Assignment 6 — Solutions[$Revision$: 1.2]

***EZ-Web* Task 1** The following table gives the total luminosity produced by the PP chain and CNO cycle, for solar-metallicity, ZAMS *EZ-Web* models spanning the mass range $1.8\,M_\odot$–$2.2\,M_\odot$:

$M/M_\odot$	$L_{\rm PP}/L_\odot$	$L_{\rm CNO}/L_\odot$
1.8	6.98	3.49
1.9	7.84	5.41
2.0	8.62	8.04
2.1	9.33	11.5
2.2	9.96	16.0

From this table, we see that the PP chain and CNO cycle generate approximately equal amounts of energy for ZAMS stars of mass $M \approx 2.0\,M_\odot$.

***EZ-Web* Task 2** The following table gives the maximum value (over the stars' whole lifetime) of the luminosity ratio between hydrogen and helium burning, for solar metallicity *EZ-Web* models spanning the mass range $0.5\,M_\odot$–$1.0\,M_\odot$.

$M/M_\odot$	$\max[L_{\rm 3He}/(L_{\rm PP}+L_{\rm CNO})]$
0.5	8.05×10^{-6}
0.6	5.27×10^{-3}
0.7	216
0.8	79.3
0.9	47.4
1.0	86.8

All models with $M \geq 0.7\,M_\odot$ clearly undergo helium ignition in a helium flash (where the helium-burning luminosity greatly exceeds the hydrogen-burning luminosity). Thus, the mass threshold for helium burning is $M \approx 0.7\,M_\odot$.

***EZ-Web* Task 3** Figs. 1 and 2 plot ∇ and $\nabla_{\rm ad}$ as function of radius for $1\,M_\odot$ and $10\,M_\odot$ solar-metallicity *EZ-Web* models half-way through their main-sequence evolution. For the $1\,M_\odot$ star the half-way model comes from structure file number 64, and for the $10\,M_\odot$ star it comes from file number 70. (These structure files are the ones that are closest to having a core hydrogen abundance $X_{\rm c} = 0.35$.)

The figures show that the $1\,M_\odot$ star has a radiative core and a convective envelope; whereas, the converse is true for the $10\,M_\odot$ star, which has a convective core and a radiative envelope. The $10\,M_\odot$ star also has a narrow convection zone near the surface; the convection in this zone is inefficient, since ∇ is rather larger than $\nabla_{\rm ad}$ there.

***EZ-Web* Task 4** Fig. 3 plots the hydrogen mass fraction X as a function of fractional radius r/R, for both models from the previous question. The $1\,M_\odot$ model shows a gradual variation between the abundance at the center ($X \approx 0.35$) and the envelope abundance $X = 0.7$. For the $10\,M_\odot$ model, however, the abundance profile is flat in the core (again, with $X \approx 0.35$), but then rises abruptly to the envelope value at $r/R \approx 0.2$.

The difference between these abundance profiles is a result of the presence or absence of mixing. In the $10\,M_\odot$ model, convection in the core (as shown in the previous question) produces rapid mixing, such that the abundance profile remains flat (uniform composition) throughout the core. This does not happen in the $1\,M_\odot$ model, which has a radiative core; instead, the abundance profile simply reflects the fact that hydrogen is consumed more rapidly as the center of the star is approached.

***EZ-Web* Task 5** Fig. 4 plots the evolution of the $1\,M_\odot$ model in the HR diagram, showing phases of significant mass loss.

