16 — Line Profiles I [$Revision: 1.2$]

- A lot of information about a stellar atmosphere is contained in the shape of spectral lines — i.e., **line profiles**
- To characterize a line profile, first calculate normalized profile

$$\tilde{F_\lambda} = \frac{F_\lambda}{F_c}$$

  where F_c is **continuum flux**, measured away from line center
- Normalized profile has four principal measurable quantities:
  - Line center λ_0, where profile is deepest
  - Depth at line center

$$d = 1 - \tilde{F}_{\lambda 0} = \frac{F_c - F_{\lambda,0}}{F_c}$$

  - Full width at half maximum $\Delta\lambda$ (FWHM), defined so that at $\lambda = \lambda_0 \pm \Delta\lambda/2$,

$$\tilde{F_\lambda} = 1 - d/2$$

  - Equivalent width

$$W = \int_0^\infty \tilde{F_\lambda} \Delta\lambda$$

    (overall measure of line strength)
- Overall shape of line profile depends on run of bound-free opacity throughout atmosphere
- From Notes 15, general bound-free cross section written as

$$\sigma(\omega) = \frac{e^2}{4\epsilon_0 mc} f \phi(\omega)$$

  or, in terms of linear frequency $\nu = \omega/2\pi$,

$$\sigma(\nu) = \frac{e^2}{4\epsilon_0 mc} f \phi(\nu)$$

  where f is oscillator strength, and $\phi(\nu)$ is profile function normalized so that

$$\int_0^\infty \phi(\nu)\,\mathrm{d}\nu = 1$$

- Profile function depends on which **broadening mechanisms** are operative
- Natural (radiation) broadening
  - For isolated single atom/ion, profile function is **Lorentzian**, as in Notes 15:

$$\phi(\nu) = \phi_{\mathrm{lor}}(\nu) \equiv \frac{\gamma/4\pi^2}{(\nu - \nu_0)^2 + (\gamma/4\pi)^2}$$

  - FWHM of Lorentzian profile is

$$\Delta\lambda = \frac{\gamma}{2\pi}$$

- Damping coefficient γ found by applying Heisenberg's uncertainty principle to energy levels involved in transition:
  $$\Delta E = \frac{\hbar}{\Delta t}$$
  where δE is uncertainty in energy, and δt is time electron spends in level.
- Since frequency ν of transition is
  $$h\nu = E_a - E_b$$
  , uncertainty in frequency is
  $$\Delta\nu = \frac{1}{2\pi}\left(\frac{1}{\Delta t_a} + \frac{1}{\Delta t_b}\right)$$
  and in wavelength (with $\Delta\lambda = \lambda^2/c\Delta\nu$) is
  $$\frac{\Delta\lambda}{\lambda} = \frac{\lambda}{2\pi c}\left(\frac{1}{\Delta t_a} + \frac{1}{\Delta t_b}\right)$$

- Pressure/collisional broadening
  - In reality, atoms are not isolated, but influenced
    * By *direct collisions* with other particles
    * By electric field from nearby particles (**pressure**)
  - Both effects act together to produce a Lorentzian profile function ϕ_{lor}
  - In this case, broadening depends on time between collisions:
    $$\Delta t \approx \frac{\langle s\rangle_{\text{part}}}{v} = \frac{1}{n\sigma_{\text{part}}}\frac{1}{\sqrt{2kT/\mu}}$$
    where $\langle s\rangle_{\text{part}}$ is mean free path for particle-particle interactions, v is speed of particles, n is particle number density, and σ_{part} is cross section for particle-particle interactions
  - Hence
    $$\frac{\Delta\lambda}{\lambda} = \frac{\lambda}{2\pi c} n\sigma_{\text{part}}\sqrt{2kT/\mu}$$
  - Note: depends on particle number density
    * Higher density $\rightarrow$ broader lines
    * Reason for MK luminosity classes!
  - To combine pressure/collisional broadening with natural broadening, simply add widths:
    $$\frac{\Delta\lambda_{\text{p/c+nat}}}{=}\frac{\Delta\lambda_{\text{p/c}}}{\lambda} + \frac{\Delta\lambda_{\text{nat}}}{\lambda}$$

- Doppler broadening
  - So far, assumed that particles are stationary
  - However, in reality particles are jostling around due to **thermal motions**
  - In LTE, velocity distribution of particles follows **Maxwell-Boltzmann** distribution
  - Probability that particle velocity projected along any given ray is in interval $(v, v + \mathrm{d}v)$ is
    $$P(v)\mathrm{d}v = \sqrt{\frac{\mu}{2\pi kT}}\mathrm{e}^{-\mu v^2/2kT}\,\mathrm{d}v$$

- This projected velocity causes *Doppler shifts* in frequency and wavelength
- This leads to a profile of the form

$$\phi(\nu) = \int_{-\infty}^{\infty} P(v)\phi_{\rm lor}(\nu + \nu v/c)\mathrm{d}v$$

  (i.e., convolution of $P(v)$ with Lorentzian profile $\phi_{\rm lor}$)
- Simple case: when Doppler broadening is very much larger than natural/collisional/pressure broadening, can approximate $\phi_{\rm lor}$ by delta function at $\nu = \nu_0$:

$$\phi(\nu) = \int_{-\infty}^{\infty} P(v)\delta(\nu + \nu v/c - \nu_0)\mathrm{d}v = P((\nu - \nu_0)c/v) = \sqrt{\frac{\mu}{2\pi kT}}\mathrm{e}^{-\mu(\nu-\nu_0)^2c^2/2kT\nu^2}$$

  This is **Gaussian** profile function; FWHM is

$$\frac{\Delta\nu_{\rm dop}}{\nu} = \frac{\Delta\lambda_{\rm dop}}{\lambda} = 2\sqrt{\ln 2}\Delta\nu_{\rm D}$$

  where

$$\Delta\nu_{\rm D} = \sqrt{\frac{2kT}{\mu}}\frac{\nu_0}{c}$$

  is **Doppler width** of line