Mid-Term Exam – Solutions [$Revision$: 1.2]

1. A G star lives longer, because its luminosity is significantly less than that of a B star, and so it burns through its nuclear fuel much more slowly.

   [2 points]

2. In O stars, the atmospheric temperature is so high that all of the hydrogen is ionized; therefore, there are no neutral atoms to absorb radiation. In K stars, the atmospheric temperature is too low for there to be any hydrogen atoms in the $n = 2$ state; therefore, no Balmer absorption transitions are possible.

   [2 points]

3. (a) Electron scattering
   (b) Bound-bound absorption
   (c) Bound-free absorption

   [3 points]

4. The flux through the atmosphere; the effective temperature is the temperature of the blackbody having the same flux as in the atmosphere (ie., $F = \sigma T_{\rm eff}^4$).

   [1 point]

5. (a) The distance is given by the distance modulus equation:

$$V - M_V = 5 \log_{10} \left(\frac{d}{10\,\mathrm{pc}} \right)$$

   Solving, $d = 13\,\mathrm{pc}$, and so the parallax is $p'' = 1/d = 0.077''$.

   [2 points]

   (b) The bolometric luminosity is calculated using the inverse-square law

$$L_{\rm bol} = 4\pi d^2 F_{\rm bol}$$

   Solving, $L_{\rm bol} = 80\,L_\odot$.

   [2 points]

   (c) Wien's law states that

$$\lambda_{\rm max} T = 0.3\,\mathrm{cm}$$

   Solving, with $T = 5,300\,\mathrm{K}$, $\lambda_{\rm max} = 5.67 \times 10^{-5}\,\mathrm{cm} = 5670\,\mathrm{Å}$.

   [2 points]

   (d) The radius is calculated from the luminosity and effective temperature:

$$L = 4\pi R^2 \sigma T_{\rm eff}^4$$

   Solving, $R = 10.6\,R_\odot$. Also

$$g = \frac{GM}{R^2};$$

   solving, $M = 2.85\,M_\odot$

   [3 points]

   (e) No; main-sequence stars follow the relation $R/R_\odot \approx (M/M_\odot)^{0.9}$. The radius is much bigger than predicted by this relation, and so Capella must be a giant.

   [2 points]

(f) Kepler's third law gives

$$M_1 + M_2 = \frac{4\pi^2}{GP^2} a^3$$

Solving, with $M_1 = 2.85\,M_\odot$, $M_2 = 2.08\,M_\odot$.

[2 points]

6. The mean intensity is

$$\langle I \rangle = \frac{1}{2}\int_{-1}^{1} I\,\mathrm{d}\mu = \frac{1}{2}(I_+ + I_-)$$

and the radiation pressure is

$$P_{\mathrm{rad}} = \frac{2\pi}{c}\int_{-1}^{1} I\mu^2\,\mathrm{d}\mu = \frac{2\pi}{c}\left(I_+\frac{1}{3} + I_-\frac{1}{3}\right)$$

Comparing the two expressions,

$$P_{\mathrm{rad}} = \frac{2\pi}{3c}\langle I \rangle$$

which is the Eddington approximation.

[5 points]