28 — Nuclear Reactions [$Revision$: 1.2]

- Proton-Proton Chain
  - Hydrogen is fused into helium in Sun via **PP Chain**
  - Net effect is to convert four hydrogen nuclei into one helium nucleus, neutrinos and radiation
  - Three variants of PP chain. All begin by forming ${}^{3}_{4}\mathrm{He}$:

$$ {}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{2}_{1}\mathrm{H} + e^{+} + \nu_{\mathrm{e}} $$

$$ {}^{2}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{H} + \gamma $$

    First reaction involves decay of proton into neutron — mediated by **weak nuclear force**. Slowest step in whole chain (takes on average 1 billion years for an individual ${}^{1}_{1}\mathrm{H}$!)
  - First reaction indirectly releases additional energy from positron annihilation:

$$ e^{+} + e^{-} \rightarrow 2\,\gamma $$

  - PP chain is completed via 3 different variants
  - PP I — two ${}^{3}_{2}\mathrm{He}$ combine:

$$ {}^{3}_{2}\mathrm{He} + {}^{3}_{2}\mathrm{He} \rightarrow {}^{4}_{2}\mathrm{He} + 2\,{}^{1}_{1}\mathrm{H} $$

    Happens in Sun 69% of time
  - PP II — ${}^{3}_{2}\mathrm{He}$ combines with ${}^{4}_{2}\mathrm{He}$:

$$ {}^{3}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{7}_{4}\mathrm{Be} + \gamma $$

$$ {}^{7}_{4}\mathrm{Be} + e^{-} \rightarrow {}^{7}_{3}\mathrm{Li} + \nu_{\mathrm{e}} $$

$$ {}^{7}_{3}\mathrm{Li} + {}^{1}_{1}\mathrm{H} \rightarrow 2\,{}^{4}_{2}\mathrm{He} $$

    Happens in Sun 31% of time
  - PP III — same as PP II, but beryllium combines with proton rather than electron

$$ {}^{7}_{4}\mathrm{Be} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{8}_{4}\mathrm{Be} + e^{+} + \nu_{\mathrm{e}} $$

$$ {}^{8}_{4}\mathrm{Be} \rightarrow 2\,{}^{4}_{2}\mathrm{He} $$

    Happens in Sun 0.09% of time; unimportant for overall energy production, but produces high-energy neutrinos
  - Total energy release by any PP chain is 26.7 MeV; but different amounts come out as γ rays and as neutrinos (for I, II & III, neutrinos carry 2%, 4% and 28% of energy)
  - Around 15 million Kelvin, overall energy generation rate by PP chain approximated as

$$ \epsilon_{\mathrm{pp}} \approx \epsilon'_{0,\mathrm{pp}}\,\rho\,X^{2}\,T_{6}^{4} $$

    where $T_6 = T/10^6\,\mathrm{K}$. Temperature dependence is modest

- CNO Cycle
  - Higher-mass stars have higher-temperature cores; dominant energy production is hydrogen burning via **CNO Cycle**

- Net effect is same as PP chain: convert four hydrogen nuclei into one helium nucleus, neutrinos and radiation
- However, reaction also involves carbon, nitrogen and oxygen as **catalysts** (assist reaction, but are not used up)
- Main cycle:

$$
\begin{aligned}
{}^{12}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H} &\rightarrow {}^{13}_{7}\mathrm{N} + \gamma \\
{}^{13}_{7}\mathrm{N} &\rightarrow {}^{13}_{6}\mathrm{C} + e^{+} + \nu_{\mathrm{e}} \\
{}^{13}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H} &\rightarrow {}^{14}_{7}\mathrm{N} + \gamma \\
{}^{14}_{7}\mathrm{N} + {}^{1}_{1}\mathrm{H} &\rightarrow {}^{15}_{8}\mathrm{O} + \gamma \\
{}^{15}_{8}\mathrm{O} &\rightarrow {}^{15}_{7}\mathrm{N} + e^{+} + \nu_{\mathrm{e}} \\
{}^{15}_{7}\mathrm{N} + {}^{1}_{1}\mathrm{H} &\rightarrow {}^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He}
\end{aligned}
$$

- There is also a secondary cycle where last reaction produces ${}^{16}_{8}\mathrm{O}$ and photon, initiating a subcycle that leads back to ${}^{14}_{7}\mathrm{N}$
- The step burning ${}^{14}_{7}\mathrm{N}$ is slowest, and so during operation C is depleted and N enriched. If cycle is interrupted (e.g., by mixing to surface), the depletion/enrichment is 'frozen in'
- Total energy release is same as PP chain, but neutrino emission different
- Around 15 million Kelvin, overall energy generatoin rate by CNO cycle approximated as

$$\epsilon_{\mathrm{CNO}} \approx \epsilon'_{0,\mathrm{CNO}}\, \rho\, X\, X_{\mathrm{CNO}}\, T_6^{19.9}$$

  Note much higher temperature sensitivity!

- Triple Alpha Process
  - After hydrogen burning, helium burns to carbon via *triple alpha process*:

$$
\begin{aligned}
{}^{4}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} &\leftrightarrow {}^{8}_{4}\mathrm{Be} \\
{}^{8}_{4}\mathrm{Be} + {}^{4}_{2}\mathrm{He} &\rightarrow {}^{12}_{6}\mathrm{C} + \gamma
\end{aligned}
$$

    (essentially, 3-body interaction, since Be nucleus is unstable)
  - Total energy release is 7.28 MeV — much smaller than hydrogen burning
  - Around 100 million Kelvin, overall energy generation rate by triple alpha approximated as

$$\epsilon_{3\alpha} \approx \epsilon'_{0,3\alpha}\, \rho^2\, Y^3\, T_8^{41.0}$$

    where $T_8 = T/10^8\,\mathrm{K}$. Note huge exponent!

- Other Reactions
  - In later evolutionary stages, elements can (possibly) burn all the way through to ${}^{56}_{26}\mathrm{Fe}$
  - Carbon burning via alpha capture:

$${}^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{16}_{8}\mathrm{O} + \gamma$$

  - Oxygen burning via alpha capture:

$${}^{16}_{8}\mathrm{O} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{20}_{10}\mathrm{Ne} + \gamma$$

  - Other reactions exist, some of which are endothermic