# Assignment 5 — due December 1$^{\mathrm{st}}$ [$Revision$ : 1.2]

1. Use *EZ Web* to calculate the following models:

   - $1\,M_{\odot}$, 5 billion years old, $Z = 0.02$
   - $10\,M_{\odot}$, 1 million years old, $Z = 0.02$

   For each model, make a line plot of $\nabla$ as a function of $\log T$. Use an appropriate range for the temperature axis, and a range $[0, 1]$ for the $\nabla$ axis. Then, using a different linestyle, overplot $\nabla_{\mathrm{rad}}$ and $\nabla_{\mathrm{ad}}$, making usre that you label the curves appropriately.

   (a) On each graph, identify zones where convection is taking place

   (b) On each graph, highlight one place where convection is efficient, and one place where it is inefficient

2. Use the Saha equation to derive an expression for the fraction $x$ of hydrogen atoms that are ionized, as a function of electron number density $n_{\mathrm{e}}$ and temperature $T$. Apply this expression to make a line plot of $x$ as a function of $\log T$ for the $1\,M_{\odot}$ model from above. You should approximate the partition function as 2 for neutral hydrogen and 1 for ionized hydrogen; the ionization potential of hydrogen is 13.6 eV.

   On the same graph, using a different linestyle, overplot the value of $x$ as determined from columns 25 and 27 of the model file. Can you think why the two curves might diverge toward the center of the star? (Hint: think what effect the high density has on energy levels).

3. For the $10\,M_{\odot}$ model from above, make a line plot of $\log \kappa$ as a function of $\log T$. Note the two prominent peaks in the opacity — one of these is due to bound-free transitions of singly-ionized helium (i.e., He II $\rightarrow$ He III), while the other is due to bound-bound transitions of iron and nickel (the so-called 'iron bump'). Find out which is which by making a line plot of the fraction $y_2$ of helium atoms that are in the twice-ionized state.

4. For the $10\,M_{\odot}$ model from above, make a line plot of $\log \epsilon$ against $\log T$; the vertical axis range (in log cgs units) should be $[-15, 5]$. On the same graph, overplot the energy generation rates due to the PP chain and the CNO cycle. Approximately below what temperature does the energy production by PP exceed that by CNO?

5. Use *EZ Web* to calculate a $100\,M_{\odot}$ model, 1 million years old, with $Z = 0.02$. Make a line plot of the Eddington factor

$$\Gamma = \frac{\kappa L_{\mathrm{rad}}}{4\pi G M_r c}$$

   as a function of $\log T$. Identify where $\Gamma$ exceeds unity; what opacity source is responsible for this 'super-Eddington' zone (use information from Q3 to answer this). In this zone, why doesn't the star blow itself apart by radiation pressure? (Hint: have a look at the behavior of the gas pressure and its gradient)