14 — Model Atmospheres [$Revision$: 1.2]

- 'Real' model atmospheres
  - Attempt to include all relevant physics, obtain **emergent spectrum** that can be compared with observations
  - Key assumptions:
    * Hydrostatic equilbrium
    * Plane parallel
    * Radiative equilibrium
    * Local thermodynamic equilibrium (LTE)
- Hydrostatic equilibrium
  - Atmosphere supports itself against downward pull of gravity by **pressure gradients**
  - Consider slab of atmosphere with vertical extent $\mathrm{d}z$ and horizontal area $\mathrm{d}A$.
    * Upward pressure force on lower side is $P(z)\mathrm{d}A$
    * Downward pressure force on upper side is $P(z+\mathrm{d}z)\mathrm{d}A$
    * Net pressure force is $[P(z) - P(z+\mathrm{d}z)]\mathrm{d}A$
    * For **hydrostatic equilibrium**, pressure force must balance gravitational force

$$[P(z) - P(z+\mathrm{d}z)]\mathrm{d}A = g(\rho \mathrm{d}A dz)$$

      where $g = GM/R^2$ is surface gravitational acceleration, and $\rho \mathrm{d}A\mathrm{d}z$ is mass of slab
    * In limit $\mathrm{d}z \to 0$,

$$\frac{\mathrm{d}P}{\mathrm{d}z} = -g\rho$$

  - For ideal gas:

$$\rho = \frac{P\mu}{kT}$$

    and so:

$$\frac{\mathrm{d}P}{\mathrm{d}z} = -g\frac{P\mu}{kT}$$

    (i.e., get pressure structure from temperature structure)
  - Approximation: assume isothermal (constant T), solve for P:

$$P = P_0 \mathrm{e}^{-g\mu z/kT} = P_0 \mathrm{e}^{-z/H_P}$$

    H_P is **pressure scale height**:

$$H_P = \frac{kT}{g\mu}$$

    (typically scale of variation of pressure)
- Plane parallel
  - Already assumed in preceding notes
  - But check: for Sun, $H_P \approx 300\,\mathrm{km} \approx 0.0004\,R_\odot$
  - So, thickness of atmosphere is much smaller than solar radius — plane parallel assumption is good
  - Same applies to other main sequence stars — but perhaps not for supergiants (low $g \to$ large H_P)

- Radiative equilibrium
  - For gray atmosphere, **radiative equilibrium** (RE) was condition that
    $$\kappa\langle I\rangle = \kappa S$$
    (amount absorbed per unit length = amount emitted)
  - For non-gray atmosphere, condition of RE must be written as
    $$\int_0^\infty \kappa_\lambda \langle I_\lambda\rangle \mathrm{d}\lambda = \int_0^\infty \kappa_\lambda S_\lambda \mathrm{d}\lambda$$
    (reduces to gray-atmosphere case when all quantities independent of wavelength)
  - This implies that
    $$\int_0^\infty \kappa_\lambda \frac{\mathrm{d}F_\lambda}{\mathrm{d}\tau_{\lambda,\mathrm{v}}}\mathrm{d}\lambda = \kappa_\lambda \frac{\mathrm{d}F}{\mathrm{d}\tau_{\lambda,\mathrm{v}}} = 0$$
    where
    $$F = \int_0^\infty F_\lambda \mathrm{d}\lambda$$
    is bolometric flux. So, bolometric flux is depth-independent even in non-gray atmosphere (although F_λ itself is not)
- Local thermodynamic equilibrium
  - In isolated enclosure, complete **thermal equilibrium** means that a single temperature T can be assigned to matter and radiation
  - This in turn means that
    * Distribution of particle velocities follows Maxwell-Boltzmann distribution
    * Excitation state of matter described by Boltzmann statistics
    * Ionization state of matter described by Saha equation
    * Radiation field described by Planck function
  - Stellar atmosphere is *not* an isolated enclosure; there is net upward flow of radiation
  - However, in most cases mean free path $\langle s\rangle \ll H_P$
  - Then, very good approximation to assume **local thermodynamic equilibrium** — single temperature T describing all matter properties
  - Differs from complete thermal equilibrium in that $I_\lambda \neq B_\lambda$ (however, still true that $S_\lambda = B_\lambda$, because source function/emissivity depends on matter properties)
- Constructing a model atmosphere
  - Assume an effective temperature T_{eff} and gravity g
  - Introduce **column mass** m (in $\mathrm{g\,cm^{-2}}$) as independent variable:
    $$\mathrm{d}m = -\rho \mathrm{d}z$$
  - Assume/guess a temperature structure $T(m)$ (e.g., from equivalent gray/Eddington model)
  - Integrate equation of hydrostatic equilibrium
    $$\frac{\mathrm{d}P}{\mathrm{d}m} = -g$$
    to find
    $$P(m) = gm + C$$
    and also $\rho(m)$ (from $P(m)$ and $T(m)$). (Note: we've ignored radiation pressure, which adds an extra term to equation of HE)

- Given T, P and ρ at each m, calculate opacity κ_λ at each m (we'll see how later)
- Solve RTE:

$$\mu \frac{\mathrm{d}I_\lambda}{\mathrm{d}\tau_{\lambda,\mathrm{v}}} = I_\lambda - S_\lambda$$

  or

$$\mu \frac{\mathrm{d}I_\lambda}{\mathrm{d}m} = \kappa_\lambda I_\lambda - \kappa_\lambda S_\lambda$$

  to find $I_\lambda(m, \mu)$ (use formal solution)
- Generally, source function S_λ depends on local temperature (thermal emission) *and* local radiation field (scattering)
- However, for simplicity we neglect scattering, and assume

$$S_\lambda = B_\lambda$$

  (i.e., LTE thermal emission)
- Calculate depth-dependent bolometric flux

$$F(m) = \int_0^\infty 2\pi \int_{-1}^1 I_\lambda(m, \mu)\mu \mathrm{d}\mu \mathrm{d}\lambda$$

- Compare $F(m)$ against 'expected' flux defined $F = \sigma T_{\mathrm{eff}}^4$
  * In radiative equilibrium, they should be equal
  * Generally, they are not same because we assumed $T(m)$
  * Apply **temperature correction procedure**, to adjust $T(m)$ at each depth so that we are closer to radiative equilibrium
  * Solve RTE again for corrected $T(m)$, repeat as necessary