6 — Binary Systems (II) [$Revision: 1.1$]

- Visual Binaries
  - In principle, can determine masses of both stars
  - Simplest case: orbit is face-on ($i = 0°$)
  - Measure semi-major axis of equivalent orbit from separation of stars at apastron (stars furthest from center of mass)
  $$a = \frac{|\mathbf{r}|}{1+e}$$
  - Since we can only measure an angular separation, need to know distance d to binary system to get the physical separation a
  - Measure period, combine with semi-major axis to get mass sum from 3$^{\mathrm{rd}}$ law:
  $$m_1 + m_2 = \frac{4\pi^2}{GP^2} a^3$$
  - Measure semi-major axis of each star's orbit at apastron:
  $$a_j = \frac{|\mathbf{r}_j|}{1+e}$$
  (for $j = 1, 2$)
  - Take ratio to get mass ratio:
  $$\frac{a_1}{a_2} = \frac{|\mathbf{r}_1|}{1+e} \frac{1+e}{|\mathbf{r}_2|} = \frac{\mu}{m_1} \frac{m_2}{\mu} = \frac{m_2}{m_1}$$
  (Don't need to know distance to calculate this ratio!)
  - Combine mass sum and mass ratio to get individual masses
  - More complicated when system isn't face-on; see O&C, eqn. (7.3)
  - If distance isn't known, can still determine a by measuring velocities using Doppler shift
- Spectroscopic binaries
  - Determine radial velocities of one or both stars from Doppler shifts:
  $$v_{jr} = \frac{\Delta\lambda_j}{\lambda_j} c$$
  ($j = 1, 2$)
  - Assuming circular orbits; orbital speeds of stars are constant:
  $$v_j = \frac{2\pi a_j}{P}$$
  - Radial velocity curves then sinusoidal; half-amplitude given by
  $$v_{jr} = v_j \sin i$$
  - Take ratio:
  $$\frac{v_{1r}}{v_{2r}} = \frac{v_1 \sin i}{v_2 \sin i} = \frac{a_1}{a_2} = \frac{m_2}{m_1}$$
  (dependence on i drops out)

- To get mass sum, use 3rd law:

$$m_1 + m_2 = \frac{4\pi^2}{GP^2} a^3$$

- But for circular orbits:

$$a = \frac{P}{2\pi}(v_1 + v_2) = \frac{P}{2\pi}(v_{1r} + v_{2r}) \sin i$$

- So mass sum depends on inclination

$$m_1 + m_2 = \frac{P}{2\pi G} \frac{(v_{1r} + v_{2r})^3}{\sin^3 i}$$

- Cannot measure inclination i directly (except eclipsing binaries, when $i \approx 90°$)
- But for large ensemble of stars, can replace $\sin^3 i$ by appropriate statistical average
- Complications: non-circular orbit; only one star visible

• Eclipsing binaries
  - We know that the orbit must be close to edge-on ($i = 90°$)
  - Also, shapes of eclipses gives:
    * radii of stars (from time between first contact and minimum light)
    * ratio of effective temperatures (from depths of eclipses)