Interior Luminosity

In *Handout* IV we introduced $m(r)$ as the mass of a star contained within the sphere with radius r. We now define the *interior luminosity* $\ell(r)$ as the total energy flowing through the surface of the same sphere, per unit time interval. By definition, $L \equiv \ell(R)$.

Fig. 1 illustrates the interior luminosity for a model of the present-day Sun. Also plotted are the radiative ($\ell_{\rm rad}$) and convective ($\ell_{\rm conv}$) interior luminosities, representing the energy transported by radiation and convection, respectively; by definition,

$$\ell \equiv \ell_{\rm rad} + \ell_{\rm conv}. \tag{1}$$

In the figure, we see that ℓ rises rapidly from zero at the center of the star, and by $r \approx 0.4\,R$ is already very close to its surface value. In the inner parts of the star ($r \lesssim 0.7\,R$), and also in a thin layer at the stellar surface (the photosphere) there is no convection: $\ell_{\rm conv} = 0$ and so $\ell_{\rm rad} = \ell$. Elsewhere, both radiation and convection contribute by varying amounts toward ℓ.

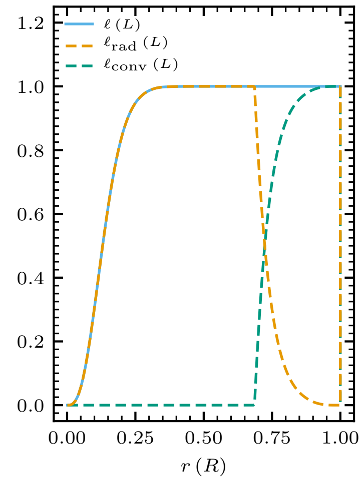

Figure 1: The interior luminosity ℓ (in units of its surface value $L = 3.86 \times 10^{33}\,{\rm erg\,s^{-1}}$), plotted as a function of radial coordinate for a *MESA* model of the present-day Sun. Also show are the contributions toward ℓ from the radiative ($\ell_{\rm rad}$) and convective ($\ell_{\rm conv}$) interior luminosities.

Thermal Equilibrium

The interior luminosity is closely linked to the generation and transport of energy within the star. To establish this link, consider a spherical shell extending from radial coordinate $r_{\rm a}$ out to radial coordinate $r_{\rm b}$. The rate $\mathrm{d}\dot{\mathcal{Q}}$ that energy is added or removed from this shell is as

$$\mathrm{d}\dot{\mathcal{Q}} = \underbrace{\Big[\ell(r_{\rm a}) - \ell(r_{\rm b})\Big]}_{\text{luminosity}} + \underbrace{\int_{r_{\rm a}}^{r_{\rm b}} 4\pi r^2 \rho \epsilon_{\rm nuc}\,\mathrm{d}r}_{\text{nuclear reactions}} - \underbrace{\int_{r_{\rm a}}^{r_{\rm b}} 4\pi r^2 \rho \epsilon_{\nu}\,\mathrm{d}r}_{\text{non-nuclear neutrinos}} \tag{2}$$

The terms on the right-hand side represent three separate processes (from left-to-right):

(I) Energy entering the shell through its lower boundary, and leaving through its upper boundary, via the interior luminosity;

(II) Energy deposited throughout the shell via nuclear reactions. The term $\epsilon_{\rm nuc}$ represents the *net*[1] energy release due to nuclear reactions, per unit time interval and mass.

(III) Energy removed throughout the shell via non-nuclear neutrino production[2]. The term ϵ_{ν} represents the energy lost as non-nuclear neutrinos, per unit time interval and mass.

Applying the fundamental theorem of calculus, the equation can also be expressed as

$$\mathrm{d}\dot{\mathcal{Q}} = \int_{r_{\rm a}}^{r_{\rm b}} \left[-\frac{\mathrm{d}\ell}{\mathrm{d}r} + 4\pi r^2 \rho(\epsilon_{\rm nuc} - \epsilon_{\nu})\right] \mathrm{d}r \tag{3}$$

[1] That is, the total energy release minus the energy of any neutrinos generated, since the latter escape from the star with no further interactions.

[2] An example here is photo-neutrino reactions: $e^- + \gamma \rightarrow e^- + \nu_{\rm e} + \bar{\nu}_{\rm e}$.

