Radiation & Matter

The interaction between electromagnetic radiation and matter involves three processes:

- *emission*, where a new photon is created by extracting energy from the matter;
- *absorption*, where an existing photon is destroyed by depositing energy into the matter;
- *scattering*, where an existing photon's direction (and possibly energy) is altered by the matter, but the photon is not destroyed.

Cross Section & Opacity

For a single matter particle situated in a radiation beam with cross section $\mathrm{d}A$, the probability that a photon in the beam interacts with the particle (by absorption or scattering) can be expressed as

$$p = \frac{\sigma}{\mathrm{d}A}. \tag{1}$$

Conceptually, you can think of the particle as blocking out a small area σ of the beam, so that a fraction $\sigma/\,\mathrm{d}A$ of all photons in the beam are absorbed or scattered.

Here, the cross section σ[1] is a quantity with units of area, which characterizes the strength of the interaction between radiation and matter.

[1] Don't confuse this with the Stefan-Boltzmann constant!

Now let's consider an infinitesimal length $\mathrm{d}l$ in the direction of propagation of the beam, containing $n\,\mathrm{d}A\,\mathrm{d}l$ particles (where, as usual, n is the number density). The interaction probability becomes

$$p = \frac{\sigma}{\mathrm{d}A} n\,\mathrm{d}A\,\mathrm{d}l = \sigma\, n\,\mathrm{d}s = \kappa\,\rho\,\mathrm{d}l, \tag{2}$$

where the final equality introduces the *opacity*

$$\kappa \equiv \frac{\sigma}{\mu \mathrm{m_H}} \tag{3}$$

as the cross section per unit mass.

To give a feel for typical values, the opacity in stars spans the range $c0.1$–$10^4\,\mathrm{cm^2\,g^{-1}}$. Compare this against a human: with a cross-sectional area $\approx 0.5\,\mathrm{m^2}$ and a mass $\approx 50\,\mathrm{kg}$, we estimate $\kappa \approx 0.1\,\mathrm{cm^2\,g^{-1}}$.

Mean Free Path & Optical Depth

The *mean free path* is the average distance a photon travels before it is absorbed or scattered. It can be calculated from the opacity and density (or cross section and number density) via

$$\langle l \rangle = \frac{1}{\sigma\, n} = \frac{1}{\kappa\,\rho}. \tag{4}$$

When dealing with stars, it is often useful to express depths below the surface layers in terms of this mean free path. We therefore introduce the *optical depth* as

$$\tau(r') \equiv \int_{r'}^{R} \frac{\mathrm{d}r}{\langle l \rangle} = \int_{r'}^{R} \kappa\,\rho\;\mathrm{d}r. \tag{5}$$

Fig. 1 shows the optical depth for a model of the Sun; note that τ is zero at the stellar surface, and increases very rapidly as we go deeper into the star, eventually reaching $\approx 3 \times 10^{12}$.

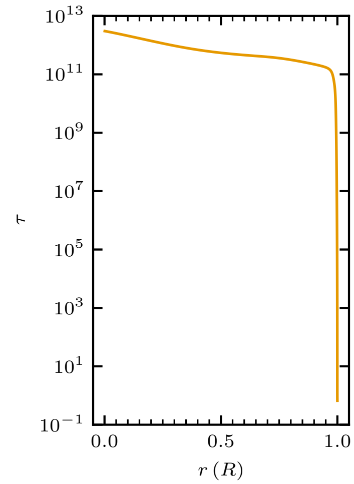

Figure 1: The optical depth τ plotted as a function of radial coordinate r for a *MESA* model of the present-day Sun.

Random Walk of Photons

Suppose a photon is emitted at radial coordinate r in the star. Assuming that it interacts with matter only by scattering (so that it is never destroyed), how long will the photon take to escape from the star?

This escape time can be written generally as

$$\tau_{\rm esc} = \frac{\Delta}{c} \tag{6}$$

where Δ represents the total distance the photon must travel in order to escape from the star. If the photon could travel in a straight line to the stellar surface, then this distance is simply $R - r$, and the escape time is

$$\tau_{\rm esc} = \frac{R-r}{c} \tag{7}$$

However, in reality the photon can only travel a distance $\langle l \rangle$ before it is scattered, randomizing its direction. Hence, the process by which the photon reaches the stellar surface is a *random walk*[2]. As an admittedly-crude simplification, if we assume $\langle l \rangle$ remains constant as the photon travels through the star, then the photon can escape by undergoing

[2] A random walk is a stochastic process where the position changes by a fixed amount $\langle l \rangle$ each step, but the direction of the change is random. After a large number of steps N, the average distance from the starting point is $\approx \langle l \rangle \sqrt{N}$.

$$N \approx \frac{(R-r)^2}{\langle l \rangle^2} \approx \tau^2 \tag{8}$$

scatterings. Here, the second equality follows from evaluating the optical depth integral (5) for constant $\langle l \rangle$. The total distance traveled by the photon is then

$$\Delta = N \langle l \rangle \approx (R-r)\tau \tag{9}$$

for an escape time

$$\tau_{\rm esc} \approx \frac{(R-r)}{c}\tau. \tag{10}$$

This is larger the straight-line escape time (7) by a factor τ.

Applying this formalism to the center of the Sun ($r = 0, R = R_{\odot}$, $\tau = 3 \times 10^{12}$), we find $N \approx 10^{25}$, $\Delta \approx 2 \times 10^{23}\,{\rm cm} \approx 70\,{\rm kpc}$[3] , and $\tau_{\rm esc} \approx 200\,000\,{\rm yr}$. We therefore see that if nuclear reactions at the center of the Sun suddenly ceased, it would take hundreds of thousands of years for this change to become apparent at the surface.

[3] Larger than the diameter of the Milky Way galaxy!

Further Reading

Kippenhahn, Weigert & Weiss, §5.1.1; *Ostlie & Carroll*, §9.3; *Prialnik*, §3.7.