38 — White Dwarfs & Neutron Stars [$Revision$: 1.1]

- Sirius B
  - Unseen companion of Sirius discovered by Bessel from star's reflex motion (arising from motion around center of mass)
  - Puzzle: if companion is massive enough to cause reflex motion, why can't we see it?
  - Companion (Sirius B) finally observed by Alvan Clark; found to have much smaller luminosity than Sirius A (compare $L_{\rm A} \approx 23\,L_\odot$, $L_{\rm B} \approx 0.03\,L_\odot$)
  - Spectrum shows very broad hydrogen lines (due to high surface gravity), but is otherwise featureless. Effective temperature $\approx 27,000\,{\rm K}$
  - Apply Stefan-Boltzmann formula:

    $$R^2 = \frac{L}{4\pi\sigma T_{\rm eff}^4}$$

    gives radius as $R \approx 5500\,{\rm km}$ — about radius of Earth
  - This tiny radius explains high surface gravity; with $M \approx 1\,M_\odot$, $g \approx 4 \times 10^8\,cms^{-2}$ — over 100,000 times what it is on Earth
  - Initially, these findings dismissed as being absurd; however, it is difficult to argue with the observations

- White Dwarf types
  - Spectral types assigned to white dwarfs all begin with 'D' (for dwarf)
  - DA dwarfs show only pressure-broadened hydrogen lines
  - DB dwarfs show only helium lines
  - DC dwarfs show no lines at all (C for continuum)
  - All characterized by very broad lines

- Physical conditions
  - Use hydrostatic equilibrium to estimate central conditions:

    $$\frac{{\rm d}P}{{\rm d}r} = -\frac{GM_r}{r^2}\rho$$

    and so

    $$\frac{P_{\rm c}}{R} \approx \frac{GM}{R^2}\rho_{\rm c} \approx \frac{GM}{R^2}\frac{M}{R^3}$$

    Using values for Sirius B, $¶_{\rm c} \approx 3 \times 10^{24}\,{\rm dyncm^{-2}}$, and $\rho_{\rm c} \approx 1 \times 10^7\,{\rm gcm^{-3}}$ (compare $\rho \approx 1\,{\rm gcm^{-3}}$ for water)
  - Assume ideal gas (wrong!) with $\mu \approx 1$; then $T_{\rm c} \approx 3 \times 10^9\,{\rm K}$
  - Recall: condition for degeneracy (in SI units):

    $$\frac{T}{\rho^{2/3}} < 1261\,{\rm K\,m^2\,kg^{-2/3}}$$

    Plug in numbers above gives $T_{\rm c}/\rho_{\rm c}^{2/3} \approx 571\,{\rm K\,m^2\,kg^{-2/3}}$ — so gas must be degenerate
  - In fact, using proper degenerate equation of state, core temperature is $\approx 10^8\,{\rm K}$

  - At such high temperatures, any hydrogen and helium in the stellar core would undergo fusion, and luminosity would be much higher than it is; therefore, white-dwarf cores are **devoid of hydrogen and helium**
  - In fact, core is composed mostly of carbon and oxygen, the ashes of prior helium burning on the horizontal branch and AGB

- Energy transport
  - Conduction is usually unimportant in main-sequence stars, but becomes important in white dwarfs
  - To include conduction, write generalized diffusion equation as

$$F_{\mathrm{rad+cond}} = -\frac{4acT^3}{3\kappa_{\mathrm{rad+cond}}\rho}\frac{\mathrm{d}T}{\mathrm{d}r}$$

    where a generalized opacity is defined

$$\frac{1}{\kappa_{\mathrm{rad+cond}}} = \frac{1}{\kappa_{\mathrm{rad}}} + \frac{1}{\kappa_{\mathrm{cond}}}$$

    κ_{rad} is ordinary radiative opacity; κ_{cond} is effective conductive opacity, and can be written as

$$\frac{1}{\kappa_{\mathrm{cond}}} = v_{\mathrm{e}}\langle s\rangle_{\mathrm{e}}$$

    where v_{e} is electron velocity, and $\langle s\rangle_{\mathrm{e}}$ is electron mean free path
  - In white dwarfs, degeneracy of gas tends to reduce scattering of electrons off other particles (because there are no free states to scatter into)
  - As a result, electron mean free path becomes large, conductive opacity becomes small, and electrons can efficiently transport energy
  - Due to efficient conduction by degenerate electrons, white dwarfs are largely isothermal
  - Surface layers are non-degenerate, however; composed of a mixture of hydrogen, helium and metals, typically with strong convection
  - Differential settling of elements responsible for different spectral types of dwarfs; e.g., DA dwarfs do contain helium, but it has settled out of the atmosphere

- The Chandrasekhar limit
  - Take central pressure expression from above:

$$P_{\mathrm{c}} \approx \frac{GM^2}{R^4}$$

    This must equal to central pressure supplied by non-relativistic degenerate electrons:

$$P_{\mathrm{c}} = K\rho^{5/3} \approx K\frac{M^{5/3}}{R^5}$$

    (where K is a constant depending only on composition of gas). So,

$$\frac{GM^2}{R^4} \approx K\frac{M^{5/3}}{R^5}$$

    or

$$R \approx K'M^{-1/3}$$

    where K' is another constant depending only on composition. Note inverse relationship — radius gets smaller as mass increases

- Suppose we steadily add mass to white dwarf; radius will get smaller and smaller until star is tiny?
- In fact, electron velocities in core will eventually get so large (due to high pressures) that they are relativistic
- Use relativistic equation of state:

$$P_{\rm c} = K_{\rm rel}\rho^{5/3} \approx K_{\rm rel}\frac{M^{5/3}}{R^5}$$

and so

$$\frac{GM^2}{R^4} \approx K_{\rm rel}\frac{M^{4/3}}{R^4}$$

Wow: the radius totally cancels out, and

$$M = K'_{\rm rel}$$

i.e., the mass is completely fixed.
- Doing the mass properly, for a C-O core composition, this fixed mass comes out around $1.44\,M_\odot$ — this is the **Chandrasekhar limit**
- Any white dwarf below the limit will be non-relativistic in its outer parts
- A white dwarf *at* the limit will be relativistic everywhere
- No stable configuration is possible beyond the limit
- In fact, right on the limit the star is unstable — because the radius cancels out from above analysis, star does not 'know' what radius it should have

• Neutron Stars

- When a white dwarf exceeds Chandrasekhar limit, it will collapse to become a neutron star
- Neutron stars supported by neutron degeneracy pressure
- Upper limit to mass of neutron star is set by how much neutron pressure can support (**Tolman-Oppenheimer-Volkoff limit**) — between 1.5 and $3\,M_\odot$
- Physical state of neutron star is very 'exotic'; density is extremely high
- Going from surface layers down to core:
  * Outer crust of heavy nuclei interspersed with degenerate electrons; surface layers are $^{56}_{26}$Fe , deeper down are neutron-rich nuclei (which don't decay via β reactions due to lack of any free electron states). $\rho \approx 10^6 - 10^9\,{\rm g\,cm^{-3}}$
  * Inner crust is mixture of nuclei like $^{118}_{36}$Kr , superfluid of free neutrons (forced out of nuclei by **neutron drip**) and relativistic degenerate electrons; $\rho \approx 10^9 - 10^{11}\,{\rm gcm^{-3}}$
  * Interior is superfluid free neutrons, with some superfluid protons and electrons; $\rho \approx 10^{11} - 10^{15}\,{\rm g\,cm^{-3}}$
  * Center is exotic mixture of neutrons, some protons and electrons, and other particles such as pions

• White dwarf binaries

• Type Ia Supernovae