

Astronomy 730

Milky Way



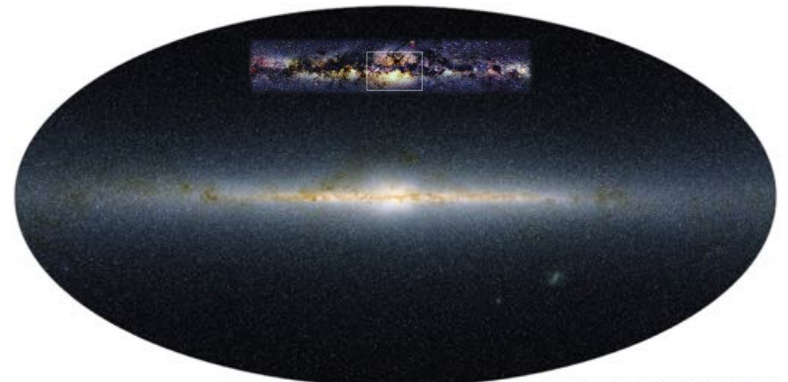
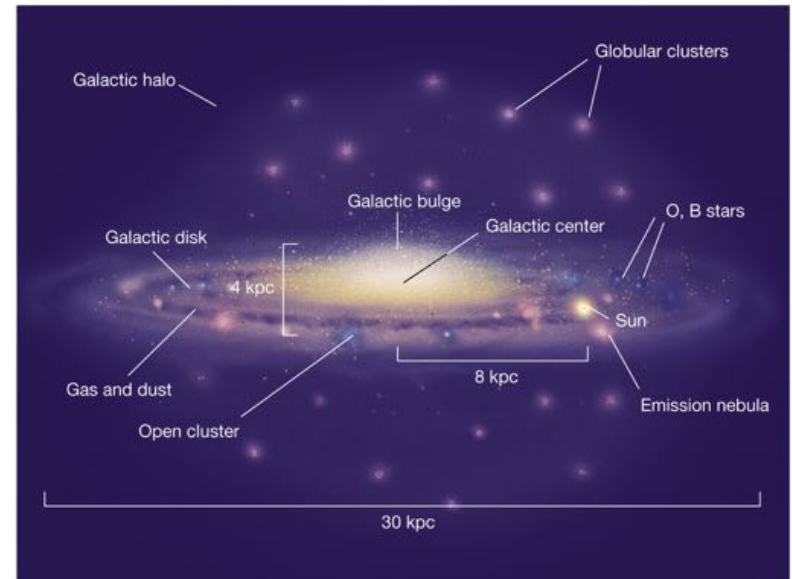
Outline

- ▶ The Milky Way
 - ▶ Star counts and stellar populations
 - ▶ Chemical cartography
 - ▶ Galactic center and bar
 - ▶ Galactic rotation
 - ▶ Departures from circular rotation



Modeling the Milky Way Galaxy (MWG)

- ▶ What is the stellar distribution?
- ▶ How big is the Milky Way?
 - ▶ Does (where does) the disk have an outer truncation?
- ▶ Does it have a bar?
- ▶ How do the stars move in the galaxy?
 - ▶ Galactic rotation and Oort's constants



Star Counts

► Formalism

- $N(M,S) = \int \Phi(M,S) D(r) r^2 dr$

- N = # of field stars of a given absolute magnitude (M) and spectral type (S) in the galaxy
- Φ = stellar luminosity function (# pc⁻³ for some spectral type)
- $D(r)$ = density distribution
 - may also depend on M, S : $D(r, M, S)$
 - Alternatively, Φ may also depend on r

► The simplest thing we can actually observe:

- $A(m)$ = # of stars of some apparent magnitude, m .
- $A(m) = \int \Phi(M) D(r) \Omega r^2 dr$
 - Ω = solid angle of survey

► If we have colors, we might get a crude $A(m,S)$

- but without distances or some luminosity indicator, we can't break the dwarf-giant degeneracy (e.g., K V vs K III).



Star Counts: Infinite Euclidean Universe

- ▶ $A(m)$ represents the differential counts, i.e., number of stars per apparent magnitude interval
- ▶ Knowing the geometry (locally Euclidean), the count slope (dA/dm) tells us about the spatial distribution of sources.
- ▶ For a *uniform* space-distribution of sources it is straightforward to show

- ▶ $d(\log A)/dm = 0.6m + \text{constant}$

- ▶ This leads to Olbers' paradox:

