Rigid-Field Models: Past, Present and Future

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MiMeS5 Meeting — June 21 2011

Rigid-Field Models

- Wind flow and field lines move together ('frozen flux')
- Magnetic energy density ≫ kinetic energy density ($\eta \gg 1$)
- Wind flow does not influence magnetic field
- Magnetic field acts as rigid conduit for wind flow

Rigid-Field Models: Past

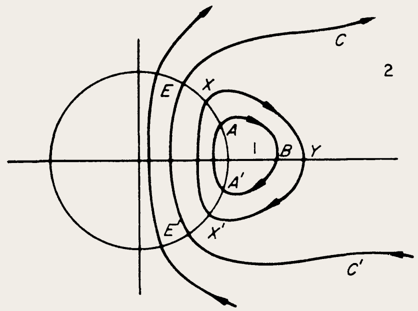


FIG. 1

$$-\frac{a^2}{\rho}\frac{\partial\rho}{\partial s}+\frac{\partial}{\partial s}\left(\frac{GM}{r}+\tfrac{1}{2}\Omega_s{}^2\varpi^2\right)=0,$$

$$\log\left(\frac{\rho}{\rho_s}\right)=l\left(\frac{r_s}{r}-1\right)+\tfrac{1}{2}\kappa l\sin^2\theta_s\left(\left(\frac{r}{r_s}\right)^3-1\right),$$

Mestel (1968)

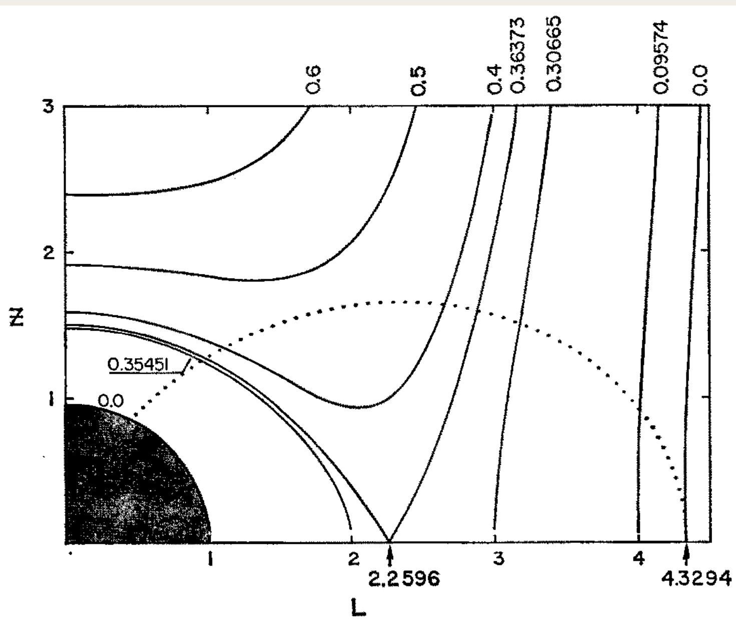


FIG. 2. LINES OF CONSTANT GEOPOTENTIAL ABOUT JUPITER.

Michel & Sturrock (1974)

