Assignment 2 — due October 3rd [$Revision : 1.2$]

1. Consider an enclosure in thermal equilibrium at temperature T, sitting in empty space.

   (a) Write down an expression for the specific intensity I_λ inside the enclosure, in terms of T and the wavelength λ

   (b) By integrating this expression over all wavelengths, calculate the bolometric intensity I.

   (c) For an infinitessimal hole in the enclosure, calculate the bolometric net flux F passing through the hole and out into the surrounding empty space. (HINT: to evaluate F, consider the specific intensity in the outward and inward directions at the hole).

   (d) What name is given to the equation you have just derived?

2. Consider a visual binary system for which the orbits have circularized due to tidal friction (i.e., $e = 0$). From Earth, the apparent motion of each star around the center of mass appears as an ellipse, due to projection effects. For each star j (with $j = 1$ denoting the primary star, and $j = 2$ the secondary star), measurements are made of the apparent semi-major axis α_j of this ellipse (in angular units of arcseconds); the corresponding apparent semi-minor axis β_j; the amplitude $v_{j\mathrm{r}}$ of the sinusoidal radial velocity variations; and the orbital period P of the system.

   (a) Derive an expression for the inclination i of the orbital plane, in terms of the measured semi-major and semi-minor axes of the primary star's ellipse (α_1 and β_1). (Recall that the inclination is defined relative to the plane of the sky; $i = 0°$ = face-on; $i = 90°$ = edge-on).

   (b) Derive an expression for the primary's orbital speed v_1, in terms of measured quantities.

   (c) Derive an expression for the actual semi-major axis a_1 (in physical length units) of the primary orbit, in terms of measured quantities.

   (d) Combine your previous answers to derive an expression for the distance d to the binary system, in terms of measured quantities.

   (e) Derive the mass sum m_1+m_2, and the mass ratio m_1/m_2, in terms of measured quantities.

3. The following list describes the spectra of various stars. For each, determine whether the star has an early spetral type (O,B); a mid type (A,F); or a late type (G,K,M) — or if there is insufficient information to decide.

   (a) The spectrum shows strong H I Balmer lines

   (b) The spectrum shows very weak H I Balmer lines

   (c) The spectrum shows strong molecular bands due to TiO

   (d) The spectrum shows weak He II lines

   (e) The spectrum shows strong Ca II H & K lines

4. Careful measurements of binary systems show that stars on the main sequence follow an approximate mass-luminosity relation $L \propto M^{7/2}$, and the mass-radius relation $R \propto M^{4/5}$.

   (a) Use these relations to plot the main sequence in a theoretical Hertzsprung-Russell diagram (HRD). Your HRD should cover the effective temperature range $\log_{10} T_{\mathrm{eff}} = 3.5 - 4.5$, and an appropriate range in $\log_{10} L/L_\odot$. Ensure that the (logarithmic) axes are properly labeled and follow the customary orientation. HINT: you may wish first to derive an expression for L in terms of T_{eff} for main-sequence stars.

   (b) On your HRD, draw and label dashed lines of constant stellar radius for $R = 0.1, 1, 10, 100\, R_\odot$.

(c) The star Betelgeuse has $T_{\text{eff}} = 3,500\,\text{K}$ and $L = 63,000\,L_{\odot}$. Mark and label its position in your HRD, and explain why it is classified as a red supergiant.

(d) Derive an expression for the main-sequence (core hydrogen burning) lifetime t of a star, in terms of its luminosity. You should assume (i) that the luminosity remains constant throughout the star's lifetime, following the mass-luminosity relation given above; (ii) that the star converts 10% of its mass from hydrogen into helium during the main sequence phase; and (iii) that hydrogen fusion is 0.7% efficient (i.e., in converting 1 gram of hydrogen into helium, 0.7% of the rest mass is released as energy).

(e) Using the expression for t, mark and label on your HRD the points on the main sequence where stars have lifetimes $t = 10$, 100, 1000, and $10000\,\text{Myr}$.

(f) The Praesepe (Beehive) open cluster exhibits no stars on the main sequence above $T_{\text{eff}} \approx 10,000\,\text{K}$. From your HRD, estimate the age of the cluster.