effective temperatures involves careful analysis of the absorption lines in their spectra, because the strength of these lines changes with T_{eff}.

Further Reading

Ostlie & Carroll, §3.4; *Prialnik*, §1.2.