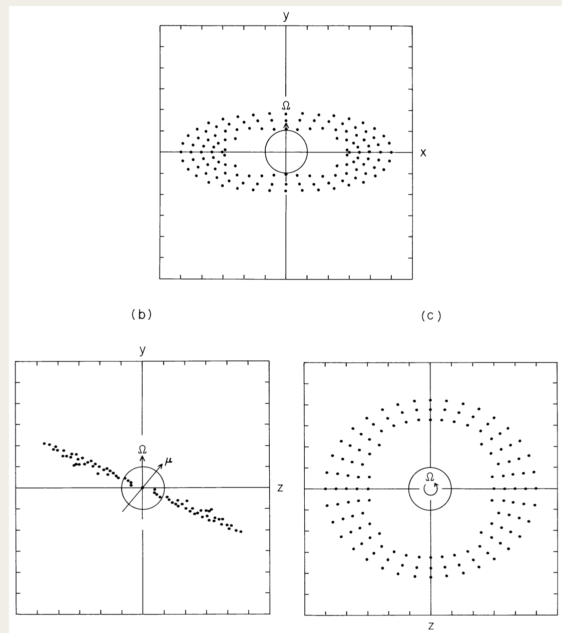


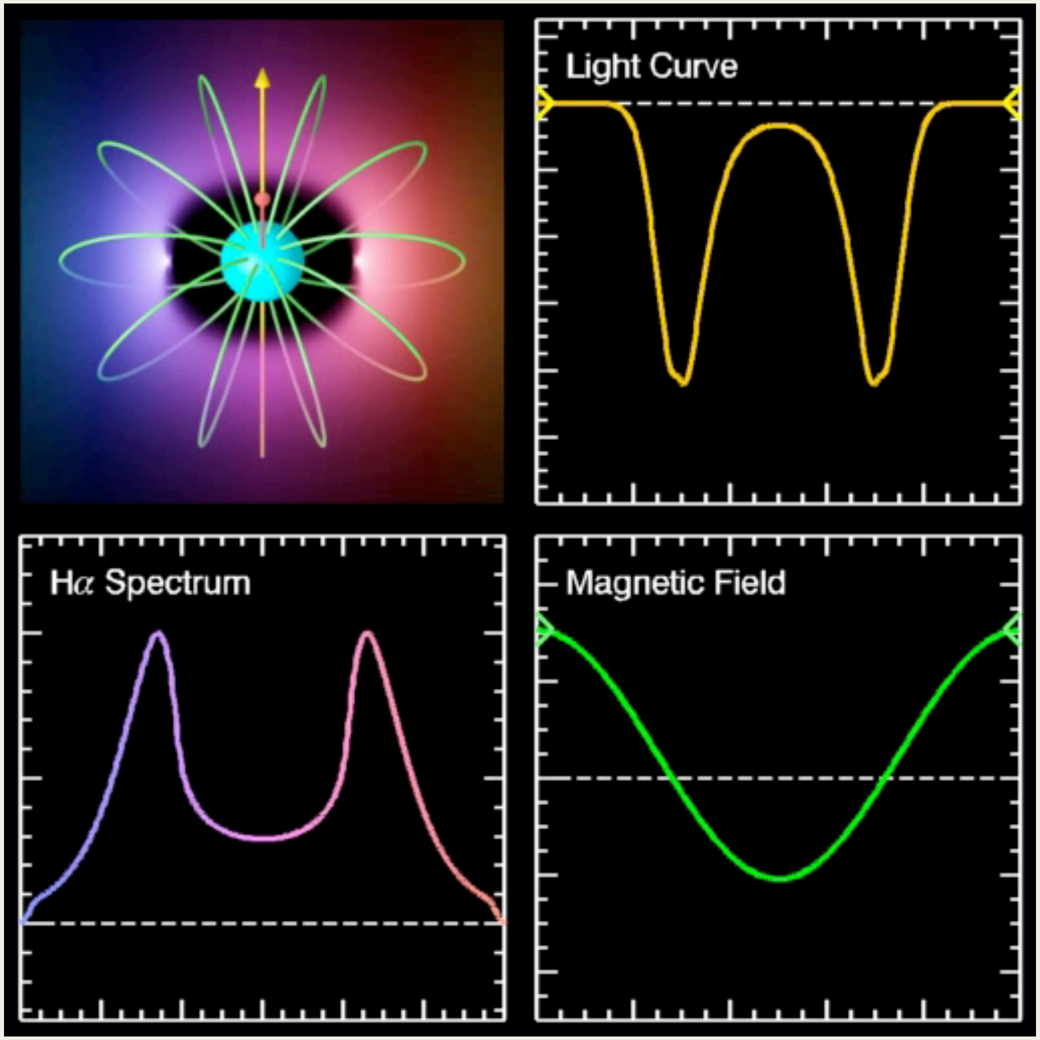
Fig. 2. Distribution of the position of potential minimum (dot point) for $\beta = 40°$. The y axis is aligned with the stellar rotation axis and the z axis to the intersection of the magnetic and rotational equatorial planes. μ shows the magnetic dipole. One scale division is the stellar radius.