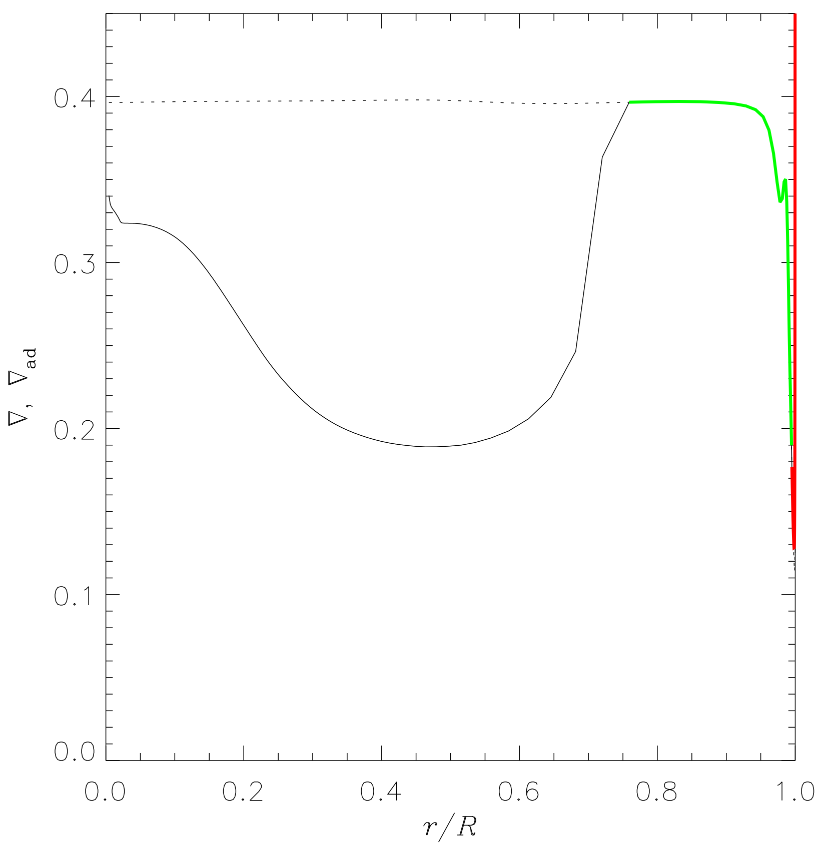

Figure 1: The temperature gradients ∇ (solid) and $\nabla_{\rm ad}$ (dotted), plotted as a function of radius for the $1\,M_{\odot}$ model. The ∇ curve is colored green where efficient convection is occurring ($\nabla \approx \nabla_{\rm ad}$), and red where inefficient convection is occurring ($\nabla > \nabla_{\rm ad}$).

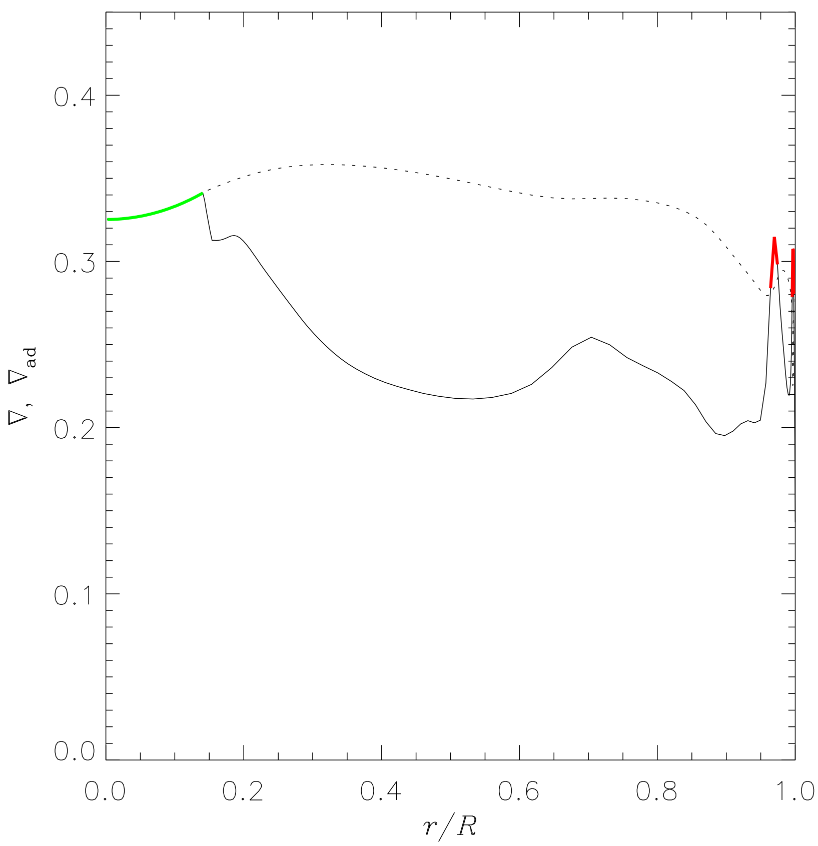

Figure 2: As in Fig. 1, but now for the $10\,M_{\odot}$ model.

