Homework Assignment 3 — Solutions

- Q3.2

  We have $L_{\rm bulb} = 100$ W, and the solar irradiance is $F_\odot = 1365$ W/m^2. At a distance r from the bulb the radiant flux is $F_{\rm bulb} = L_{\rm bulb}/4\pi r^2$. Setting $F_{\rm bulb} = F_\odot$ and solving for r gives $r = 0.08$ m.

- Q3.3

  $p = 0.379''$.

  (a). By definition, the distance in parsecs is given by one over the parallax angle measured in arcseconds: $(d/1\ \text{pc}) = (1/p'')$. For Sirius we have (i) $d = 1/0.379$ pc $= 2.64$ pc, which corresponds to (ii) 2.64 pc $\times$ 3.26 light-years/pc $=$ 8.60 light-years, or to (iii) 2.64 pc $\times\ 2.06 \times 10^5$ AU/pc $= 5.44 \times 10^5$ AU, or to (iv) 2.64 pc $\times\ 3.09 \times 10^{16}$ m/pc $= 8.16 \times 10^{16}$ m.

  (b). The distance modulus for Sirius is $m - M = 5\ \log_{10}(d/10\ \text{pc}) = 5\ \log_{10}(0.264) = -2.89$.

- Q3.4

  $m = -1.53$ and $m - M = -2.89$, so $M = m + 2.89 = +1.36$. The Sun's absolute magnitude is $M_\odot = +4.74$. The ratio of the luminosities of Sirius and the Sun is related to the difference of their absolute magnitudes by $M - M_\odot = -2.5\ \log_{10}(L/L_\odot) = -3.38$. Solving for L/L_{odot} gives $L/L_{odot} = 22.5$.

- Q3.8

  $A = 1.4$ m^2, $T_{\rm skin} = 306$ K, and $T_{\rm room} = 293$ K.

  (a). $L_{\rm emitted} = A\,\sigma\,T_{\rm skin}^4 = 700$ W.

  (b). $\lambda_{\rm max} \approx 0.0029\ \text{m}\,\text{K}/T_{\rm skin} = 9.5 \times 10^{-6}$ m, which is in the infrared. This is why mosquitoes have evolved infrared vision.

  (c). $L_{\rm absorbed} = A\,\sigma\,T_{\rm room}^4 = 590$ W.

  (d). $L_{\rm net} = L_{\rm absorbed} - L_{\rm emitted} = 590\ \text{W} - 700\ \text{W} = -110$ W. The average person loses a net energy of 110 J per second due to blackbody radiation.

- Q3.9

  $T = 28,000$ K, $R = 5.16 \times 10^9$ m, and $r = 123$ pc.

  (a). $L = 4\pi R^2 \sigma T^4 = 1.2 \times 10^{31}$ W.

  (b). $M = M_\odot - 2.5 \log_{10}(L/L_\odot) = -6.5$.

  (c). $m = M + 5\ \log_{10}(r/10\ \text{pc}) = -1.0$.

  (d). $m - M = 5.5$.

  (e). $F_{\rm surface} = \sigma T^4 = 3.5 \times 10^{10}$ W/m^2.

  (f). $F_\oplus = \sigma T^4 \times (R/r)^2 = 6.5 \times 10^{-8}$ W/m^2. This is much less than the solar irradiance $F_\odot = 1365$ W/m^2.

  (g). $\lambda_{\rm max} \approx 0.0029\ \text{m}\,\text{K}/T = 1.0 \times 10^{-7}$ m, which is in the ultraviolet.

- Q3.14

  (a). We have

  $$L = \int_0^\infty \frac{8\pi^2 R^2 hc^2/\lambda^5}{e^{hc/\lambda kT} - 1} d\lambda.$$

  Let $u = hc/\lambda kT$, so that $du = -(hc/kT\lambda^2)d\lambda$. Then

  $$L = \int_\infty^0 -\frac{8\pi^2 R^2 hc^2}{e^u - 1} u^3 \left(\frac{kT}{hc}\right)^4 du = 8\pi^2 R^2 hc^2 \left(\frac{kT}{hc}\right)^4 \int_0^\infty u^3 du = \frac{8\pi^6 R^2 hc^2}{15} \left(\frac{kT}{hc}\right)^4.$$

(b).

$$L = 4\pi R^2 \sigma T^4 = \frac{8\pi^6 R^2 hc^2}{15} \left(\frac{kT}{hc}\right)^4,$$

so

$$\sigma = \frac{2\pi^5 hc^2}{15} \left(\frac{k}{hc}\right)^4 = \frac{2\pi^5 k^4}{15\ h^3 c^2}$$

(c). Plugging in values for the various constants, I get $\sigma = 5.676 \times 10^{-8}$ W m^{-2} K^{-4}. This is very close to the listed value, and presumably differs slightly because I only took values to 4 significant figures for h, c, and k.