Nakajima (1985)

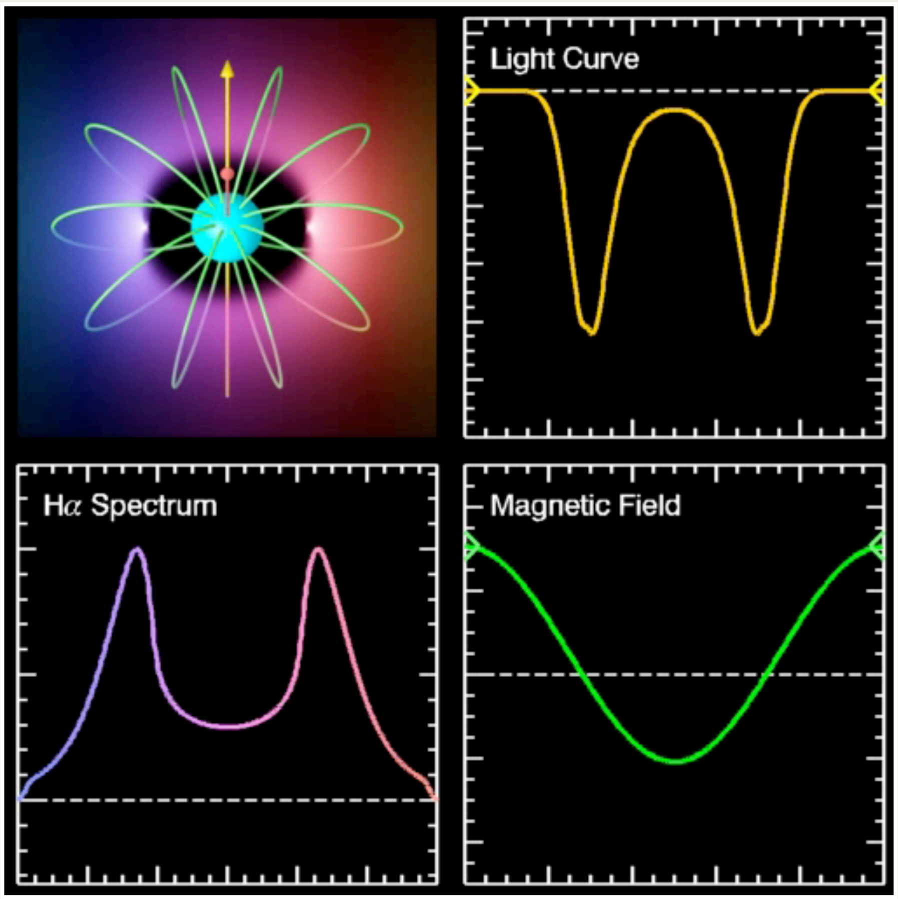
Rigid-Field Models: Present

- *Rigidly Rotating Magnetosphere (RRM)*
  - Townsend & Owocki (2005) — formulation
  - Townsend, Owocki & Groote (2005) — application to σ Ori E
  - Townsend (2008) — photometric modeling
- *Rigid-Field Hydrodynamics (RFHD)*
  - Townsend, Owocki & ud-Doula (2007) — formulation