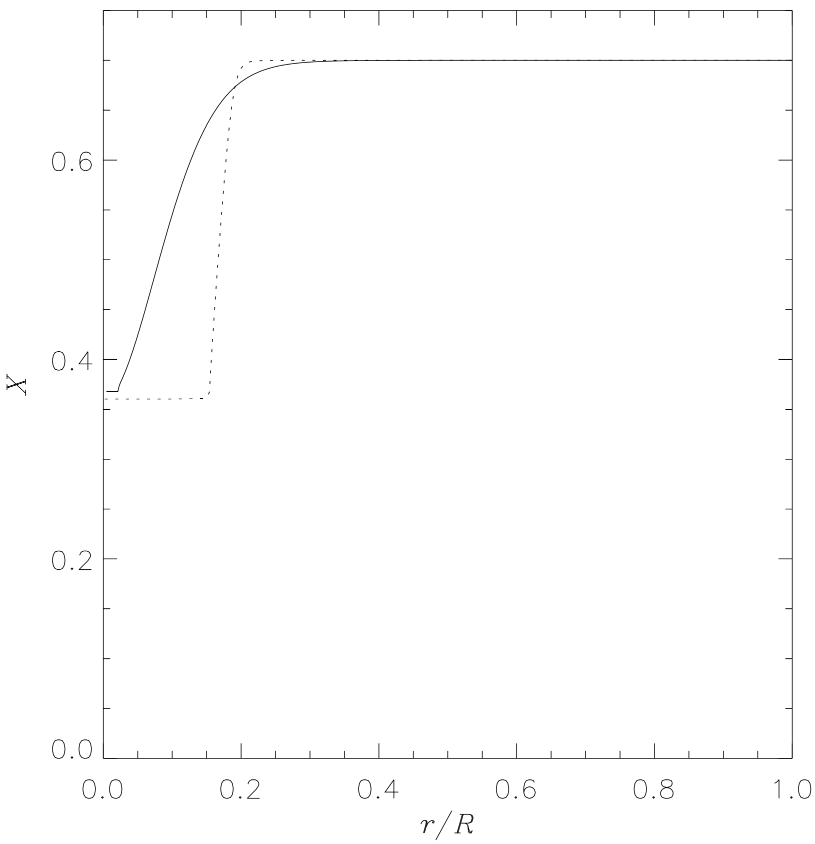

Figure 3: The hydrogen mass fraction X, plotted as a function of radius for the $1\,M_\odot$ (solid) and $10\,M_\odot$ (dotted) models.

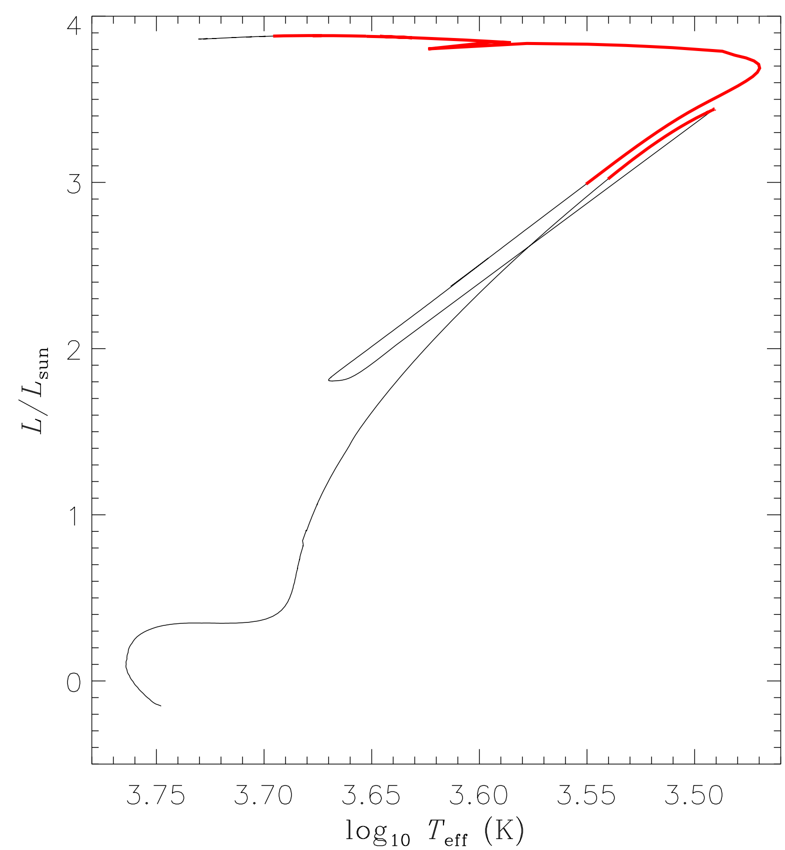

Figure 4: The evolutionary trajectory of the $1\,M_{\odot}$ model in the HR diagram. Phases of significant mass loss are shown in red, and occur on the RGB and AGB.

