## *Opacity Sources*

The opacity $\kappa$ of stellar material arises from a number of distinct physical processes, known collectively as *opacity sources*:

(a) *bound-bound absorption*, where a photon is absorbed by a bound electron, excited the electron to a higher (but still bound) energy level.

(b) *bound-free absorption*, where a photon is absorbed by a bound electron, freeing the electron from the atom. This is the same process as *ionization*.

(c) *free-free absorption*, where a photon is absorbed by a free electron in the vicinity of an atom, raising the electron to a higher (and still free) energy state. This is the inverse process of *bremsstrahlung*.

(d) *electron scattering*, where a photon is scattered by a free electron.

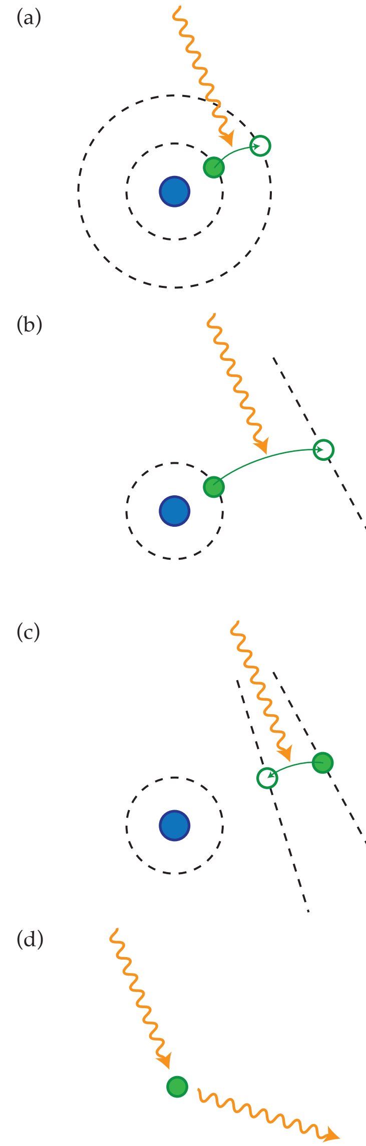


Figure 1: Pictorial representation of the four opacity sources in stars: (a) bound-bound absorption, (b) bound-free absorption, (c) free-free absorption, (d) electron scattering. The blue circle indicates the atomic nucleus, the green circle an electron.

Fig. 1 sketches these processes. Because free-free absorption and electron scattering require the presence of free electrons, they are absent in completely neutral material. Conversely, because bound-bound and bound-free absorption require the presence of bound electrons, they are absent in fully ionized material.

The dependence of opacity on temperature and density is generally quite complex. However, in stellar interiors there are three main regimes where only a single or couple of opacity sources make a significant contribution toward $\kappa$. At high temperatures ($T \gtrsim 10^6\,\mathrm{K}$) electron scattering is dominant, and the opacity is insensitive to $T$ or $\rho$. At intermediate temperatures ($10^4\,\mathrm{K} \lesssim T \lesssim 10^4\,\mathrm{K}$) bound-free and free-free absorption are dominant, and the opacity follows *Kramers' law*,

$$\kappa_{\mathrm{Kramers}} \sim \rho T^{-7/2}. \tag{1}$$

Finally, at low temperatures ($T \lesssim 10^4\,\mathrm{K}$) so-called *H-minus* opacity becomes dominant. This is bound-free absorption by $\mathrm{H}^-$ ions — hydrogen atoms with a second, loosely-bound electron. The opacity then behaves as

$$\kappa_{\mathrm{H}^-} \sim \rho^{1/2} T^{15/2}. \tag{2}$$

Fig. 2 demonstrates these three regimes for a model of the Sun.