The RRM Model of σ Ori E

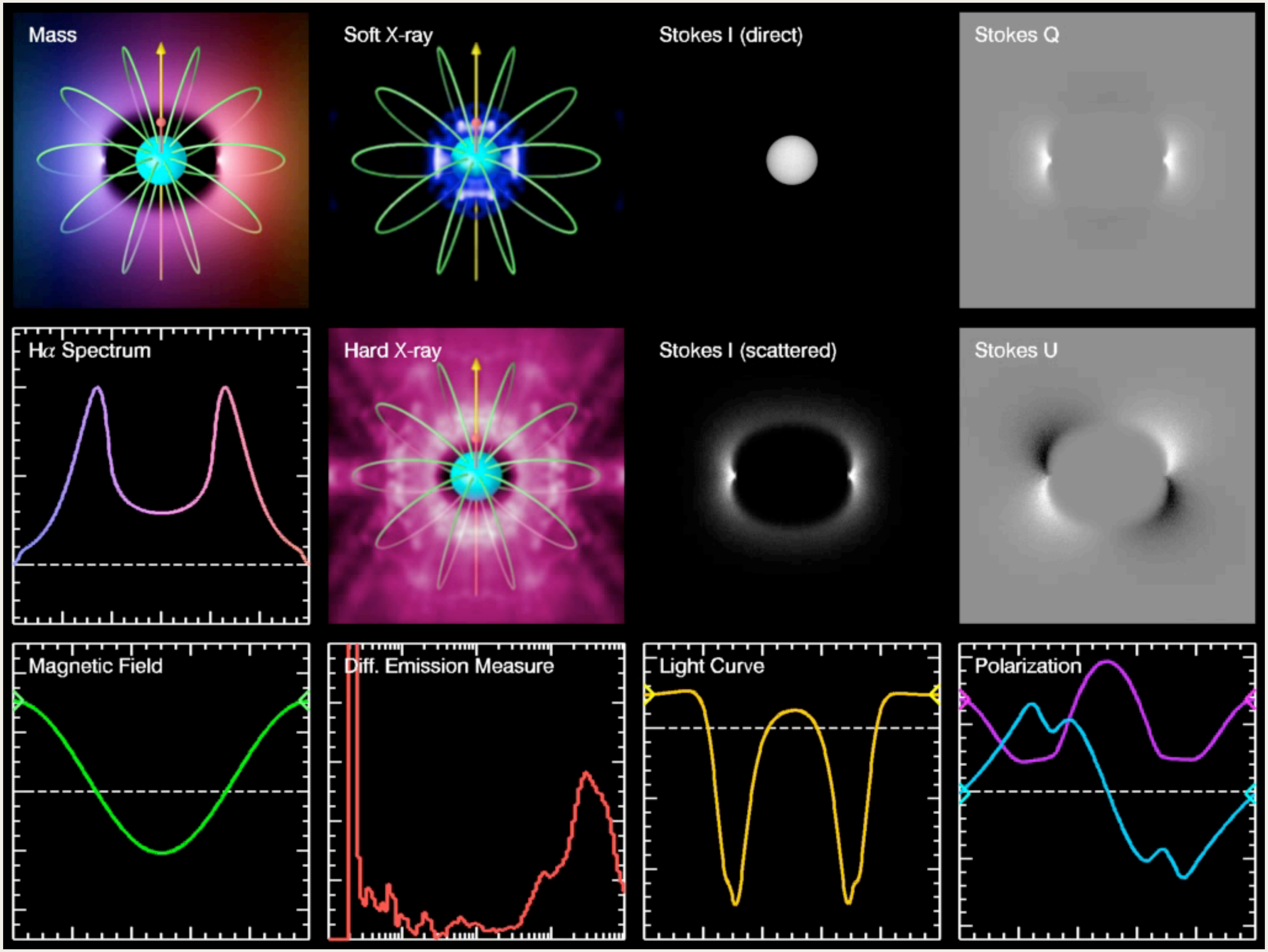


The RRM Model: Too Simplistic?