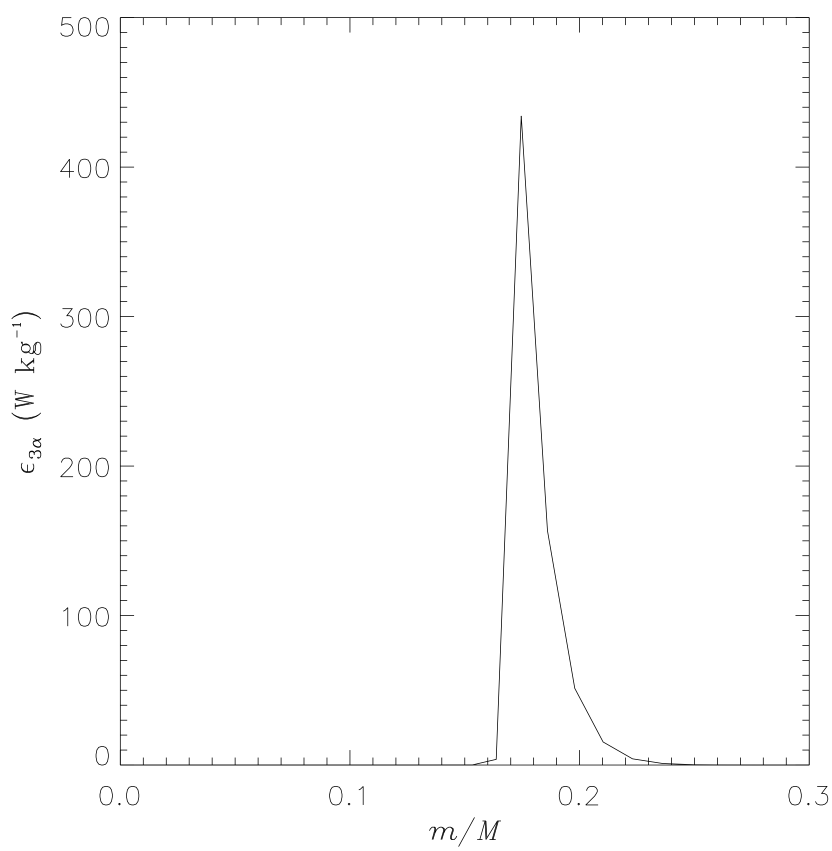

Figure 5: The triple-alpha energy generation rate $\epsilon_{3\alpha}$ plotted as a function of fractional mass M_r/M over the inner regions of the helium-flashing $1\,M_\odot$ model.

***EZ-Web* Task 6**

1. Fig. 5 plots the triple-alpha energy generation rate $\epsilon_{3\alpha}$ as a function of fractional mass M_r/M for the helium-flashing $1\,M_\odot$ model (model number 490). The sharp peak at $M_r/M \approx 0.17$ clearly indicates off-center helium ignition.
2. Fig. 6 plots the temperature as a function of fractional mass. The peak in $\epsilon_{3\alpha}$ coincides with a local temperature maximum at the same location, $M_r/M \approx 0.17$.
3. Fig. 7 plots the neutrino energy loss rate $\epsilon_{\rm nu}$ as a function of fractional mass. Besides the sharp peak caused by the helium ignition at $M_r/M \approx 0.17$, a non-negligible energy loss can be seen in the center of the core. This loss cools the center, causing a temperature inversion and a consequent off-center temperature maximum.

