15 — Opacity Sources [$Revision: 1.3$]

- So far, assumed that the opacity is some known quantity at each depth in atmosphere
- To evaluate opacity as part of model atmosphere calculation, must add together contributions from various sources. Usually, work in terms of cross section rather than opacity:

$$\sigma = \kappa\mu$$

  (recall: μ is mean molecular (particle) weight)

- Bound-bound opacity
  - Arises when photon is absorbed by atom/ion, causing bound electron to transition from one energy level to another (higher) energy level
  - Opacity is non-vanishing only over narrow range of wavelengths centered on the wavelength of transition

$$\lambda_{\rm ab} = \frac{hc}{E_{\rm b} - E_{\rm a}}$$

    (here, $E_{\rm a}$ and $E_{\rm b}$ are energies of levels involved)
  - Classical treatment
    * incoming electromagnetic wave causes electron to oscillate
    * oscillating electron accelerates, and so radiates energy
    * net effect: wave is scattered into different directions
    * treat scattering as **damped harmonic oscillator**; cross section varies with angular frequency $\omega = 2\pi c/\lambda$ as

$$\sigma_{\rm bb}(\omega) = \frac{e^2}{4\epsilon_0 mc}\phi(\omega)$$

      (here, m is electron mass)
    * $\phi(\omega)$ is **Lorentz profile function**:

$$\phi(\omega) = \frac{\gamma}{(\omega - \omega_{\rm ab})^2 + (\gamma/2)^2}$$

      where

$$\gamma = \frac{e^2\omega_{\rm ab}^2}{6\pi\epsilon_0 mc^3}$$

      is **classical damping constant**, and $\omega_{\rm ab} = 2\pi c/\lambda_{\rm ab}$ is central frequency of transition
    * Frequency-integrated cross section:

$$\sigma_{\rm tot} = \int_0^\infty \sigma_{\rm bb}(\omega)\,{\rm d}\omega = \frac{e^2}{4\epsilon_0 mc}$$

      (note: units of area times angular frequency)
  - Classical cross-section is for scattering, depends only on $\omega_{\rm ab}$; typically, wrong
  - However, quantum-mechanical treatment of bb absorption usually expressed in terms of classical result:

$$\sigma_{\rm bb}(\omega) = \frac{e^2}{4\epsilon_0 mc} f_{\rm ab}\phi(\omega)$$

    where $f_{\rm ab}$ is **oscillator strength** —classical scattering cross section times (fractional) number of classical oscillators involved in $E_{\rm a} \rightarrow E_{\rm b}$ transition

- Bound-free opacity
  - Arises when photon is absorbed by atom, causing bound electron to be removed (**photoionization**)
  - Can only occur if photon has energy exceeding ionization energy χ_n from starting level n:

$$\lambda \leq \lambda_{\rm n} = \frac{hc}{E_{\rm n}}$$

  - So, bf absorption causes **continuum opacity** blueward of **ionization edge** at $\lambda_{\rm n}$.
  - For hydrogen atom in state n

$$\sigma_{\rm bf} = 1.31 \times 10^{-15} \frac{1}{n^5} \left(\frac{\lambda}{5000\text{Å}} \right)^3$$

    (must multiply this by fraction of atoms in state n when calculating opacity)
  - Inverse process is known as bf emission, or recombination radiation (good example: Hα emission)
- Free-free opacity
  - Arises when photon is absorbed by free electron in vicinity of an ion, causing the electron to gain kinetic energy. (Note that free electrons cannot absorb unless an ion is nearby)
  - Causes continuum opacity at all wavelengths
  - Inverse process is known as ff emission, or **Bremsstrahlung** — 'braking radiation'
- Electron scattering
  - Arises when photon is scattered by free electron, changing direction but retaining (approximately) same energy
  - Not quite the same as absorption opacity discussed above; must be handled separately in solving RTE
  - Simple Thomson formula for cross section:

$$\sigma_{\rm es} = \frac{8\pi}{3} \left(\frac{e^2}{4\pi\epsilon_0 mc^2} \right)^2$$

    (note: related to classical radius of electron)