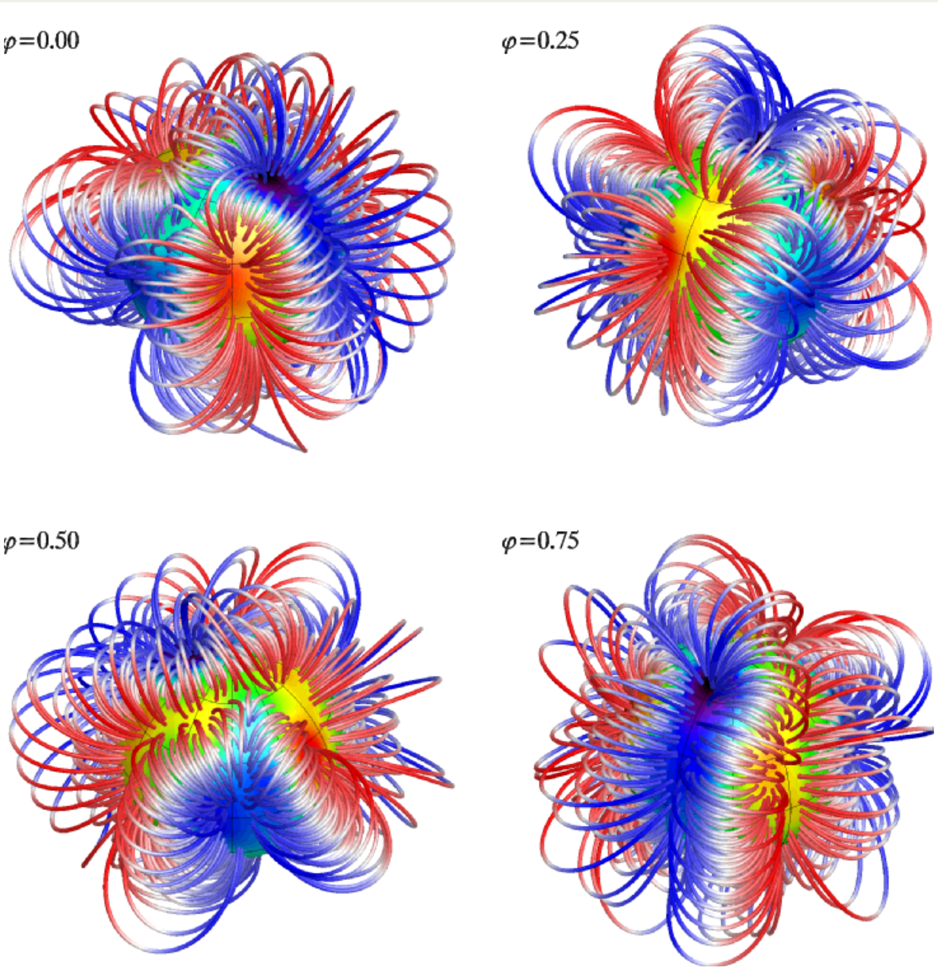


- No treatment of wind & shock zones
- Simplistic magnetic topologies
- No atomic or thermal physics
- When densities reach breakout point, field cannot be rigid - inconsistency

Wind & Shock Zones: RFHD



Complex Fields



Kochukhov et al. (2011)

- New specpol allows us to measure surface field strength and orientation
- Reconstruct circumstellar field using extrapolation
- Use reconstructed field in rigid-field models

Potential Field Extrapolation

- Assume field is derived from a scalar potential:

