1 GYRE stuff

1.1 Rotation inversion

First, let's gather some definitions. From GYRE's documentation,

$$U = \frac{\mathrm{d}\ln m}{\mathrm{d}\ln r} \tag{1}$$
$$= \frac{4\pi r^3\rho}{m} \tag{2}$$
$$c_1 = \frac{r^3}{R^3}\frac{M}{m} \tag{3}$$
$$\Rightarrow \frac{U}{c_1} = \frac{4\pi R^3\rho}{M} \tag{4}$$

Now, reading from GYRE's code and re-arranging,

$$\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{(\xi_r^2 + \lambda\xi_h^2)Ux^2}{c_1} \tag{5}$$
$$= \frac{4\pi R\rho}{M}(\xi_r^2 + \lambda\xi_h^2)r^2 \tag{6}$$
$$\Rightarrow \frac{M}{4\pi R}\frac{\mathrm{d}E}{\mathrm{d}x} = (\xi_r^2 + \lambda\xi_h^2)\rho r^2 \tag{7}$$
$$\Rightarrow \int_0^R (\xi_r^2 + \lambda\xi_h^2)\rho r^2 dr = \frac{ME}{4\pi} \tag{8}$$
$$\frac{\mathrm{d}\beta}{\mathrm{d}x} = \frac{4\pi(\xi_r^2 + (\lambda-1)\xi_h^2 - 2\xi_r\xi_h)Ux^2}{c_1 E} \tag{9}$$
$$= \frac{4\pi}{E}\frac{4\pi R\rho}{M}(\xi_r^2 + (\lambda-1)\xi_h^2 - 2\xi_r\xi_h)r^2 \tag{10}$$
$$\Rightarrow \frac{ME}{(4\pi)^2 R}\frac{\mathrm{d}\beta}{\mathrm{d}x} = (\xi_r^2 + (\lambda-1)\xi_h^2 - 2\xi_r\xi_h)\rho r^2 \tag{11}$$

Let's pile all this into the definition of the rotational splitting, from Aerts, Christensen-Dalsgaard & Kurtz (2010), eq. (3.354),

$$\delta\omega = m\int_0^R J\Omega dr \tag{12}$$

where

$$J = \frac{(\xi_r^2 + (\lambda-1)\xi_h^2 - 2\xi_r\xi_h)\rho r^2}{\int_0^R (\xi_r^2 + \lambda\xi_h^2)\rho r^2} \tag{13}$$
$$= \frac{ME}{(4\pi)^2 R}\frac{\mathrm{d}\beta}{\mathrm{d}x}\cdot\frac{4\pi}{ME} \tag{14}$$
$$= \frac{1}{4\pi R}\frac{\mathrm{d}\beta}{\mathrm{d}x} \tag{15}$$

The factor R is, I believe, absorbed by the fact that the integral is taken over x rather than R. That leaves just the factor 4π, which is what I've found by inserting a constant rotation rate $2\pi\,\mathrm{rad.s}^{-1}$ into a Sun-like model.