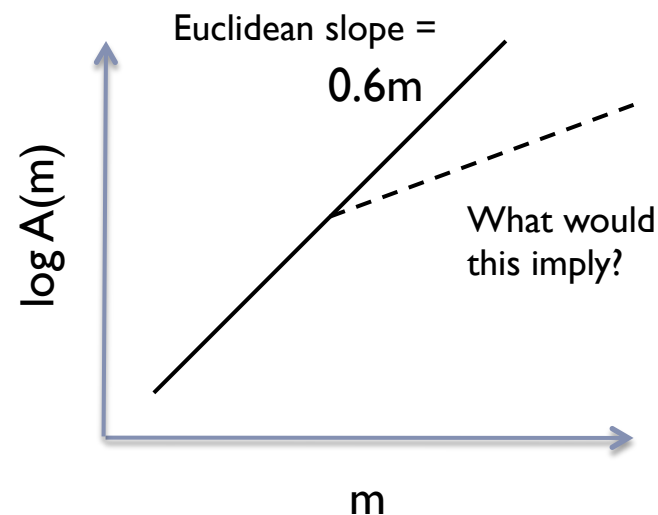
- ▶ $I(m) = I_0 \exp(-0.4m)$

- ▶ $L(m) = I(m)A(m)$

- ▶ $L_{\text{tot}} = \int I(m)A(m) dm = \infty$

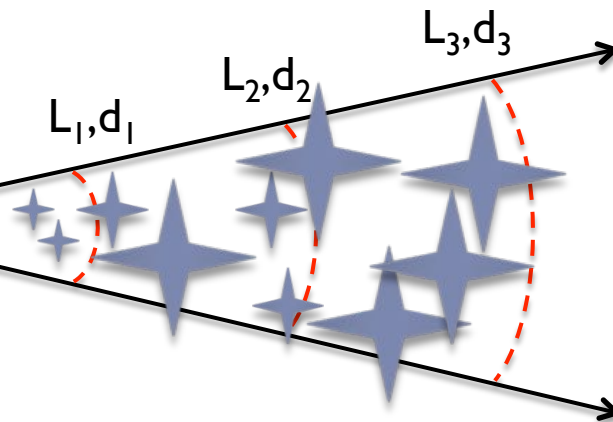
- ▶ The distribution of stars in the galaxy must be spatially finite

- ▶ *What about the universe of galaxies?*



The Malmquist Bias & the Night Sky

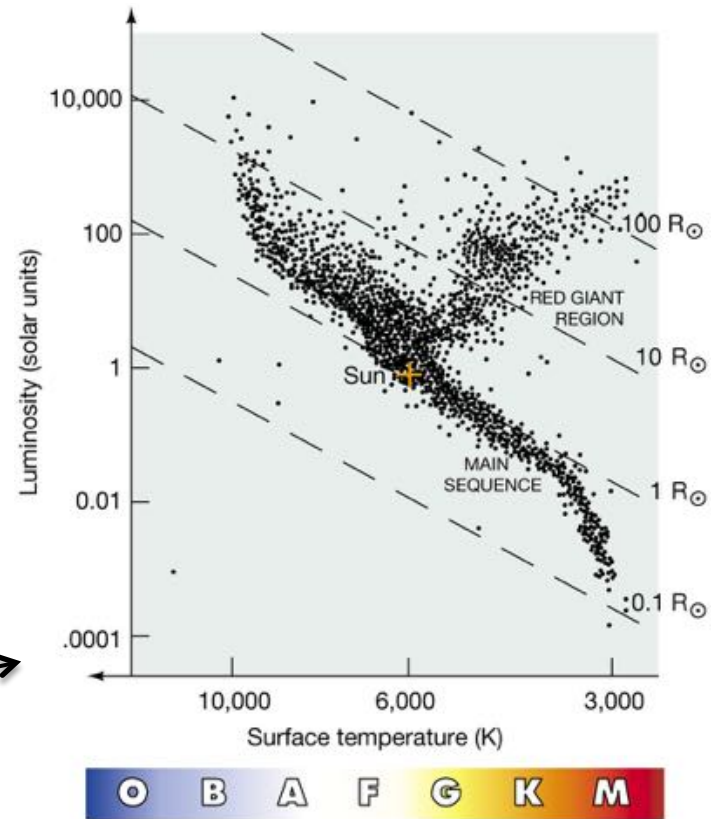
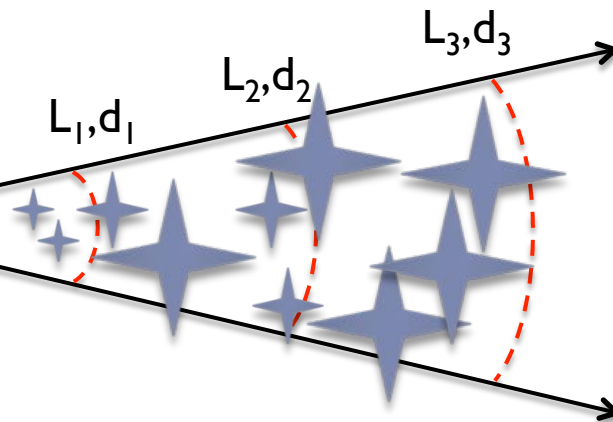
- ▶ What stars do you see when you look up at the night-time sky?
- ▶ Does this make sense given what you know about the HR diagram
- ▶ Malmquist bias:
 - ▶ You can see brighter objects farther away to a fixed m
 - ▶ Volume increases as d^3



For a uniform space-density, the observer is biased toward finding intrinsically luminous objects.

The Malmquist Bias & the Night Sky

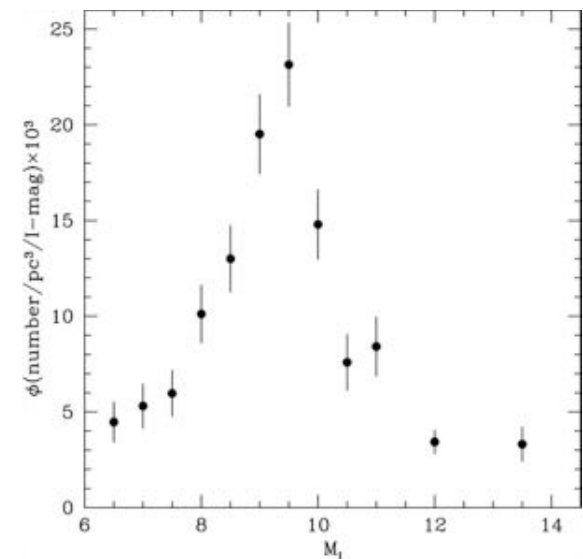
- ▶ What stars do you see when you look up at the night-time sky?
- ▶ Does this make sense given what you know about the HR diagram
- ▶ Malmquist bias:
 - ▶ You can see brighter objects farther away to a fixed m
 - ▶ Volume increases as d^3



For a uniform space-density, the observer is biased toward finding intrinsically luminous objects.