$$\mathbf{B} = \nabla \Phi$$

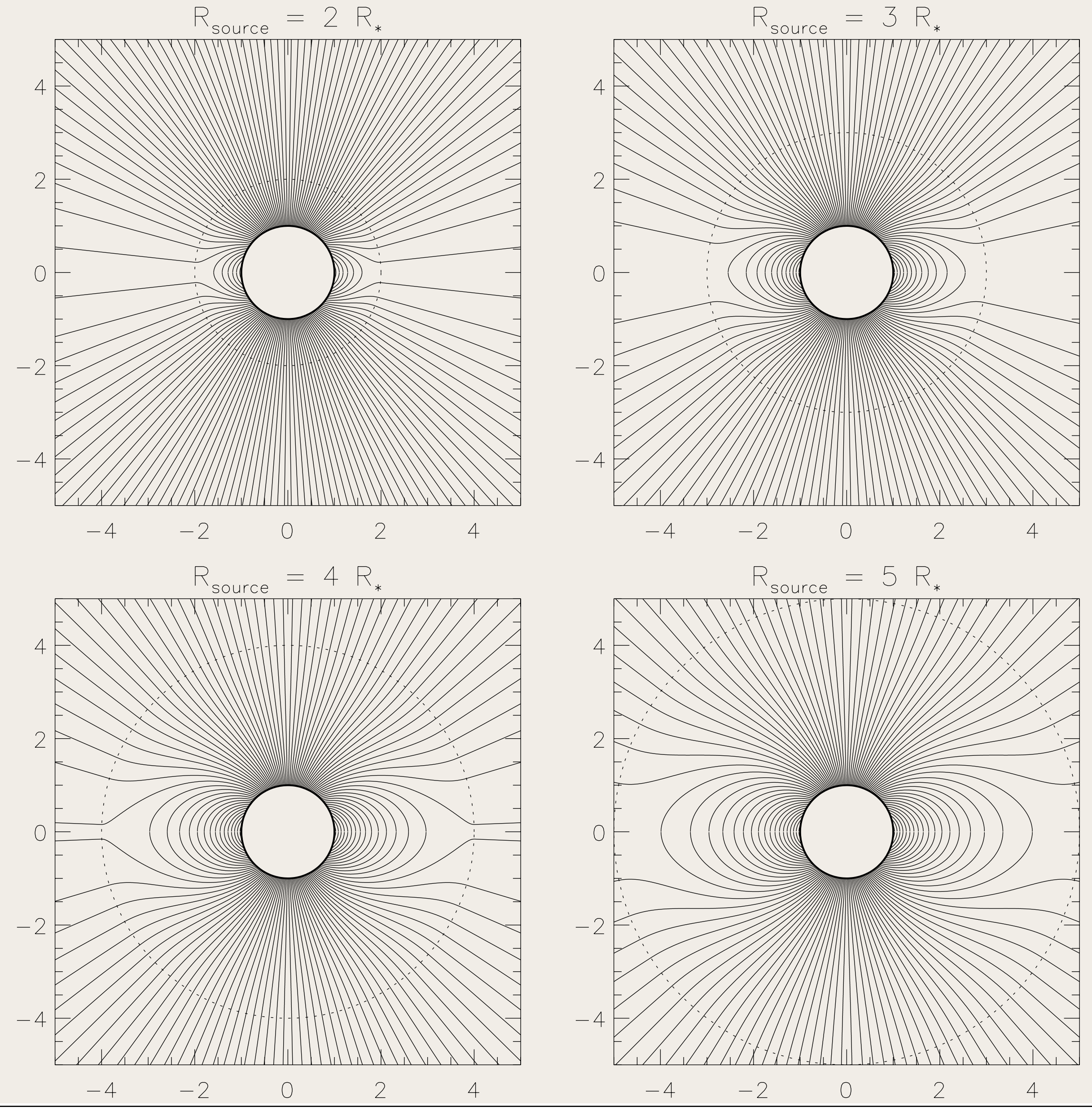
- Express potential as multipole expansion

$$\Phi(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-\ell}^{\ell} \left[a_{\ell m} r^{\ell} + b_{\ell,m} r^{-\ell-1} \right] Y_{\ell}^{m}(\theta, \phi)$$