## *Radiative Diffusion*

In stellar interiors, the mean free path of photons is much shorter than the characteristic length scales over which the temperature changes. This allows us to treat the transport of heat by radiation as *diffusive* process, with a radiative flux $F_{\rm rad}$ given by

$$F_{\rm rad} = -k_{\rm rad} \frac{\mathrm{d}u_{\rm rad}}{\mathrm{d}r}. \tag{3}$$

Here, the *radiation energy density* is

$$u_{\rm rad} = aT^4, \tag{4}$$

with $a \equiv 4\sigma/c$ being the *radiation constant*, and the *radiative diffusivity* is

$$k_{\rm rad} = \frac{c\langle l \rangle}{3} = \frac{c}{3\kappa\rho}. \tag{5}$$

The radiative flux is in turn related to the radiative interior luminosity (see *Handout* x) via $\ell_{\rm rad} \equiv 4\pi r^2 F_{\rm rad}$; putting these expressions together, we arrive at an expression for the radiative luminosity:

$$\ell_{\rm rad} = -\frac{16\pi r^2 acT^3}{3\kappa\rho} \frac{\mathrm{d}T}{\mathrm{d}r}. \tag{6}$$

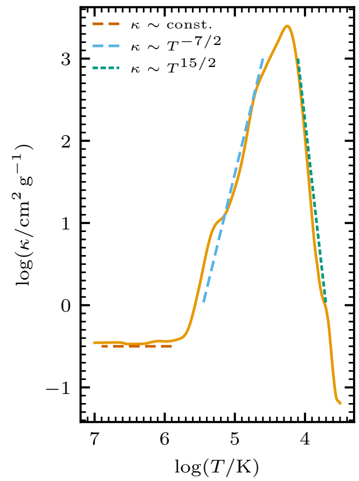


Figure 2: The opacity $\kappa$ plotted as a function of temperature $T$, for a fixed density $\rho = 10^{-6}\,\mathrm{g\,cm^{-3}}$. The data are taken from *MESA* for solar composition. The lines shows the three regimes discussed in the text: $\kappa \sim$ const. (electron scattering), $\kappa \sim T^{-7/2}$ (bound-free and free-free absorption) and $\kappa \sim T^{15/2}$ (bound-free absorption due to H-minus).

## *Temperature Gradients*

Expressed in the form (6), the radiative diffusion equation allows us to calculate $\ell_{\rm rad}$ given $\mathrm{d}T/\mathrm{d}r$ and other quantities. However, we often need to solve the converse problem: what temperature gradient is required to transport a given radiative luminosity? In such cases, we rewrite the diffusion equation as

$$\nabla_T \equiv \frac{\mathrm{d}\ln T}{\mathrm{d}\ln P} = \frac{3}{16\pi acG} \frac{\kappa \ell_{\rm rad} P}{mT^4}, \tag{7}$$

where the first equality serves to define the *dimensionless temperature gradient* $\nabla_T$, and the second equality follows from applying the equation of hydrostatic equilibrium (eqn. 6 of *Handout* v).

A related quantity of interest is the *radiative temperature gradient* $\nabla_{\rm rad}$, which quantifies the hypothetical temperature gradient that could transport *all* of the interior luminosity by radiation alone:

$$\nabla_{\rm rad} = \frac{3}{16\pi acG} \frac{\kappa \ell P}{mT^4} \tag{8}$$

When $\nabla_{\rm rad} = \nabla_T$, all the interior luminosity is transported by radiation (by definition); whereas when $\nabla_{\rm rad} > \nabla_T$, a fraction $\nabla_T/\nabla_{\rm rad}$ is transported by radiation, and the remainder by convection. Fig. 3 plots the two temperature gradients for a model of the Sun.

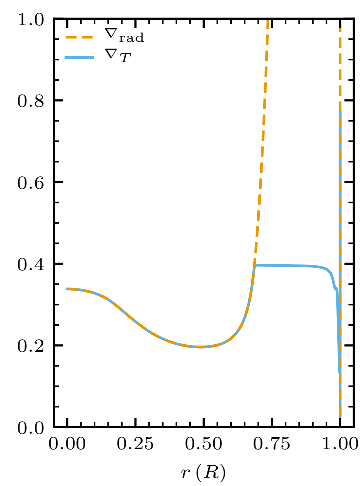


Figure 3: The dimensionless temperature gradient $\nabla_T$ and radiative temperature gradient $\nabla_{\rm rad}$, plotted as a function of radial coordinate $r$ for a *MESA* model of the present-day Sun. Energy transport is by radiation alone in regions where $\nabla_{\rm rad} = \nabla_T$; and by radiation and convection together in regions where $\nabla_{\rm rad} > \nabla_T$.

## *Further Reading*

*Kippenhahn, Weigert & Weiss*, §§5.1,5.2; *Ostlie & Carroll*, §10.4; *Prialnik*, §3.7.