Star Counts & The Malmquist Bias

- ▶ What's the mean magnitude of stars with apparent magnitude, m ?
 - ▶ $M(m) = \int M \Phi(M) D(r) r^2 dr / \int \Phi(M) D(r) r^2 dr$
 - ▶ Recall $A(m) = \int \Phi(M) D(r) \Omega r^2 dr$
- ▶ Assume the stellar luminosity function (LF) is Gaussian for a given spectral type:
 - ▶ $\Phi(M, S) = \Phi_0 / (2\pi)^{1/2} \sigma \exp[-(M - M_0)^2 / 2 \sigma^2]$
 - ▶ M_0 = mean magnitude
 - ▶ Φ_0 = # pc⁻³ for some spectral type
 - ▶ σ is the distribution width



Zheng et al. (2004, ApJ, 601 500):
MW disk M-dwarfs, *I*-band, *HST*

Star Counts & The Malmquist Bias

- ▶ Push through the integral for $M(m)$ and move things around a bit....

- ▶ $M(m) = M_0 - [\sigma^2/A(m,S)] [dA(m,S)/dm]$

- ▶ Or:

$$M(m) - M_0 = -\sigma^2 d \ln A / dm$$

This is for the specific case of a Gaussian LF, but it can be generalized.

- ▶ If there are more stars at faint magnitudes, then the stars at some m are more luminous than the average for all stars in a given volume
 - ▶ This will come back to bite us with a vengeance when considering distant galaxy counts

A note on logarithmic derivatives:

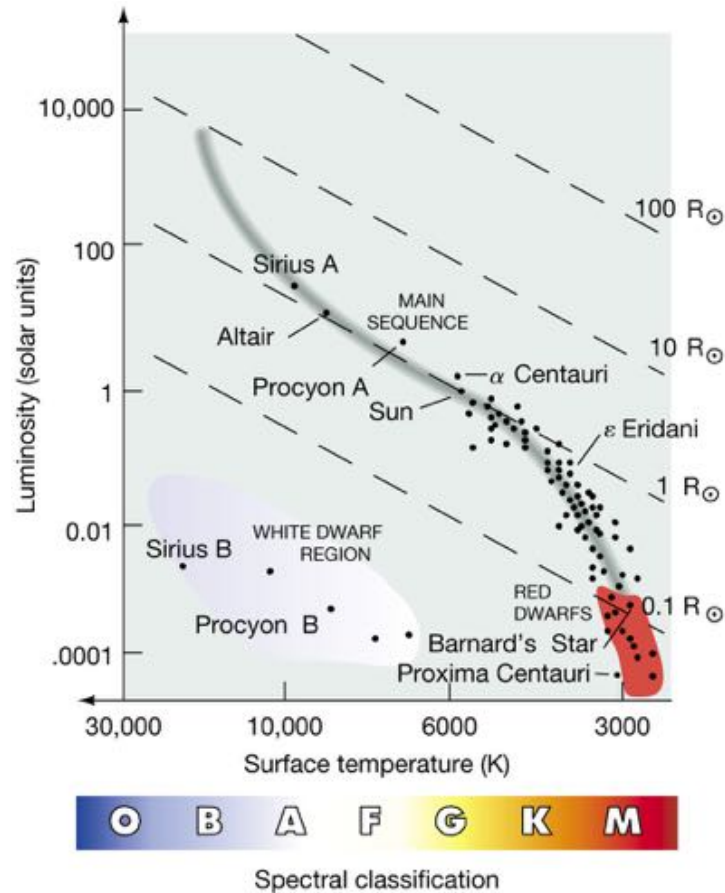
$$d \ln x = dx / x$$

This is nice because it normalizes the gradient to the amplitude of the signal, i.e., dimensionless.