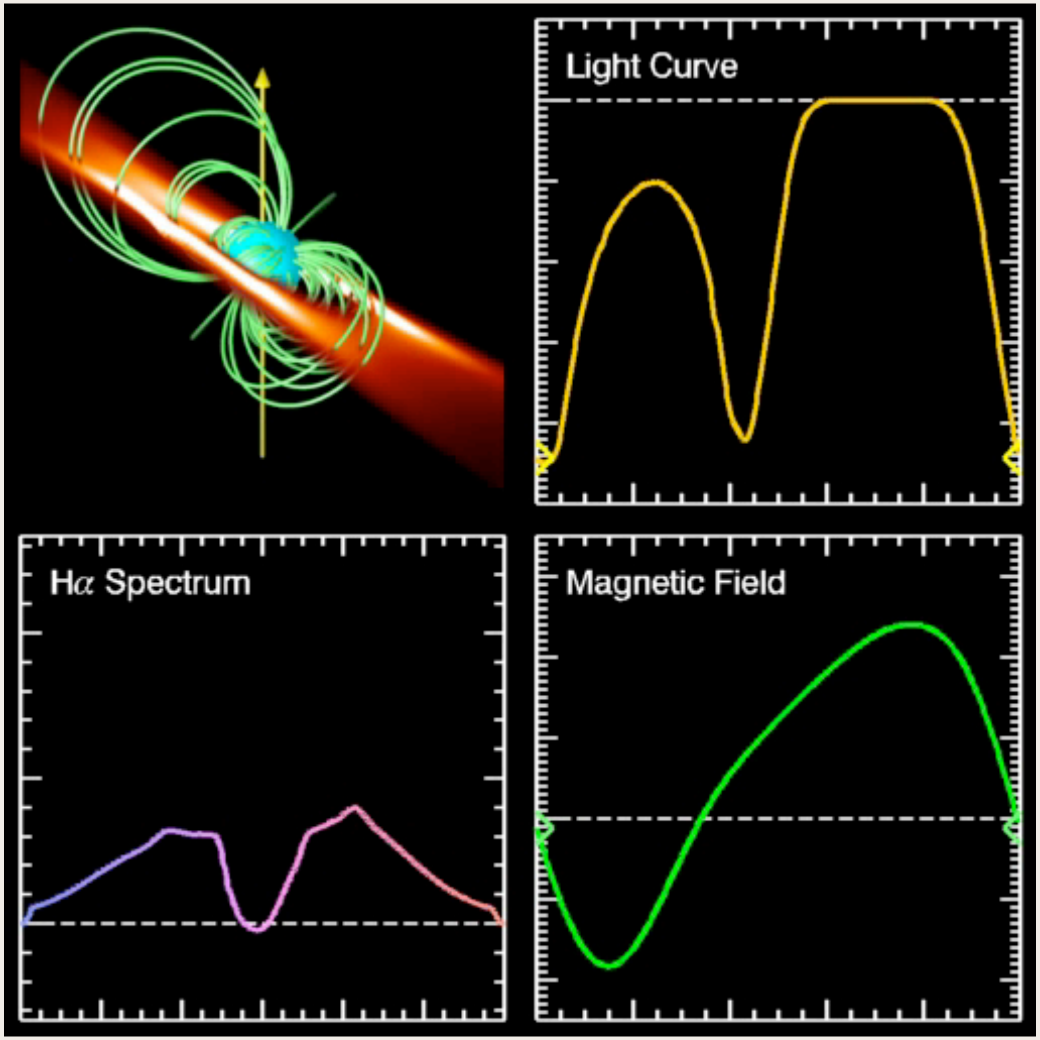
- Determine expansion coefficients by
  - matching B_r at stellar surface
  - requiring a purely radial field at R_{source} (‘source surface’)

- This approach has problems (see later)