- Q3.16

  $T = 9600$ K.

  A word of caution: we can use Wien's displacement law to find the wavelength at which the spectral energy distribution peaks (I get $\lambda_{\rm max} = 302$ nm), but this does *not* tell us through which filter Vega would appear brightest to a photometer: the brightness measured by the photometer is the integral of the spectral energy distribution over the wavelength range (bandwidth) of the filter, so even though the Planck function has a larger value at the center of the U filter than at the center of the B filter, the fact that the B filter has a larger bandwidth means that it is still in the running for having the brightest appearance to a photometer.

  The brightness measured by the photometer is determined by the total flux received through the filter, which is

$$F_{\rm filter} = \int_{\lambda_{\rm filter\,min}}^{\lambda_{\rm filter\,max}} F_\lambda d\lambda,$$

  where F_λ is the monochromatic flux, and $\lambda_{\rm filter\,min}$ and $\lambda_{\rm filter\,max}$ are the wavelengths at the ends of the filter's wavelength range. For the UBV filters, the relevant integrals are

$$F_U = \int_{331\,\rm nm}^{398\,\rm nm} \pi B_\lambda d\lambda,$$

$$F_B = \int_{391\,\rm nm}^{489\,\rm nm} \pi B_\lambda d\lambda,$$

  and

$$F_V = \int_{505.5\,\rm nm}^{594.5\,\rm nm} \pi B_\lambda d\lambda.$$

  Because the wavelength ranges in each integral are relatively small, we can approximate each integral as

$$F_{\rm filter} \approx \pi B_{\lambda\,\rm center} \Delta\lambda_{\rm filter},$$

  where $\lambda\,$center is the central wavelength of the filter and $\Delta\lambda_{\rm filter}$ is its bandwidth. This approximation is used in Example 3.6.2 in the text; using the notation employed there, we have

$$F_U \approx \pi B_{365} \Delta\lambda_U = 6.6 \times 10^7\ \text{W m}^{-2},$$

$$F_B \approx \pi B_{440} \Delta\lambda_B = 7.6 \times 10^7 \text{ W m}^{-2},$$

and

$$F_V \approx \pi B_{550} \Delta\lambda_V = 4.6 \times 10^7 \text{ W m}^{-2}.$$

The total flux through the B filter is greater than the fluxes through the U or V filters, so Vega appears brightest through the B filter.

If we want to be careful, we can check our approximation by calculating the values of B_λ at the endpoints of each filter's wavelength range and checking that B_λ does not vary significantly over each wavelength range, so that our approximation of taking a constant value for B_λ corresponding to the central value is good.

- Q3.19

  $V = 1.62$ and $T = 22,000$ K.

  (a). $C_{U-B} = -0.87$ and $C_{B-V} = 0.65$, so

$$\begin{aligned} U - B &= -2.5 \log_{10}\left(\frac{F_U}{F_B}\right) + C_{U-B} \approx -2.5 \log_{10}\left(\frac{B_{365}\Delta\lambda_U}{B_{440}\Delta\lambda_B}\right) + C_{U-B} \\ &= -2.5 \log_{10}\left(\frac{2.5 \times 10^8}{2.1 \times 10^8}\right) + C_{U-B} = -1.07 \end{aligned}$$

  and

$$\begin{aligned} B - V &= -2.5 \log_{10}\left(\frac{F_B}{F_V}\right) + C_{B-V} \approx -2.5 \log_{10}\left(\frac{B_{440}\Delta\lambda_B}{B_{550}\Delta\lambda_V}\right) + C_{B-V} \\ &= -2.5 \log_{10}\left(\frac{2.1 \times 10^8}{9.2 \times 10^7}\right) + C_{B-V} = -0.24 \end{aligned}$$

  The measured values for the color indices are both more positive than the calculated values, probably due to reddening by interstellar dust.

  (b). $p = 0.00464''$, so the distance to Shaula is $r = 215$ pc. The absolute visual magnitude M_V is related to the apparent visual magnitude V by $V - M_V = 5 \log_{10}(r/10\,\text{pc})$. Plugging in V and r gives $M_V = -5.04$.