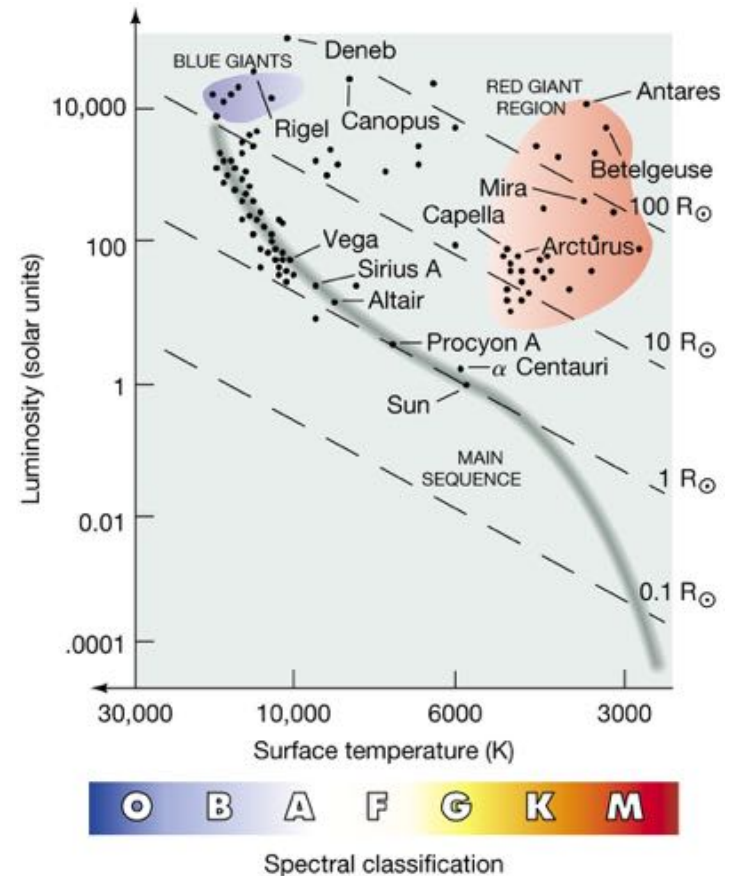
Therefore often used in Astronomy

Space Densities: Local Neighborhood

Volume-limited



Brightness-limited



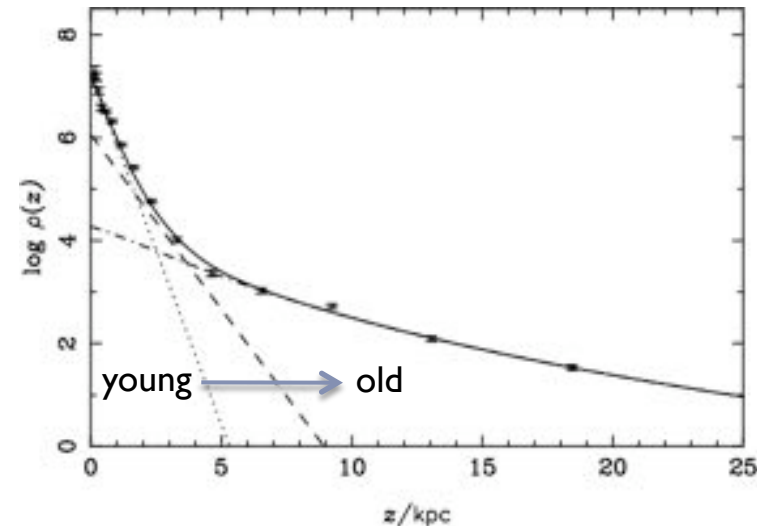
Stellar Luminosity Function

- ▶ Measure for a distance(volume)-limited sample
- ▶ Bahcall & Soneira (1980) used:
 - ▶ $\Phi(M) = [n_* 10^{\beta(M-M_*)}] / [1 + 10^{-(\alpha - \beta)\delta(M-M_*)}]^{1/\delta}$
 - ▶ $n_* = 4.03 \times 10^{-3}$
 - ▶ $M_* = 1.28$
 - ▶ $\alpha = 0.74, \beta = 0.04, 1/\delta = 3.40$
- ▶ See also Figure 2.4 in S&G.
- ▶ Basic results
 - ▶ 10^5 times more G stars than O stars
 - ▶ Nearby stars tend to be
 - ▶ low-luminosity and apparently faint
 - ▶ Average $M/L = 0.67 (M/L)_{\odot}$



Star Counts: The Disk

- ▶ Star counts (z direction)
 - ▶ $n(z)$ proportional to $\exp(-z/z_0(m))$,
 - ▶ $z_0(m)$ is the scale height (and, yes, it does vary with magnitude)
 - ▶ More importantly, z_0 varies with spectral type:
 - ▶ young : old \rightarrow small : large
 - ▶ **WHY?**
 - ▶ Reid & Majewski (1993)
 - ▶ Thin disk with $z_0 = 325$ pc – Pop I
 - ▶ Thick disk with $z_0 = 1200$ pc – Pop II
- ▶ The Disk
 - ▶ $I(R) = I(0)\exp[-R/h_R]$
 - ▶ tough to measure in our own galaxy, but measurable in other disks pretty easily.
- ▶ Disk is really a double exponential:
 - ▶ $I(R,z) = I(0,0)\exp(-z/z_0 - R/h_R)$