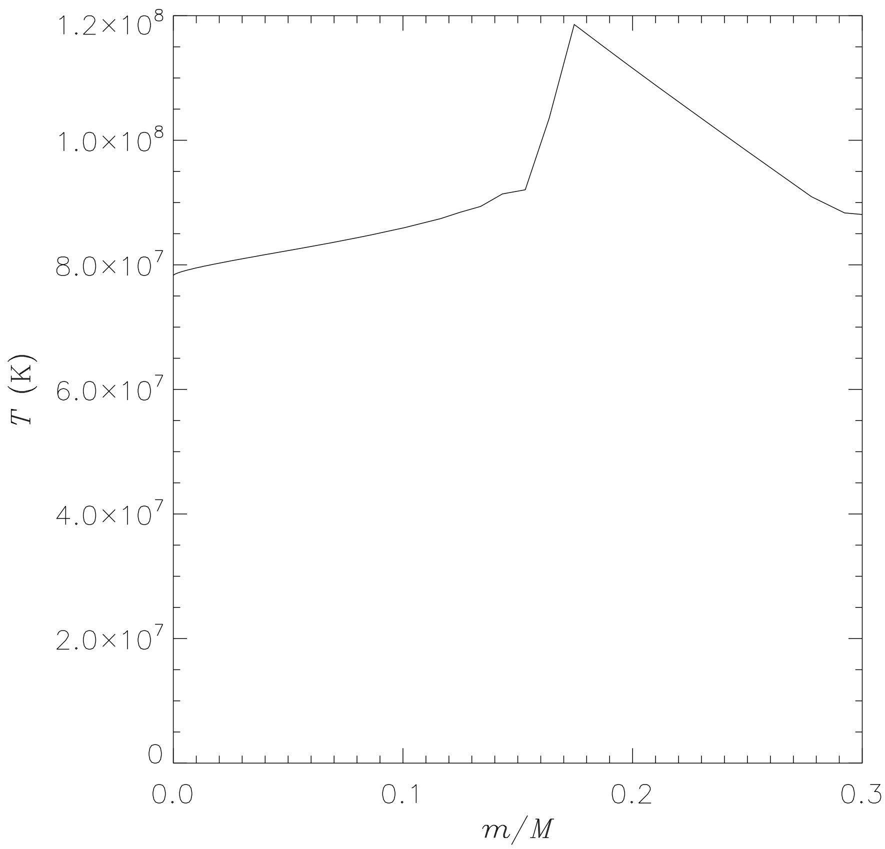

Figure 6: The temperature T plotted as a function of fractional mass M_r/M over the inner regions of the helium-flashing $1\,M_\odot$ model.

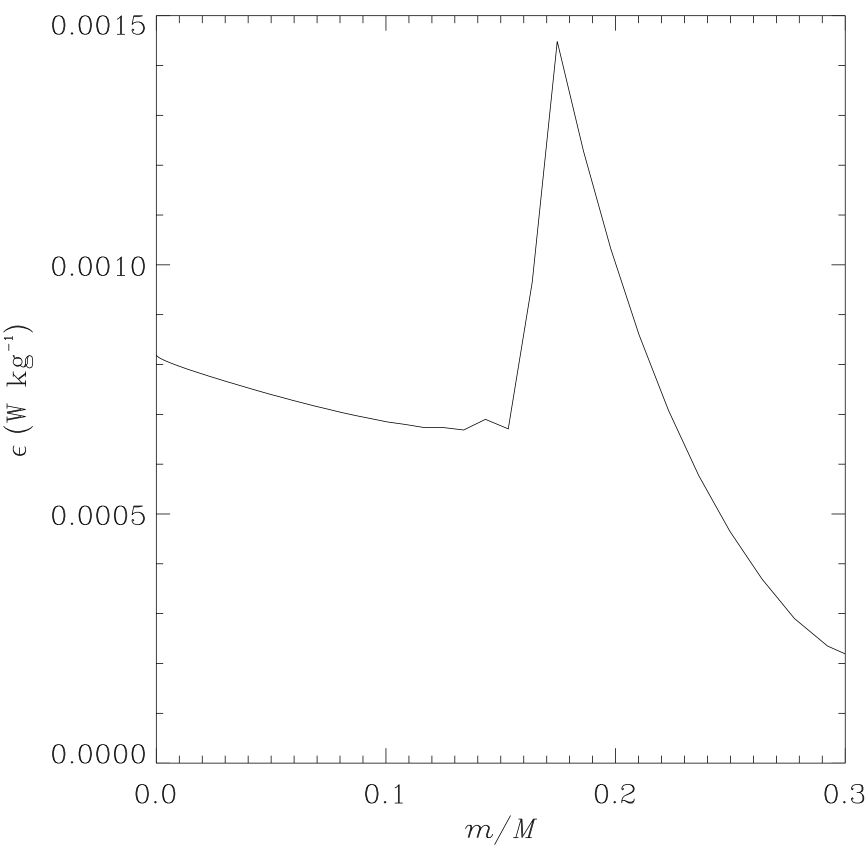

Figure 7: The neutrino energy loss rate ϵ_ν plotted as a function of fractional mass M_r/M over the inner regions of the helium-flashing $1\,M_\odot$ model.