The Source Surface: Fudging Departures From Rigidity

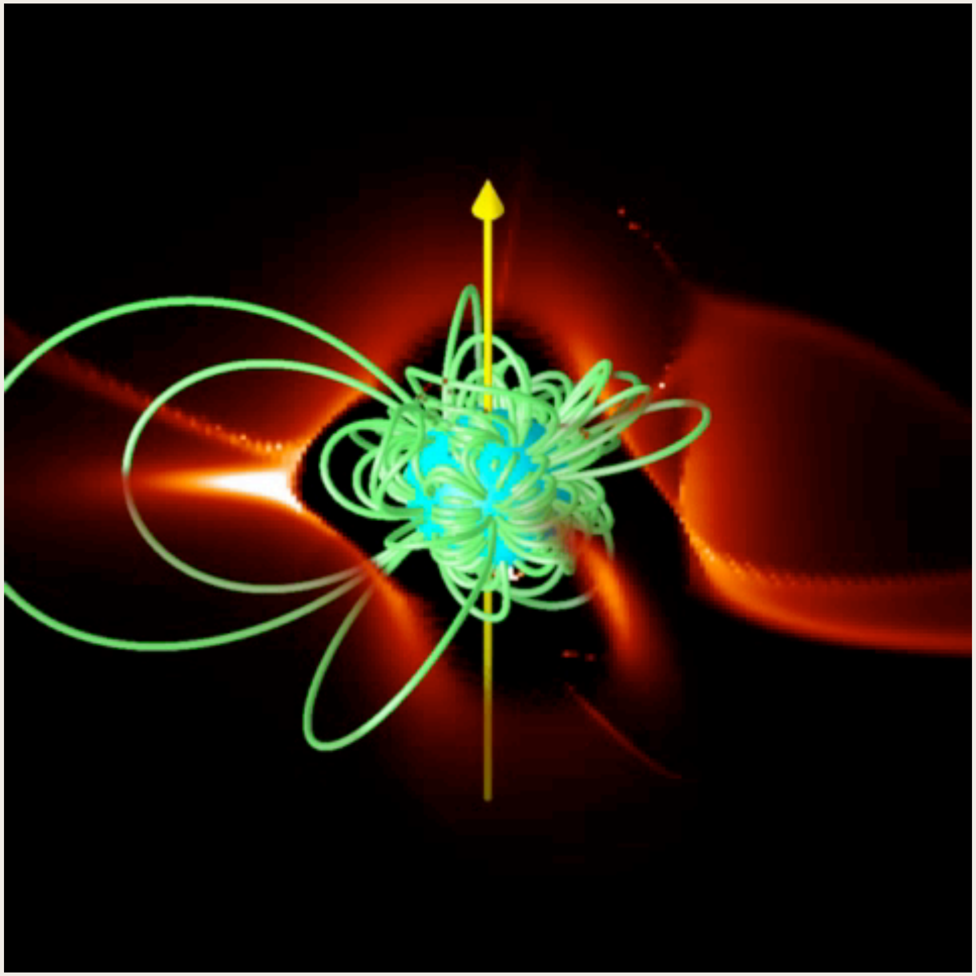


An A-RRM Model of σ Ori E



field model from Oksala et al. (2011)

An A-RRM Model of HD 37776



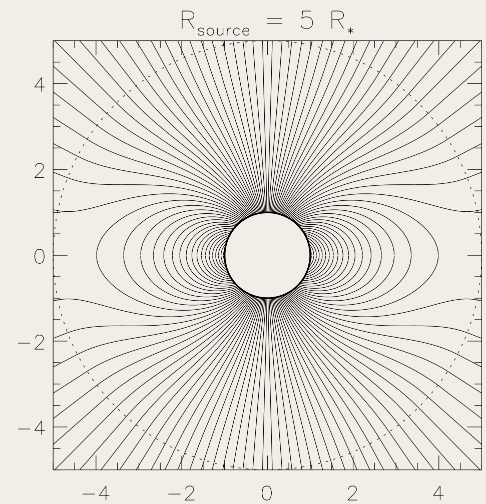
field model from Kochukhov et al. (2011)

A-RFHD Models

(see Chris Bard's talk)

Beyond Potential Fields

- Potential extrapolation is problematic
  - Source surface requires monopoles!
  - Does not correctly describe field in $r < R_{\text{source}}$ region



- Introduce vector potential:

$$\mathbf{B} = \nabla\Phi + \nabla \times \mathbf{A}$$

- The vector potential generates the Lorentz force:

$$\mathbf{F}_{\text{lor}} = (\nabla \times \mathbf{B}) \times \mathbf{B} = (\nabla \times [\nabla \times \mathbf{A}]) \times (\nabla\Phi + \nabla \times \mathbf{A})$$

- Represents the field's response to being stressed by the wind flow
- Full formalism laid out by Mestel (1968)

Work in Progress