Overview

In this handout we're going to build a simple model for *fully convective stars* — that is, stars where convection is occurring throughout the entire interior, with only a thin radiative layer[1] at the surface.

[1] The surface layers of a star — the photosphere — must always be radiative, because $\nabla_{\rm rad} \propto P/T^4$ inevitably becomes smaller than $\nabla_{\rm ad}$ in these layers.

Interior Solutions

We start by developing a formula for how the temperature T behaves with respect to pressure P throughout the stellar interior. Assuming that the convection is efficient, we have

$$\nabla_T = \frac{d\ln T}{d\ln P} = \nabla_{\rm ad}. \tag{1}$$

Let's also assume an ideal-gas equation of state applies, so that $\nabla_{\rm ad} = 2/5$. Then, we integrate this equation to find

$$\ln T - \ln T_{\rm c} = \frac{2}{5} \left(\ln P - \ln P_{\rm c}\right). \tag{2}$$

To eliminate the central temperature $T_{\rm c}$ and pressure $P_{\rm c}$, we use the scaling relations[2] $P_{\rm c} \sim M^2/R^4$ and $T_{\rm c} \sim M/R$, yielding

$$\ln T = \frac{2}{5} \ln P + \frac{1}{5} \ln M + \frac{3}{5} \ln R + C, \tag{3}$$

where C is a constant. Fig. 1 plots these interior solutions for a star with fixed M and three different choices $R_1 < R_2 < R_3$ of radius.

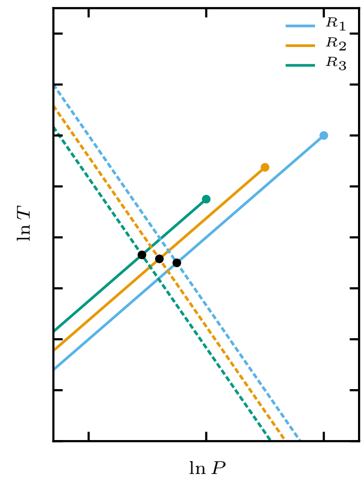

Figure 1: Interior solutions (solid lines) and photosphere solutions (dotted lines) for a fully convective star with three different choices $R_1 < R_2 < R_3$ of the stellar radius. The colored filled circles mark the center of the star, and the black filled circles (where interior and photosphere solutions intersect) mark the surface of the star.

Photosphere Solutions

As already mentioned, stellar photospheres are always radiative, and we must use a different $\ln T - \ln P$ relation to describe them. To develop this relation, we restate the outer boundary conditions from *Handout* XIII:

$$T = T_{\rm eff}, \qquad P = \frac{GM}{R^2\kappa}\frac{2}{3}. \tag{4}$$

[2] The first relation comes from eqn. (8) of *Handout* V, and the second follows from the ideal-gas EOS.

Let's now assume that the opacity depends on pressure and temperature via the generic relation

$$\kappa = \kappa_0 P^a T^b, \tag{5}$$

where κ_0, a and b are constants. Combining this with the pressure boundary condition leads, after some algebra. to the desired relation:

$$\ln T = -\frac{1+a}{b} \ln P + \frac{1}{b} \ln M - \frac{2}{b} \ln R + C' \tag{6}$$

where C' is another constant. Fig. 1 plots these photosphere solutions for the same three radius choices as before.

Matching Solutions

At the outer boundary of the star, the interior solution (3) must match the photosphere solution (6). Setting these two equations equal, we solve to find the photospheric pressure as

$$\ln P_{\rm phot} = \frac{(10-3b)\ln R + (5-b)\ln M - 5bC + 5bC'}{5+5a+2b}. \qquad (7)$$

The corresponding photospheric temperature, which by eqn. (4) is also the effective temperature, is therefore

$$\ln T_{\rm phot} = \ln T_{\rm eff} = \frac{(3a-1)\ln R + (3+a)\ln M + 5(1+a)C + 2bC'}{5+5a+2b}. \qquad (8)$$

Fig. 1 also marks the matching points between interior and photosphere solutions.

Fully Convective Star in the HR Diagram

For a fully convective star with a given mass, eqn. (8) indicates that its effective temperature is a function only of its radius. This means that the star must lie on a well-defined line in the Hertzsprung-Russell diagram. To determine this line, we use the stellar Stefan-Boltzmann equation (see eqn. 8 of *Handout* II) to eliminate $\ln R$ in favor of $\ln L$ and $\ln T_{\rm eff}$; this yields

$$\ln L = \frac{(6+22a+4b)\ln T_{\rm eff} - (6+2a)\ln M + C''}{3a-1}, \qquad (9)$$

where C'' is yet another constant. The slope of this line is

$$\frac{{\rm d}\log L}{{\rm d}\log T_{\rm eff}} = \frac{{\rm d}\ln L}{{\rm d}\ln T_{\rm eff}} = \frac{6+22a+4b}{3a-1}. \qquad (10)$$

In cool stars ($T_{\rm eff} \lesssim 10^4\,{\rm K}$), H^- opacity dominates throughout much of the star, leading to an empirical scaling $a \approx 1$ and $b \approx 3$ and a steep slope ${\rm d}\log L/\,{\rm d}\log T_{\rm eff} \approx 20$. So, fully convective stars lie on almost vertical lines in the HR diagram.

We've already encountered these vertical lines for stars during their pre-main sequence evolution: they are Hayashi tracks (see Fig. 2). Initially, all PMS stars are fully convective; as they undergo Kelvin-Helmholtz contraction they move vertically down in the HR diagram. Eventually, for stars with $M \gtrsim 0.4\,M_\odot$, a radiative region develops in the interior; then, the star pulls off its Hayashi track and evolves blueward along a Henyey track.

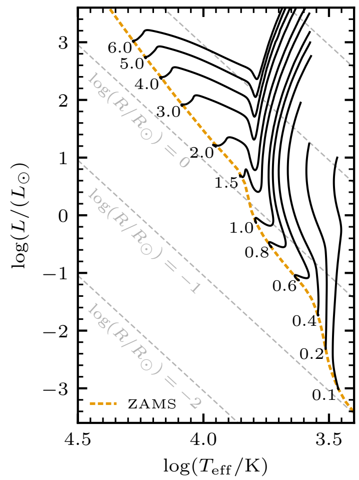

Figure 2: Evolutionary tracks in the Hertzsprung-Russell diagram for stars in the pre-main sequence phase, calculated using *MESA*. Each track is labeled at the zero-age main sequence (ZAMS) with its initial mass in $M_\odot$. With increasing mass, the vertical Hayashi tracks are displaced toward hotter effective temperatures — exactly as predicted by eqn. (9). Taken from Fig. 1 of *Handout* VIII).

Further Reading

Kippenhahn, Weigert & Weiss, §24; *Prialnik*§9.1.