Homework Assignment 1 -Solutions

• Q1.1

First consider a superior planet. Let f be the fraction of its orbit the planet has traversed since its last conjunction with the Earth:

$$f = \frac{\Delta t}{P},$$

where Δt is the time elapsed since the last conjunction, and P the planet's sidereal period. Likewise, for the Earth,

$$f_{\oplus} = \frac{\Delta t}{P_{\oplus}}$$

At the next conjunction the Earth will pass the the planet on the inside, having completed one more orbit than the planet; that is, $f_{\oplus} = f + 1$, or

$$\frac{\Delta t}{P_{\oplus}} = \frac{\Delta t}{P} + 1$$

Rearranging,

$$\frac{1}{\Delta t} = \frac{1}{P_{\oplus}} - \frac{1}{P}$$

But the time from one conjunction to the next is just the synodic period. Thus, setting $S = \Delta t$,

$$\frac{1}{S} = \frac{1}{P_{\oplus}} - \frac{1}{P}$$

which is the desired result.

Similar reasoning is used for inferior planets; however, at the next conjunction it is the planet which passes the Earth on the inside. Therefore, $f = f_{\oplus} + 1$, leading to

$$\frac{\Delta t}{P} = \frac{\Delta t}{P_{\oplus}} + 1$$

Following through with the algebra gives

$$\frac{1}{S} = \frac{1}{\Delta t} = \frac{1}{P} - \frac{1}{P_\oplus}$$

which is the desired result.

• Q1.3

(a). Applying the above expressions to Venus (inferior), with $P_{\oplus} = 365.25 \,\mathrm{d}$,

$$\frac{1}{583.9\,\mathrm{d}} = \frac{1}{P} - \frac{1}{365.25\,\mathrm{d}} \implies P = 224.7\,\mathrm{d}$$

Likewise, for Mars (superior),

$$\frac{1}{779.9\,\mathrm{d}} = \frac{1}{365.25\,\mathrm{d}} - \frac{1}{P} \implies P = 687.0\,\mathrm{d}$$

(b). Neptune (or Pluto, if still counted as a planet). This is because it is the slowest to orbit the Sun, and therefore the Earth can catch up with it in the shortest amount of time (relative to the other superior planets).

• Q1.4

At the vernal equinox, $\alpha_{\odot} = 0^{h}0^{m}$ and $\delta_{\odot} = 0^{\circ}0'$ (from the definition of the vernal equinox).

- At the summer solstice, $\alpha_{\odot} = 6^{h}0^{m}$ (a quarter revolution around from the vernal equinox) and $\delta_{\odot} = +23^{\circ}30'$ (northern hemisphere, equal to the Earth's axial tilt).
- At the autumnal equinox, $\alpha_{\odot} = 12^{h}0^{m}$ (a half revolution around from the vernal equinox) and $\delta_{\odot} = 0^{\circ}0'$ (on the celestial equator)
- At the winter solstice, $\alpha_{\odot} = 18^{h}0^{m}$ (three quarters of a revolution around from the vernal equinox) and $\delta_{\odot} = -23^{\circ}30'$ (southern hemisphere, equal to the Earth's axial tilt).

These answers can also be trivially read off from Fig. 1.11 of Ostlie & Carroll.

• Q1.5

- (a). The first day of summer is the summer solstice, when the Sun is at a declination $+23^{\circ}30'$ above the celestial equator (see Fig. 1.12). For an observer at 42° N latitude, the Sun will appear $42^{\circ} 23^{\circ}30' = 18^{\circ}30'$ from the zenith, corresponding to an altitude of $71^{\circ}30'$ above the horizon.
- (b). The first day of winter is the winter solstice, when the Sun is at a declination $-23^{\circ}30'$ below the celestial equator. The Sun will be highest in the sky when it crosses the meridian; at this point, the zenith distance is $42^{\circ} (-23^{\circ}30') = 65^{\circ}30'$, corresponding to an altitude of $24^{\circ}30'$.

(a). From eqn. (1.8) of Ostlie & Carroll:

$$(\Delta\theta)^2 = (\Delta\alpha\cos\delta)^2 + (\Delta\delta)^2.$$

For α Centauri and Proxima Centauri, $\Delta \alpha = 0^{h}9^{m}53.55^{s} = 2.473^{\circ}$ and $\Delta \delta = -1^{\circ}50'43.8'' = 1.846^{\circ}$. Plugging these values into the above formula, plus the declination of α Centauri (it doesn't matter much which star we choose), gives $\Delta \theta = 2.20^{\circ}$.

(b). Using the small-angle approximation, the distance r, separation Δd and angular separation $\Delta \theta$ are related by $\Delta d = r\Delta \theta$, where $\Delta \theta$ must be measured in radians. With $r = 4.0 \times 10^{16}$ m and $\Delta \theta = 0.0385$ rad, we find that $\Delta d = 1.54 \times 10^{15}$ m.

• Q1.9

(a). For the epoch J2010.0, the time lapse in eqn. (1.4) of Ostlie & Carroll is T = 0.1. Therefore, using eqns. (1.2) and (1.3), we find that

$$\Delta \alpha = 0.1936^{\circ} = 0^{h} 0^{m} 46.46^{s}, \qquad \Delta \delta = -0.04421^{\circ} = -0^{\circ} 2' 39.2''.$$

This gives the J2010.0 coordinates as

$$\alpha = 14^{h}30^{m}29.41^{s}, \qquad \delta = -62^{\circ}43'25.3''.$$

(b). Over the 10 years between J2000.0 and J2010.0, the total change in position is $\Delta \theta = 38.4''$. With a position angle $\phi = 282^{\circ}$, eqns. (1.6) and (1.7) give

$$\Delta \alpha = -0.02273^{\circ} = -0^{h} 0^{m} 5.46^{s}, \qquad \Delta \delta = 0.002218^{\circ} = 0^{\circ} 0' 8.0''.$$

(c). Precession makes by far the largest contribution to changes in the coordinates of Proxima Centauri.