Du et al. 2003 A&A 407 541,
stellar density perpendicular to
the plane

Star Counts: The Halo

- ▶ Halo stars

- ▶ Halo stars are faint, need an easy to find tracer

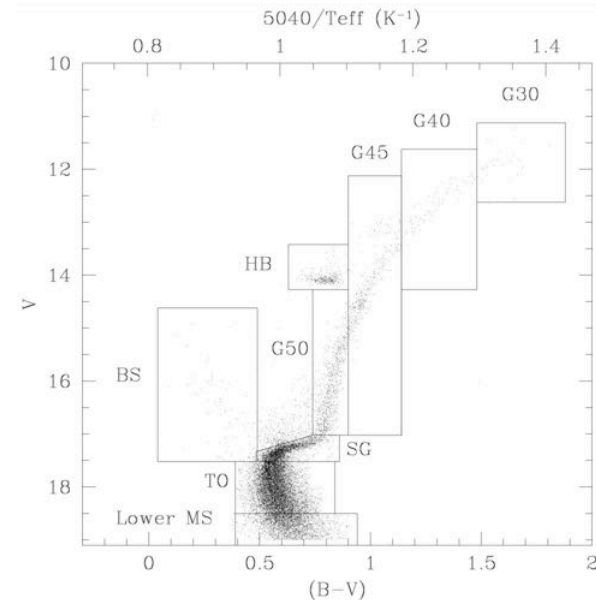
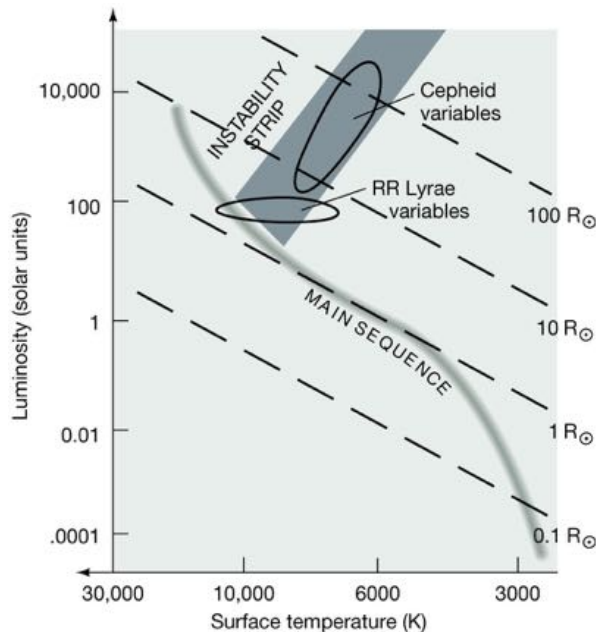
RR Lyrae stars

- ▶ Stellar density falls off as r^{-3}
 - ▶ Looks like the distribution of globular clusters, which you also get from RR Lyrae stars
- ▶ We will come back to this density fall-off when we consider the rotation curve.



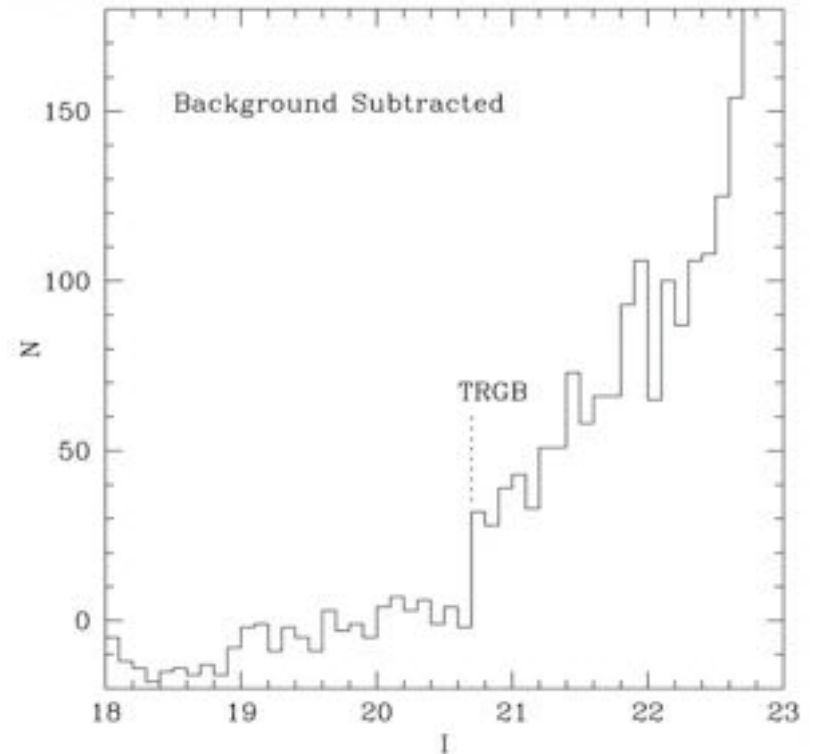
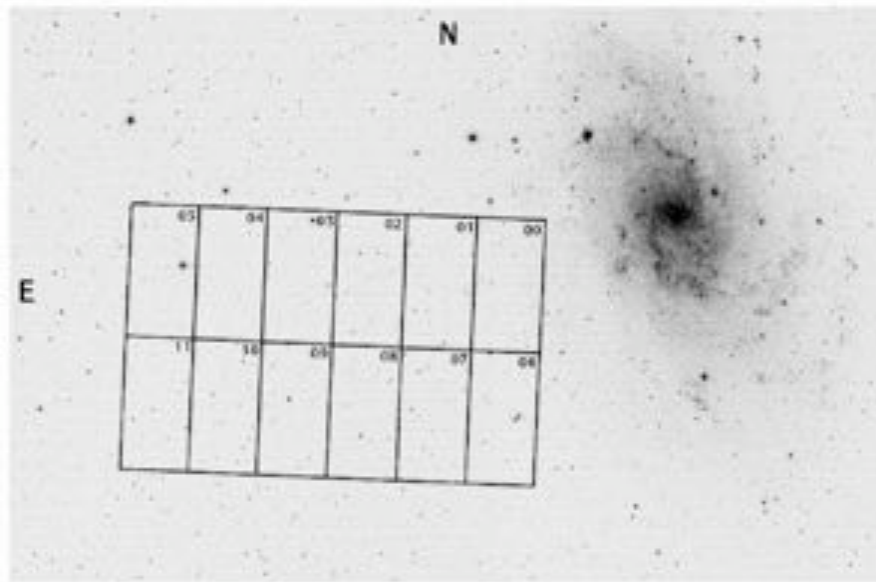
Recall: RR Lyrae Stars

