35 — Helium Burning and Beyond [$Revision$: 1.1]

- Low-mass stars
  - As low-mass stars evolve off main sequence, degeneracy pressure supports isothermal core and prevents Schönberg-Chandrasekhar contraction
  - So, as stars reach Hayashi line, core still 'cool', no Helium buring possible
  - As core mass increases, temperature in H-burning shell starts to climb:
    * To find shell temperature, apply simple two-point analysis, assuming that envelope has negligible density, temperature and pressure
    * Hydrostatic equilibrium has

      $$\frac{P_{\text{shell}}}{\Delta r_{\text{shell}}} \sim \rho_{\text{shell}} \frac{M_{\text{core}}}{R_{\text{core}}^2}$$

      where Δr_{shell} is the shell thickness
    * Apply ideal gas law in shell to eliminate pressure and density:

      $$\frac{T_{\text{shell}}}{\Delta r_{\text{shell}}} \sim \frac{M_{\text{core}}}{R_{\text{core}}^2}$$

    * Assume that shell thickness is some fraction q of core radius:

      $$T_{\text{shell}} \sim \frac{M_{\text{core}} q}{R_{\text{core}}}$$

    * Treat core as non-relativisitic, degenerate gas; since $P \sim \rho^{5/3}$, core behaves as polytrope with $n = 3/2$
    * For polytropes, mass-radius relation is

      $$R \sim M^{\frac{1-n}{3-n}}$$

      and so in this case $R_{\text{core}} \sim M_{\text{core}}^{-1/3}$
    * Thus, eliminating R_{core},

      $$T_{\text{shell}} \sim M_{\text{core}}^{4/3} q$$

    * So, as core gets more massive, temperature in shell must increase
  - Increase in shell temperature implies a corresponding increase in luminosity. Since star on Hayashi line, this increase in luminisity appears almost vertical in HRD — the **Red Giant Branch** (RGB)
  - Convection zone dips deep enough to bring nucleosynthetic products (mainly Helium) to surface — **first dredge-up**
  - Hot shell warms up core until temperature eventually reaches point where Helium igition occurs ($T \approx 10^8$ K)
  - Typically, helium ignition not at $r = 0$ since central regions of core are cooled by neutrino losses
  - Helium ignition initially is run-away, because the core is denerate; for a few seconds, total luminosity reaches $\sim 10^{11} L_\odot$
  - Once temperature high enough for degeneracy to be lifted, core and envelope expand
  - Expansion cools off shell source, so that luminosity drops
  - Once steady, non-degenerate shell burning has set in, star settles onto **horizontal branch**

  - Horizontal branch is in some ways a **helium-burning main sequence**
  - Initial position on horizontal branch depends primarily on mass of star (more massive means closer to Hayashi line), and to some extent on composition
  - As core helium burning nears its end, stars evolve back toward the Hayashi line, and begin to climb the **Asymptotic Giant Branch** (this time powered by both hydrogen and helium shell sources)
  - Convection zone brings carbon and oxygen to surface — **second dredge-up**
  - Shell helium burning is unstable, switching on and off over timescale $\sim 10^5$ yr — **thermal pulses**
  - Core is now inert, made up of carbon and oxygen, and degenerate; soon, it will become a **white dwarf**

- Higher-mass stars
  - Schönberg-Chandrasekhar contraction serves to heat up core
  - So, soon after Hayashi line reached, helium ignition occurs in non-degenerate core
  - Expansion of shell source causes modest drop in luminosity
  - Star loops toward blue; multiple loops are possible during core helium burning, depending on details of convection
  - Eventually, as helium begins to run out, star again moves toward the Hayashi line and climbs the AGB