22 — Energy Conservation [$Revision$: 1.1]

- Energy conservation
  - To close mechanical equations of stellar structure (hydrostatic equilibrium, mass-radius relation), need to know T
  - Must therefore consider thermal energy distribution in star
  - Consider spherical shell inside star, extending from radius r to radius $r+\mathrm{d}r$ (i.e., thickness is $\mathrm{d}r$)
    * Energy flowing into shell through inner surface is $L_r(r)$ (here, L_r indicates **interior luminosity**, a function of radius inside star)
    * Energy flowing into shell through outer surface is $-L_r(r + \mathrm{d}r)$
    * If energy generation rate per unit mass is ϵ, energy generated in shell is $4\pi r^2 \rho \mathrm{d}r \epsilon$
    * But for **energy conservation**, no net energy can be created:

      $$L_r(r) - L_r(r + \mathrm{d}r) + 4\pi r^2 \rho \mathrm{d}r \epsilon = 0$$

      (strictly speaking, applies only in thermal equilibrium)
    * Divide through by $\mathrm{d}r$, take limit $\mathrm{d}r \to 0$:

      $$\frac{\mathrm{d}L_r}{\mathrm{d}r} = -4\pi r^2 \rho \epsilon$$

      This is **luminosity gradient equation** — one of the fundamental equations of stellar structure

- Energy transport
  - Can express interior luminosity in terms of interior flux,

    $$L_r = 4\pi r^2 F,$$

    allowing calculation of F throughout star once ϵ is known
  - F in turn can then be used to determine temperature distribution throughout star, by considering processes responsible for **energy transport**:
    * Radiation
    * Conduction (not too important)
    * Convection