- ▶ HB stars in “the instability strip”
 - ▶ Solar mass
 - ▶ Opacity driven pulsations yield variability which is correlated with M
 - ▶ $M = -2.3 \pm 0.2 \log(P) - 0.88 \pm 0.06$ (with some additional variation due to metallicity)
 - ▶ Old, low mass stars (hence good tracers of the halo)
- ▶ Higher mass (farther up the instability strip you’ll find Cepheids)



M33 LF

- ▶ Outer fields of M33
 - ▶ (Brooks et al. 2004, AJ, 128, 237)



Counts proportional to luminosity function of *all* stars in M33 halo, corrected for contamination and completeness. TRGB at $I = 20.7$ gives distance modulus.

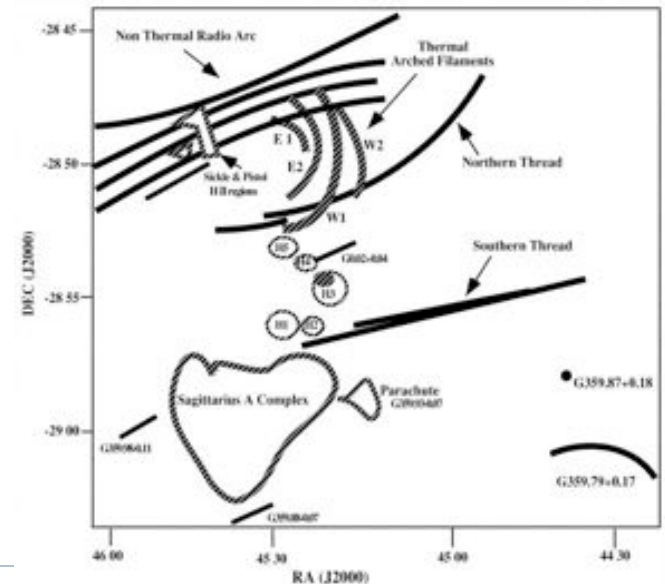
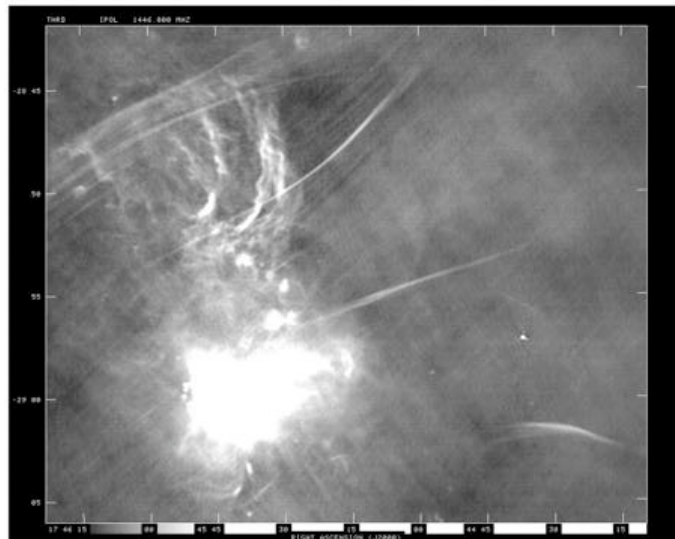
Galactic Center

- ▶ We'll talk about the center again when we discuss AGNs

- ▶ The optical view:



- ▶ The VLA 1.4 Ghz view:



