Position

To specify the position of an object in 3-dimensional space, we require three coordinates. Astronomers use the *Celestial coordinate system*, which is a spherical system centered on the Earth. Two angular coordinates — the *right ascension* α and *declination* δ[1] — describe the position of a star or other celestial object in the sky; the third coordinate gives the distance d to the object.

Distances are typically specified in *astronomical units*[2], *light years*[3], or most commonly in *parsecs*. A parsec is defined so that a star at 1 pc distance will appear to move by an angle ± 1 *arcseconds*[4] relative to the background of more-distant stars, due to the orbit of the Earth around the Sun. More generally, a star at a distance of d will show parallax motion $\pm p$, where

$$d/1\,\mathrm{pc} = \frac{1}{p/1''}. \qquad (1)$$

(see Fig. 1). This formula, which can be derived from the small-angle approximation of trigonometry, allows us to calculate the distance to nearby stars (out to a few kpc) from their parallax motion.

[1] Right ascension (often abbreviated RA) is analogous to longitude on the Earth's surface, and declination (abbreviated Dec) is likewise analogous to latitude.

[2] The average distance from the Earth to the Sun; $1\,\mathrm{au} = 1.496 \times 10^{13}$ cm.

[3] The distance traveled by light in a year; $1\,\mathrm{ly} = 9.461 \times 10^{17}$ cm.

[4] A small unit of angular measure, denoted by double apostrophes; $1'' = \frac{1}{3600}{}^\circ$.

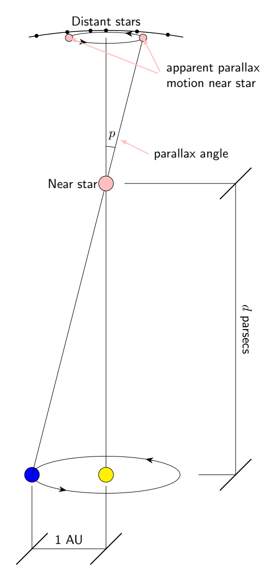

Figure 1: Parallax motion due to the Earth's orbit around the Sun.

Brightness

When we observe a celestial object, its brightness depends on how much energy (in the form of electromagnetic radiation) we receive from it. This energy ΔE in turn depends on the exposure time Δt and the area ΔA of the device collecting the energy.[5] We define the energy flux of the object as

$$F \equiv \frac{\Delta E}{\Delta t\, \Delta A} \qquad (2)$$

The flux from the Sun — known as the *solar constant* — is 1.36×10^6 $\mathrm{erg\,cm^{-2}\,s^{-1}}$. By contrast, the flux from Sirius (the brightest star in the sky) is only $1.05 \times 10^{-4}\,\mathrm{erg\,cm^{-2}\,s^{-1}}$.

[5] For the human eye under dark conditions, $\Delta t \approx 0.02$ s and $\Delta A \approx 0.5\,\mathrm{cm}^2$.

The reason why Sirius is much dimmer than the Sun, is that it's much further away. The energy flux from an object varies inversely with the square of the distance — the *inverse square law of light*. Mathematically, we can express this as

$$F = \frac{L}{4\pi d^2}. \qquad (3)$$

The quantity L is the *luminosity* of the object, which does not depend on distance. To understand physically what L means, imagine building a spherical shell around the object with radius r. The total surface area of the shell is $\Delta A = 4\pi r^2$, and so the energy received in time interval Δt is

$$\Delta E = F\,\Delta t\,\Delta A = \left(\frac{L}{4\pi r^2}\right)\left(4\pi r^2\right)\,\Delta t = L\,\Delta t$$

By conservation of energy, the energy received by the shell must match the energy emitted by the object. Hence, we arrive at the meaning of luminosity: it is the total amount of energy emitted by an object per unit time interval.

Returning to the Sun and Sirius, we can use eqn. (3) to calculate their respective luminosities from their measured fluxes and distances ($d_\odot = 1\,\mathrm{au}$; $d_{\mathrm{Sirius}} = 2.64\,\mathrm{pc}$). We find the solar luminosity[6] is $3.828 \times 10^{33}\,\mathrm{erg\,s^{-1}} \equiv 1\,\mathrm{L_\odot}$, while the luminosity of Sirius is $8.75 \times 10^{34}\,\mathrm{erg\,s^{-1}} = 22.9\,\mathrm{L_\odot}$; hence, although appearing much dimmer when viewed from Earth, Sirius is in fact quite a bit more luminous than the Sun.

[6] Both $d_\odot$ and $\mathrm{L_\odot}$ follow the standard notation that the subscript $\odot$ (a circle with a dot inside it) indicate a value pertaining to the Sun.

Magnitude

For historical reasons, Astronomers often use a logarithmic scale to quantify the flux from celestial objects. The *apparent magnitude* m of an object is defined by the equation

$$m = -2.5 \log F + c \tag{4}$$

Here, and throughout these notes, 'log' is the base-10 logarithm, and 'ln' the natural logarithm.

where F is the flux from the object, as measured on Earth, and c a constant. This definition, which was first proposed by the 19th-Century astronomer Norman Robert Pogson[7] , means that

- brighter stars have less positive (or more negative) magnitudes than dimmer stars;
- stars which differ in apparent magnitude by an additive offset of 5, differ in flux by a multiplicative factor of 100.

[7] Pogson drew inspiration from ancient Greek astronomers, most prominently Hipparchus of Nicaea, who used a system where the brightest stars were assigned a magnitude '1', the next-brightest stars a magnitude '2', and so on through to the dimmest stars visible to the naked eye, with a magnitude '6'.

A logarithmic scale can also be used to quantify the luminosity of celestial objects. The *absolute magnitude* M of an object is defined as

$$M = -2.5 \log L + C, \tag{5}$$

where C is another constant chosen so that the absolute magnitude of an object at a distance 10 pc equals its apparent magnitude.

Further Reading

Ostlie & Carroll, §§3.1,3.2; *Prialnik*, §1.2.