# Assignment 6 — due Wednesday December 2$^{\text{nd}}$ $[Revision : 1.2]$

This assignment comprises the following *EZ-Web* calculation tasks:

1. Use *EZ-Web* to establish the approximate mass (to within $0.1\,M_\odot$) at which ZAMS, solar-metallicity stars generate equal amounts of nuclear energy from the PP chain and the CNO cycle. (Above this mass, more energy is produced by the CNO cycle, below this mass more is produced via the PP chain). Explain how you arrive at your result.

2. Use *EZ-Web* to establish the approximate mass threshold (to within $0.1\,M_\odot$) for a solar-metallicity star to burn Helium at some point during its lifetime. Explain how you arrive at your result.

3. The criterion for the onset of convection, given by eqn. (10.95) of Ostlie & Carroll, is often expressed in terms of the quantities $\nabla$ and $\nabla_{\rm ad}$ (pronounced 'nabla' and 'nabla-ad'). The first of these,
$$\nabla \equiv \frac{d\ln T}{d\ln P}$$
is the (logarithmic) temperature gradient measured with pressure as the independent variable — a positive, dimensionless quantity that depends on the temperature and pressure gradients at each point in the star. Conversely,
$$\nabla_{\rm ad} \equiv \left.\frac{d\ln T}{d\ln P}\right|_{\rm ad} = \frac{\gamma - 1}{\gamma}$$
(where the second equality holds for an ideal gas) is a measure of the thermodynamic properties of the gas during adiabatic changes; again, it is positive and dimensionless.

   When $\nabla < \nabla_{ad}$, the temperature gradient is sub-adiabatic, and no convection will occur. However, if this inequality is not satisfied, then convection *will* occur. If the convection is very efficient, then $\nabla = \nabla_{ad}$ (i.e, the temperature gradient is adiabatic); however, if the convection is inefficient, then $\nabla \gg \nabla_{ad}$ (i.e., the temperature gradient is super-adibatic).

   Use *EZ-Web* to construct detailed structure files for $1\,M_\odot$ and $10\,M_\odot$models that are approximately half-way through their main-sequence lifetimes (as determined from a core hydrogen mass fraction $X_{\rm c} \approx 0.35$). For each model, plot a graph showing both $\nabla$ and $\nabla_{\rm ad}$ as a function of fractional radius $r/R$. Identify the regions where convection is occurring, and indicate whether the convection is efficient or not.

4. For both models in the previous question, plot (on the same graph) the hydrogen mass fraction $X$ as a function of fractional radius $r/R$. Describe and explain the differences between the composition distributions in the two models.

5. Use *EZ-Web* to construct a summary file for the entire evolution of a $1\,M_\odot$, solar-metallicity model. Plot the evolutionary trajectory of the star in the HR diagram. On this diagram, mark the phases of the star's evolution where it is undergoing significant mass-loss.

6. Use *EZ-Web* to construct detailed structure files for a $1\,M_\odot$, solar-metallicity model up to the commencement of helium burning. Determine the specific file where the helium-burning luminosity first exceeds the hydrogen-burning luminosity (i.e., $L_{3\alpha} > L_{PP} + L_{CNO}$). Using the data in this file, plot appropriate graphs to confirm:

   (a) that the helium ignition is occurring off-center;

   (b) that the off-center ignition is a consequence of a local temperature maximum at $M_r/M \approx 0.17$; and

   (c) that the reason for this local temperature maximum is slow cooling of the central core by neutrino losses.