Galactic Center: Distance

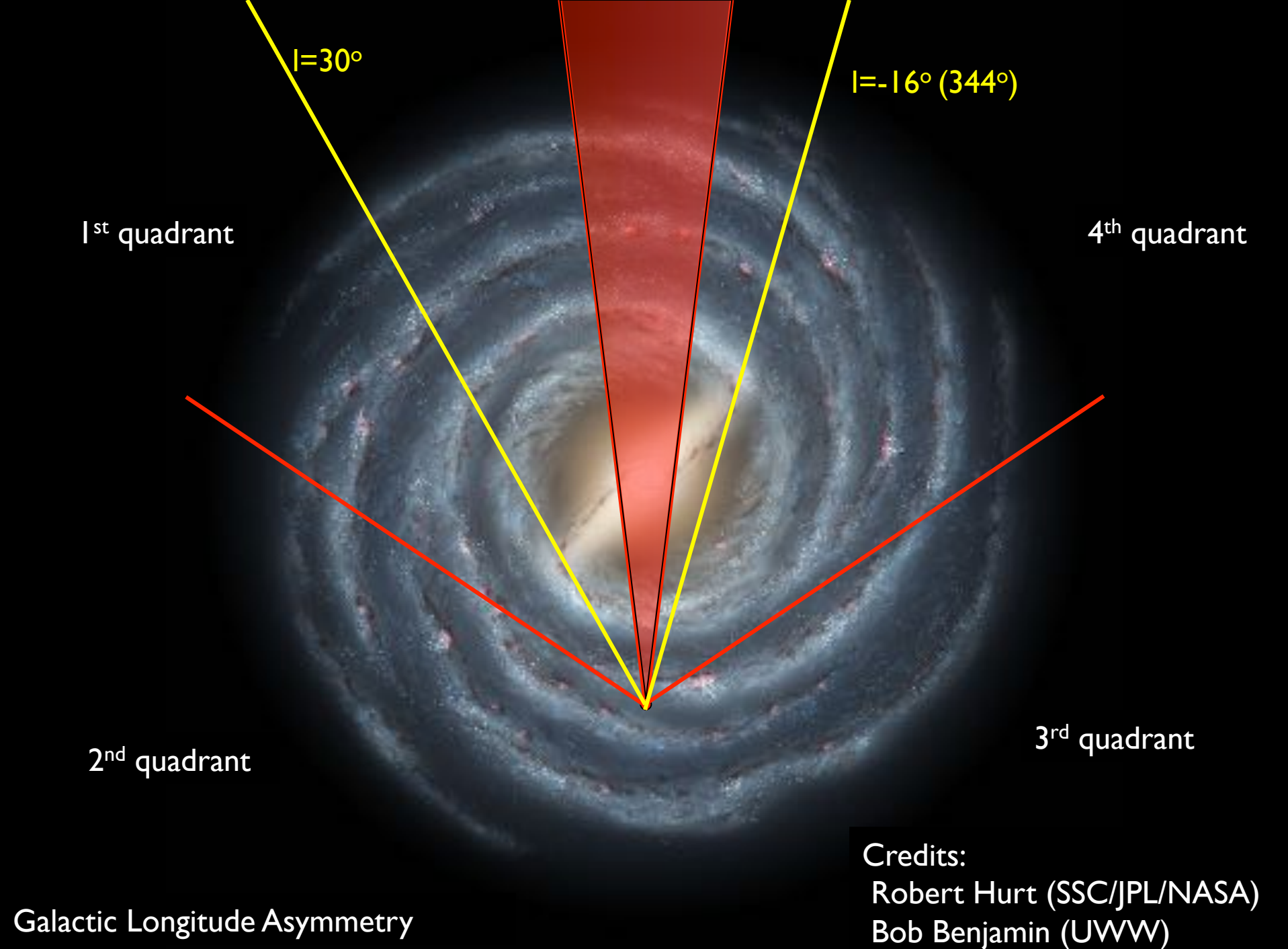
- ▶ Use RR Lyraes + other stellar tracers
 - ▶ Use the globular cluster population, OH/IR stars in the bulge
 - ▶ Get mean distances → 8.5 kpc
- ▶ Proper motion studies of Sgr A*
 - ▶ Look for maser emission
 - ▶ Follow maser proper motion + observed velocity
→ distance (7.5 kpc)



Galactic Bar

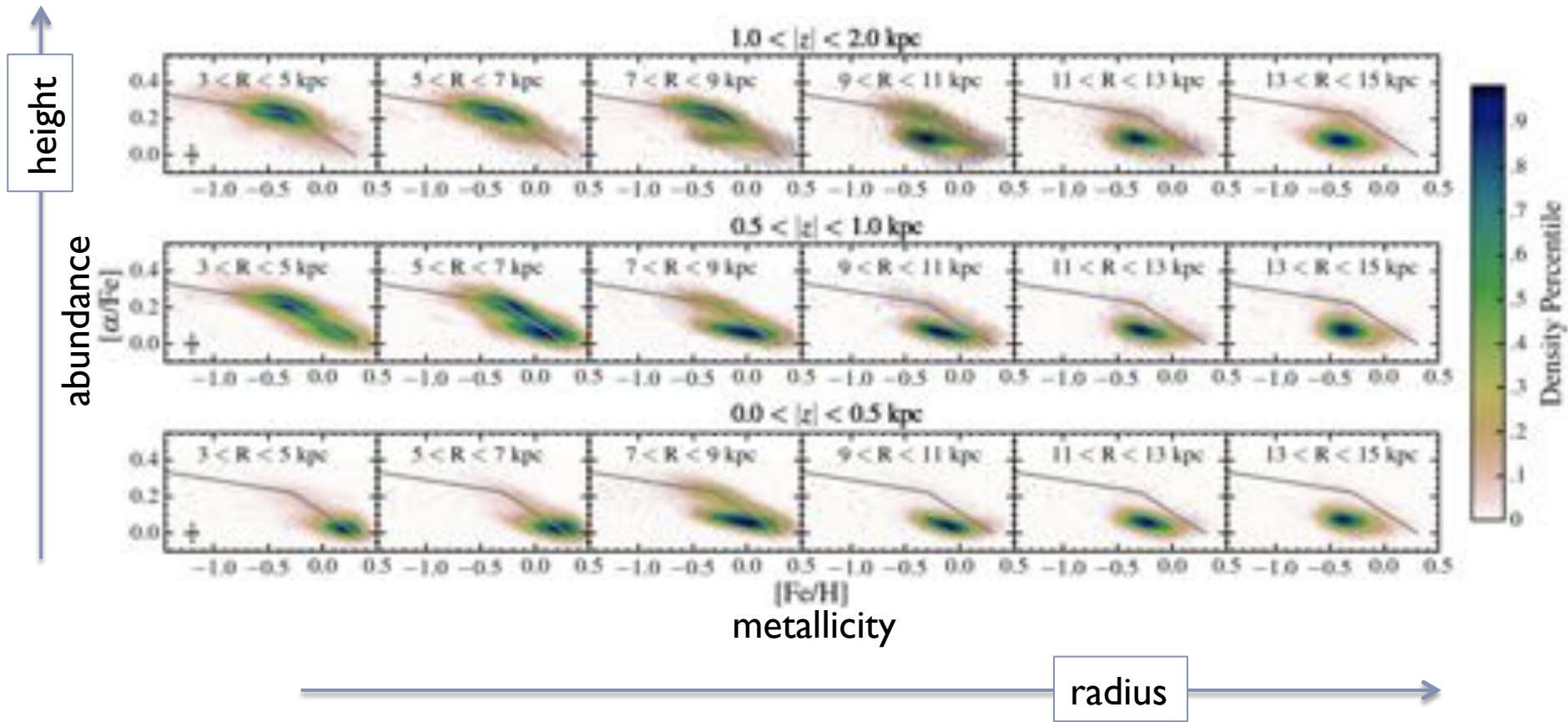
- ▶ Lots of other disk galaxies have a central bar (elongated structure). Does the Milky Way?
- ▶ Photometry – what does the stellar distribution in the center of the Galaxy look like?
 - ▶ Bar-like distribution: $N = N_0 \exp(-0.5r^2)$, where $r^2 = (x^2+y^2)/R^2 + z^2/z_0^2$
 - ▶ Observe $A(m)$ as a function of Galactic coordinates (l,b)
 - ▶ Use N as an estimate of your source distribution:
 - ▶ counts $A(m,l,b)$ appear bar-like
 - ▶ Sevenster (1990s) found overabundance of OH/IR stars in 1st quadrant. Asymmetry is also seen in RR Lyrae distribution.
- ▶ Gas kinematics: $V_c(r) = (4\pi G \rho / 3)^{1/2} r$
 - ➔ we should see a straight-line trend of $V_c(r)$ with r through the center (we don't).
- ▶ Stellar kinematics – again use a population of easily identifiable stars whose velocity you can measure (e.g. OH/IR stars).
 - ▶ Similar result to gas.





