23 The Red Giant Branch

Evolution onto the Red Giant Branch

When a star runs out of hydrogen at its center, its time on the main sequence has reached an end, and it embarks on a series of dramatic changes. For low- and intermediate-mass stars like the Sun, hydrogen burning continues in a shell around the core (now composed predominantly of helium), and the star evolves to the right in the Hertzsprung-Russell diagram. This phase is known as the *subgiant branch* (see Fig. 23.1).

Eventually, the star becomes mostly convective and it transitions to more-vertical evolution in the HR diagram. This phase is known as the *red giant branch (RGB)*, because the star becomes very large ($R \gtrsim 100\,\mathrm{R}_\odot$) and appears red due to its low effective temperature.

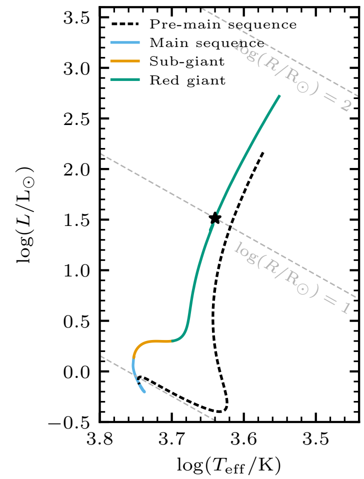

Figure 23.1: Evolutionary track in the Hertzsprung-Russell diagram for a *MESA* model of the Sun, spanning the pre-main sequence phase through the main sequence and subgiant phases to the RGB. The asterisk marks the case plotted in Figs. 23.2 and 23.3.

Core-Envelope Dichotomy on the RGB

A characteristic property of stars on the RGB is the division of the star into dichotomous regions:

- A high-density *radiative core* composed primarily of helium (plus a small amount of metals). The core can encompass a significant fraction of the star's mass, but spans only a small fraction of the star's radius.
- A surrounding low-density *convective envelope* composed of hydrogen-rich material. The envelope contains the remainder of the star's mass, and spans almost all of the star's radius.

Fig. 23.2 illustrates the marked contrast between the core and the envelope, for a solar model about halfway up the RGB. Note how the density at the top of the core is 5 orders of magnitude larger than the density at the bottom of the envelope. For this model, the core contains around 25% of the star's mass, but spans only $\sim$ 0.3% of its radius. The envelope encompasses almost all of the remaining mass and radius. Because 95% by radius of the star is convective, its evolution in the HR diagram (Fig. 23.1) closely resembles a Hayashi track (see *Handout* 8 and *Handout* 20).

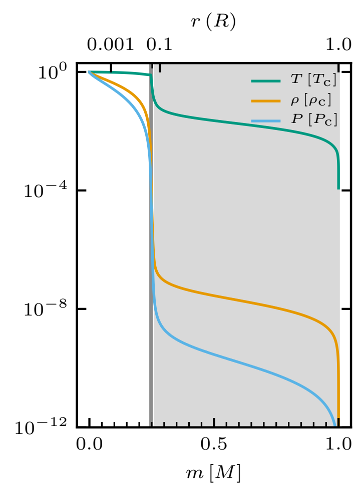

Figure 23.2: The temperature T, density ρ and pressure P (in units of their central values $T_c = 3.65 \times 10^7\,\mathrm{K}$, $\rho_c = 1.91 \times 10^5\,\mathrm{g\,cm^{-2}}$, $P_c = 2.01 \times 10^{21}\,\mathrm{Ba}$), plotted as a function of interior mass m for a *MESA* model of the Sun at a radius $R = 10\,\mathrm{R}_\odot$ on the RGB. The tick marks at the top indicate the position of layers with radial coordinates $r = 0.001, 0.01, 0.1$ and $1\,R$. Light-grey shading indicates convection, and dark-grey shading indicates the (very narrow) hydrogen burning shell. Compare against Fig. 6.1.

Shell Energy Generation

The hydrogen burning shell sits in the narrow region of rapidly varying density between the core and the envelope. Within the shell, the strong energy generation (primarily by the CNO cycle) results in a very steep luminosity gradient (see Fig. 23.3). Interior and exterior to the shell, however, the absence of any significant energy generation leads to a flat luminosity gradient there. Hence, the interior luminosity shows a characteristic step-like profile, as seen in the lower panel of Fig. 23.3, with the step coinciding with the burning shell.

