# Assignment 1 — due September 18$^{\mathrm{th}}$ [$Revision$ : 1.2]

1. Use the full (quantum-mechanical) formula for the blackbody energy density

$$u_\lambda \mathrm{d}\lambda = \frac{8\pi hc/\lambda^5}{e^{hc/\lambda kT} - 1}\,\mathrm{d}\lambda$$

   to derive the classical expression (see class notes) for the Rayleigh-Jeans tail (Note: the blackbody formula given in eqn. 3.22 of O&C is for the specific intensity of a blackbody, and *not* the energy density; although the two are related, they are not the same quantity!).

2. For this question, assume that the effective temperature of the Sun is 5780 K; the radius of the Sun is $R_\odot = 6.96 \times 10^{10}$ cm; and the radius of the Earth is 6360 km.

   - Calculate the bolometric solar flux at 1 AU from the Sun.
   - The average albedo of the Earth is about 0.3, meaning that it reflects back into space 30% of the radiation hitting it. Work out the total amount of solar radiation absorbed by the Earth every second.
   - Assuming that the the Earth behaves as a blackbody, find the equilibrium temperature $T_{\mathrm{eq}}$ at which it will radiate energy at the same rate it absorbs it.
   - What environmental effect is responsible for raising Earth's average temperature ($\sim$ 288 K) above the $T_{\mathrm{eq}}$ value you have calculated?

3. Vega has a parallax of 129 milliarcsec; a radius of 2.73 $R_\odot$; an effective temperature of 9600 K; a visual magnitude of 0.03; and colors $U - B = -0.01$ and $B - V = 0.00$. Calculate the constants $C_U$, $C_B$, $C_V$ that set the zero point of the Johnson magnitude system. (You may assume that the Johnson passbands are narrow enough that

$$\int_0^\infty S_X(\lambda) F_\lambda \mathrm{d}\lambda \approx \Delta\lambda F_{\lambda_0},$$

   where $\lambda_0$ and $\Delta\lambda$ are the central wavelength and *full* width of the transmission function $S_X(\lambda)$. You may also assume, when evaluating $F_{\lambda_0}$, that Vega is a blackbody).

4. Polaris, also known as the pole star (because it is situated almost at the north celestial pole), is a Cepheid variable — a star that changes brightness regularly, with a period $P$ that depends on the star's absolute visual magnitude via the approximate relation

$$M_V \approx -2.81 \log_{10} P - 1.43$$

   where $P$ is measured in days.

   - Use Simbad[1] to look up the apparent visual magnitude of Polaris (just do a basic query for 'Polaris').
   - The period of Polaris is 3.97 days. Calculate its distance in parsecs using the period/magnitude relation above.
   - Compare this distance against the value determined by *Hipparcos*, which measured a parallax of 7.56 milliarcsec. Which value is more reliable?
   - Imagine that an alien lives on a planet orbiting Polaris. What's the angular distance it would measure between the Earth and the Sun?

[1]Simbad is an on-line database of many bright/important stars; reach it at `http://http://simbad.u-strasbg.fr/simbad/sim-fbasic`.

- Assuming that the Sun and Earth are both blackbodies, and using data given in the preceding questions (in particular, the $C_X$ values for the Johnson system), calculate the apparent Johnson magnitudes $(U, B, V)$ of the Sun and Earth, as seen by the alien.
- In which passband does the alien have the best chance of seeing the Earth? Should they be building ultraviolet or infra-red telescopes?