Assignment 4 — due Friday October 30$^{\mathrm{th}}$ [$Revision$: 1.1]

This assignment comprises questions **9.7**, **9.10**, **9.18**, **9.19**, **9.21** and **9.22** from Chapter 9 of Ostlie & Carroll, plus the following *EZ-Web* calculation tasks:

1. - Use *EZ-Web* to calculate a structure model for a $20\,M_\odot$ star that is 1 million years old.
   - For this structure model, plot the Rosseland mean opacity $\bar{\kappa}$ as a function of $\log T$.
   - On your diagram, clearly mark the opacity peak around $T \approx 200{,}000\,\mathrm{K}$. This peak, the so-called 'iron bump', is caused by millions of bound-bound transitions of iron and nickel, and is responsible for driving pulsations (periodic variations in size, shape and brightness) of massive stars.
   - On the same diagram, plot the Thomson scattering opacity for a fully-ionized gas,

     $$\kappa_{\mathrm{es}} = 0.02(1 + X)\,\mathrm{m^2 kg^{-1}},$$

     where X is the hydrogen mass fraction.
   - Your plot should indicate that $\bar{\kappa}$ is smaller than κ_{es} in the very innermost parts of the star. This is due to relativistic departures from the Thomson-scattering formula above, which arise when photon energies become non-negligible ($\gtrsim 2\%$) compared to the electron rest-mass energy $m_{\mathrm{e}}c^2$. To confirm this explanation, make a separate plot of $\epsilon_{\mathrm{ph}}/m_{\mathrm{e}}c^2$ as a function of $\log T$, where ϵ_{ph} is the energy of photons having wavelengths at the Wien peak of the Planck function $B_\lambda(T)$.

2. - Use *EZ-Web* to calculate zero-age main sequence (ZAMS) models having the following masses: $M = 0.2, 0.5, 1, 2, 5, 10, 20$ and $50\,M_\odot$. This will involve multiple submissions to the *EZ-Web* server, each with 'Maximum Age' set to zero but 'Maximum Number of Steps' set to one. Do *not* check 'Create Detailed Structure Files', since you will only need to use the summary files produced by *EZ-Web*. These files will each contain a single line of data, giving the general properties of the ZAMS model (see the *EZ-Web* page for a description of the summary-file format).
   - Using these ZAMS models, create a theoretical Hertzsprung-Russell diagram.
   - Also create the following two plots:
     - $\log L/L_\odot$ as a function of $\log M/M_\odot$
     - $\log R/R_\odot$ as a function of $\log M/M_\odot$

     For each plot, fit a straight line through the data, and from the slope of the line detemine the exponents α and β in the mass-luminosity relation $L/L_\odot = (M/M_\odot)^\alpha$ and the mass-radius relation $R/R_\odot = (M/M_\odot)^\beta$.
   - Assume that over their lifetimes all stars convert 0.1% of their rest mass into energy. Use your fitted mass-luminosity relation to derive an expression for the lifetime τ of a stars as a function of their mass (you should assume the luminosity remains constant throughout stars' lives). Use your expression to estimate how much shorter the lifetime of a $50\,M_\odot$ star is, compared to the Sun.
   - Use your fitted mass-radius and mass-luminosity relations to derive a mass-effective temperature relation. Based on this relation, estimate the masses of the following main-sequence stars:

     (a) σ Ori A ($T_{\mathrm{eff}} = 32{,}000\,\mathrm{K}$)
     (b) Regulus ($T_{\mathrm{eff}} = 10{,}300\,\mathrm{K}$)
     (c) Procyon ($T_{\mathrm{eff}} = 6{,}650\,\mathrm{K}$)
     (d) ϵ Indii ($T_{\mathrm{eff}} = 4{,}280\,\mathrm{K}$)