During RGB evolution, the stellar luminosity grows significantly. To understand why, let's create simple scaling relations for the properties of the hydrogen-burning shell based on the following assumptions:

(i) the mass Δm and radial extent Δr of the shell are small compared to the mass M_{c} and radius R_{c} of the core;

(ii) the temperature, density and pressure at the top of the shell are negligible compared to their values at the bottom of the shell, and vice versa for the interior luminosity;

(iii) energy is transported within the shell by radiation, with an opacity that's independent of density and temperature;

(iv) neutrino losses are negligible within the shell, and nuclear energy generation follows the scaling $\epsilon_{\mathrm{nuc}} \propto \rho^{\alpha} T^{\beta}$.

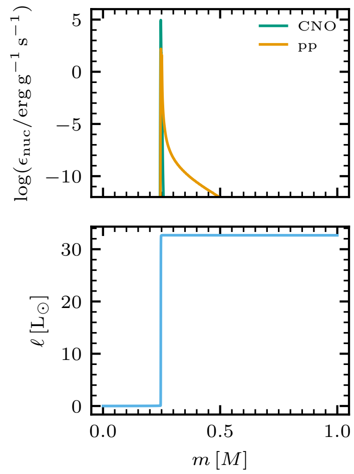

Figure 23.3: The logarithm of the nuclear energy generation rate ϵ_{nuc} from the pp chains and the CNO cycle (upper panel), and the interior luminosity ℓ (lower panel), plotted as a function of interior mass m for the same *MESA* model presented in Fig. (23.2).

Then, by applying the stellar structure equations [18.1]–[18.4] across the shell, we obtain[1]

$$\frac{\Delta r}{\Delta m} \propto \frac{1}{R_{\mathrm{c}}^2 \rho_{\mathrm{b}}}, \qquad \frac{P_{\mathrm{b}}}{\Delta m} \propto \frac{M_{\mathrm{c}}}{R_{\mathrm{c}}^4},$$
$$\frac{\ell_{\mathrm{t}}}{\Delta m} \propto \rho_{\mathrm{b}}^{\alpha} T_{\mathrm{b}}^{\beta}, \qquad \frac{T_{\mathrm{b}}}{\Delta m} \propto \frac{\ell_{\mathrm{t}}}{R_{\mathrm{c}}^4 T_{\mathrm{b}}^3}, \qquad [23.1]$$

[1] To derive the upper-left scaling relation from the mass equation [18.1], on the left-hand side approximate $\partial r/\partial m$ with $\Delta r/\Delta m$; and on the right-hand side, approximate r with R_{c} and ρ with ρ_{b}. Similar approaches give the other scaling relations.

where the subscripts 'b' and 't' refer to the bottom and top of the shell, respectively. We need two further equations to close this system. For one, we adopt the ideal-gas scaling $P_{\mathrm{b}} \propto \rho_{\mathrm{b}} T_{\mathrm{b}}$; for the other we assume that the shell's radial extent is a fixed fraction of the core radius, so that $\Delta r \propto R_{\mathrm{c}}$. Then, we solve the equations to find the following scalings[2]:

$$\ell_{\mathrm{t}} \propto M_{\mathrm{c}}^{\frac{4\alpha+\beta+4}{\alpha+2}} R_{\mathrm{c}}^{\frac{-3\alpha-\beta}{\alpha+2}}, \quad T_{\mathrm{b}} \propto M_{\mathrm{c}} R_{\mathrm{c}}^{-1}, \quad \Delta m \propto M_{\mathrm{c}}^{\frac{-\beta+4}{\alpha+2}} R_{\mathrm{c}}^{\frac{3\alpha+\beta}{\alpha+2}} \qquad [23.2]$$

[2] The scalings for P_{b} and ρ_{b} can also be derived, but are less interesting.

At the high temperatures encountered in hydrogen-burning shells, $\alpha = 1$ and $\beta \approx 13$. With these values, the luminosity and mass scalings above become

$$\ell_{\mathrm{t}} \propto M_{\mathrm{c}}^{7} R_{\mathrm{c}}^{-16/3}, \quad \Delta m \propto M_{\mathrm{c}}^{-3} R_{\mathrm{c}}^{16/3} \qquad [23.3]$$

As we'll see in the next handout, the core radius varies inversely with core mass. Therefore, these expressions indicates that the stellar luminosity[3] grows extremely rapidly with increasing core mass; and the shell mass likewise shrinks.

[3] Because there is no energy generation above the shell (see lower panel of Fig. 23.3), the stellar luminosity L matches the interior luminosity ℓ_{t} at the top of the shell.

Further Reading

Kippenhahn, Weigert & Weiss, §§33.1,33.2; *Ostlie & Carroll*, §13.2; *Prialnik*, §9.4.