For stars on the main sequence, $\mathrm{d}\dot{Q}$ vanishes for all possible choices of r_a and r_b. This condition is known as *thermal equilibrium*; it arises due to precise balance between the transport, generation and loss of energy. Setting the integrand to zero in the eqn. (3), we find the condition for thermal equilibrium:

$$\frac{\mathrm{d}\ell}{\mathrm{d}r} = 4\pi r^2 \rho (\epsilon_\mathrm{nuc} - \epsilon_\nu). \tag{4}$$

Fig. 2 demonstrates thermal equilibrium in action for a model of the Sun.

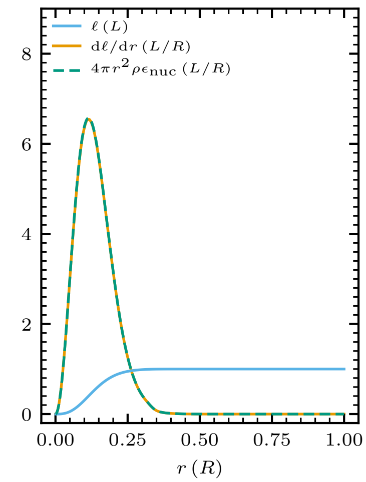

Figure 2: The interior luminosity ℓ (in units of L), its gradient $\mathrm{d}\ell/\,\mathrm{d}r$ and the nuclear energy release rate per unit radius $4\pi r^2\rho\epsilon_\mathrm{nuc}$ (both in units of L/R), plotted as a function of radial coordinate r for the *MESA* model of the present-day Sun (cf. Fig. 2). With ϵ_ν being negligible, the close match between $\mathrm{d}\ell/\,\mathrm{d}r$ and $4\pi r^2\rho\epsilon_\mathrm{nuc}$ indicates that the model is in thermal equilibrium.

Departures from Thermal Equilibrium

In certain phases of stellar evolution, thermal equilibrium does not hold. Then, eqn. (4) must be replaced by the energy conservation equation

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{1}{4\pi r^2 \rho}\left[\frac{\mathrm{d}\ell}{\mathrm{d}r} + 4\pi r^2 \rho(\epsilon_\mathrm{nuc} - \epsilon_\nu)\right] + \frac{P}{\rho^2}\frac{\mathrm{d}\rho}{\mathrm{d}t}. \tag{5}$$

This is a form of the *first law of thermodynamics* (FLOT), which relates the change in internal energy of a thermodynamic system to the heat added to the system (the first term on the right hand side, which involves the integrand from eqn .3), and the work done on the system (the second term).

Fig. 3 gives an example of a star that's not in thermal equilibrium. As with all stars on the PMS, the nuclear energy generation in this star is negligible (as are the neutrino losses), and so there's no way it can satisfy eqn. (4). As we already know, the star will undergo Kelvin-Helmholtz contraction and heat up.

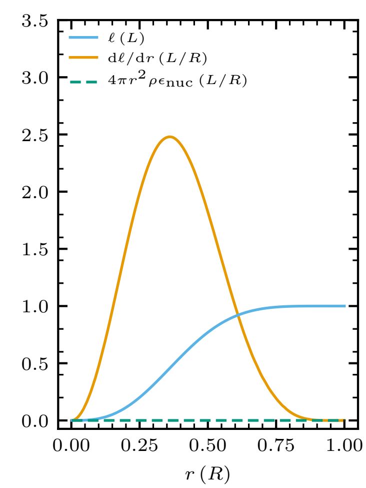

Figure 3: As in Fig. 2, but for a *MESA* model of the Sun in the pre-main sequence phase, when $L = 280\,\mathrm{L}_\odot$. Because there is no energy generation in the star, $\mathrm{d}\ell/\,\mathrm{d}r$ does not match $4\pi r^2\rho\epsilon_\mathrm{nuc}$, and so the star isn't in thermal equilibrium.

The Thermal Timescale

In *Handout* 5, we introduced the dynamical timescale τ_dyn as the characteristic timescale over which a star re-establishes hydrostatic equilibrium. A corresponding quantity, the *thermal timescale* τ_thm, measures how long it takes for a star to establish thermal equilibrium. We already have encountered this timescale — it's the Kelvin-Helmholtz timescale:

$$\tau_\mathrm{thm} \equiv \tau_\mathrm{KH} \equiv \frac{GM^2}{RL} \tag{6}$$

(see eqn. 4 of *Handout* VIII).

Further Reading

Kippenhahn, Weigert & Weiss, §4.4; *Ostlie & Carroll*, §10.3; *Prialnik*, §2.1.