17 — Line Profiles II $[Revision : 1.2]$

- The Voigt function
  - In most general case, shape of line profiles is governed by a combination of natural/pressure/collisional broadening and Doppler broadening:
$$\phi(\nu) = \sqrt{\frac{\mu}{2\pi kT}} \int_{-\infty}^{\infty} \mathrm{e}^{-\mu v^2/2kT} \frac{\gamma/4\pi^2}{(\nu + \nu v/c - \nu_0)^2 + (\gamma/4\pi)^2} \mathrm{d}v$$
  - Looks messy; but let
$$u = \frac{(\nu - \nu_0)}{\Delta\nu_{\mathrm{D}}}$$
(measure of distance from line center, in units of Doppler FWHM),
$$y = v\sqrt{\frac{\mu}{2kT}}$$
(measure of particle speed),
$$a = \frac{\gamma}{4\pi\Delta\nu_{\mathrm{D}}}$$
(measure of Lorenz width vs. Doppler width)
  - Then, profile is
$$\phi(\nu) = \frac{\sqrt{\pi}}{\Delta\nu_{\mathrm{D}}} H(a, u)$$
where
$$H(a, u) = \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{\mathrm{e}^{-y^2}}{(u - y)^2 + a^2} \mathrm{d}y$$
is the **Voigt function**
  - Voigt function looks like Gaussian for small u (i.e., near line center), and like Lorentzian for large u (i.e., out in line wings)

- Schuster-Schwarzschild model
  - Simple model for the formation of spectral lines
  - Assumes lines are formed by cold **reversing layer** sitting above hot background source of radiation
  - In reversing layer, $S_\lambda \to 0$, and so emergent intensity is
$$I_\lambda = I_0 \mathrm{e}^{-\tau_\lambda}$$
where I_0 is background continuum intensity, and τ_λ is optical thickness of reversing layer
  - Assume reversing layer is uniform with thickness Δ_z; then
$$\tau_\lambda = \kappa_\lambda \rho \Delta_z = n\sigma(\nu)\Delta_z = \sigma(\nu)N$$
where $N = n\Delta_z$ is **column density** — number of particles per unit area that participate in the bound-free absorption responsible for a given spectral line
  - Using general expression
$$\sigma(\nu) = \frac{e^2}{4\epsilon_0 mc} f\phi(\nu),$$
with $\phi(\nu)$ given by Voigt-function expression, result is
$$I_\lambda = I_0 \exp\left[\frac{e^2}{4\epsilon_0 mc} fN \frac{\sqrt{\pi}}{\Delta\nu_{\mathrm{D}}} H(a, u)\right]$$

- General behavior
  - When N is small, damping coefficient γ is also small (i.e., little collisional/pressure broadening), and so profile looks Gaussian
  - As N gets bigger, the optical depth at line center becomes so large that $I_\lambda = 0$ there (**saturation**)
  - As N continues to get bigger, amount of collisional/pressure broadening increases, and so Lorentzian-dominated **damping winds** appear

- The Curve of Growth
  - From Schuster-Schwarzschild model, the equivalent width W of a profile shows three distinct regimes:
    - $W \propto N$ for small column densities (profile not yet saturated)
    - $W \propto \sqrt{\ln N}$ for intermediate column densities (profile saturated)
    - $W \propto \sqrt{N}$ for large column densities (damping wings appear)
  - A plot of $\log_{10} W$ against $\log_{10} N$ shows these three regimes as the **curve of growth** (i.e., how the strength of a line grows as the column density increases)
  - Curve-of-growth analysis can be used to determine number of absorbing atoms of given type in atmosphere — useful for determining temperatures or abundances