2 — Distance & Magnitude [$Revision$: 1.6]

- Equatorial Coordinates
  - Coordinate system used to describe angular positions of stars
  - Generalization of latitude & longitude onto the **celetial sphere**
  - Celestial poles coincide with terrestrial poles (Earth's rotation axis)
  - Celestial equator in same plane as terrestrial equator
  - **Right Ascension** α is equivalent of longitude (0 hours → 24 hours)
  - **Declination** δ is equivalent of latitude (-90° (south) → 90° (north))
  - Zero point $\alpha = 0$ is known as the **first point of Aries**, equivalent to the prime Meridian
  - Equatorial coordinates (α, δ) can change slowly over time
    * due to motion of the star itself
    * due to changes in the direction of the Earth's rotation axis (**precession of the equinoxes**
  - Planet/Sun/Moon coordinates change much more rapidly

- Distances to stars
  - Measured for nearby stars using **trigonometric parallax**
  - Principle: angular position of object changes when viewing point is moved:

    $$d = \frac{B}{\tan p}$$

    where d is distance, $2B$ is baseline of triangle, $2p$ is apex angle of triangle (note factors of 2; see O&C, Figs. 3.1 & 3.2)
  - For small p *measured in radians*,

    $$d \approx \frac{B}{p}$$

  - For stellar distances, $B = 1$ **Astronomical Unit** (AU) $= 1.496 \times 10^{13}$ cm — radius of Earth's orbit around Sun. (AU measured historically by parallax of Venus & asteroids — **bootstrapping**; more recently by radar & telemetry)
  - Convention to measure p in **arcseconds** ($''$); $1\,\mathrm{rad} = 57.3^\circ = 57.3 \times 60 \times 60'' = 2.06 \times 10^{5}{}''$
  - Convention to measure d in **parsecs**; $1\,\mathrm{pc} = 2.06 \times 10^5\,\mathrm{AU} = 3.09 \times 10^{18}\,\mathrm{cm} = 3.26\,\mathrm{ly}$

    $$d = \frac{1}{p''}\,\mathrm{pc}$$

  - Modern-day parallaxes from *Hipparcos* astrometry satellite (1989–1993), accurate down to $\sim 0.001''$ (i.e., 1 kpc). Around 1,000,000 objects in catalog; *GAIA* will find 1,000,000,000.
  - Parallax can also be used to measure radius of the Sun from its angular diameter $\approx 32'$

- Stellar magnitudes
  - Originally (Hipparchus) assigned **apparent magnitudes**: $m = 1 \rightarrow$ brightest, $m = 6 \rightarrow$ dimmest
  - Now standardized on logarithmic scale: 5 magnitudes ↔ factor 100 brightness difference

- Typical values: Sun: -26.7; Full moon: -12.6; Max Venus, ISS: -4.7; Naked eye by day: -3.9; Sirius: -1.47; Vega: 0; Naked eye, perfect dark: 6.5; Binoculars: 9.5; Pluto: 13.6; Ground-based 8-m telescope: 27; Hubble Space Telescope: 30; OWL: 38
- Apparent magnitude is a measure of total **flux** F: energy received from source per unit area per unit second ($\mathrm{erg\,cm^{-2}\,s^{-1}}$)

$$m = -2.5 \log_{10} F + C$$

(note minus sign!). Measuring F can be tricky!
- C is constant that sets zero-point of magnitude scale; most often, we work with magnitude differences, and C drops out

$$m_1 - m_2 = -2.5 \log_{10} F_1 + 2.5 \log_{10} F_2 = -2.5 \log_{10} \frac{F_1}{F_2}$$

- Flux depends on total **luminosity** L ($\mathrm{erg\,s^{-1}}$) and distance d of source.
- Consider spherical shell at *any* radius r:

$$L = 4\pi r^2 F$$

(flux $\times$ total surface area of shell must equal luminosity, to conserve energy)
- Hence, for a star at distance $r = d$

$$F = \frac{L}{4\pi d^2}$$

$$m = -2.5 \log_{10} \frac{L}{4\pi d^2} + C = -2.5 \log_{10} L + 5 \log_{10} d + C'$$

- **Absolute magnitude** M: equal to the apparent magnitude the star would have *if* it were 10 pc away

$$m - M = 5 \log_{10} d - 5 \log_{10}(10\,\mathrm{pc}) = 5 \log_{10} \frac{d}{10\,\mathrm{pc}}$$

($m - M$ is sometimes known as the **distance modulus**)
- For some **pulsating** stars, we can obtain L directly. From m and L, then find d — **standard candle**!