Chemical Cartography

► The APOGEE-I view from SDSS-III



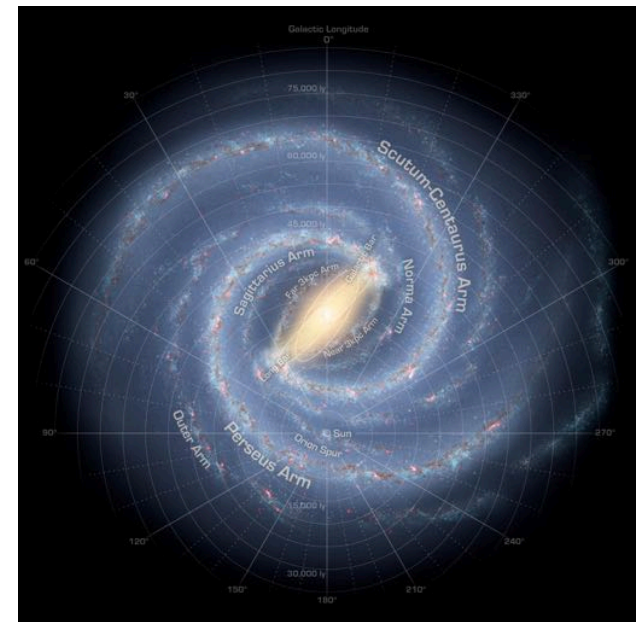
► Also: RAVE, SEGUE, GALAH/HERMES, APOGEE-2

Hayden+ '15

Galactic Rotation: A Simple Picture

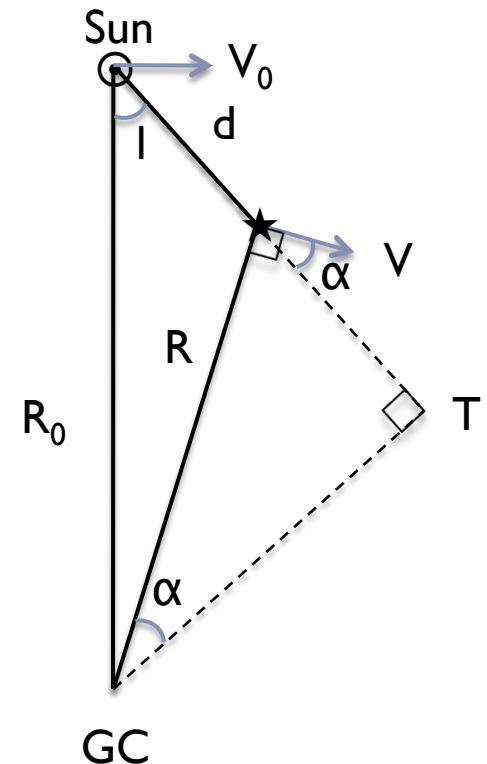
- ▶ Imagine two stars in the Galactic disk; the Sun at distance R_0 , the other at a distance R from the center and a distance, d , from the Sun. The angle between the Galactic Center (GC) and the star is l , and the angle between the motion of the stars and the vector connecting the star and the Sun is α . The Sun moves with velocity, V_0 , and the other star moves with velocity, V .

- ▶ See Figure 2.19 in S&G.



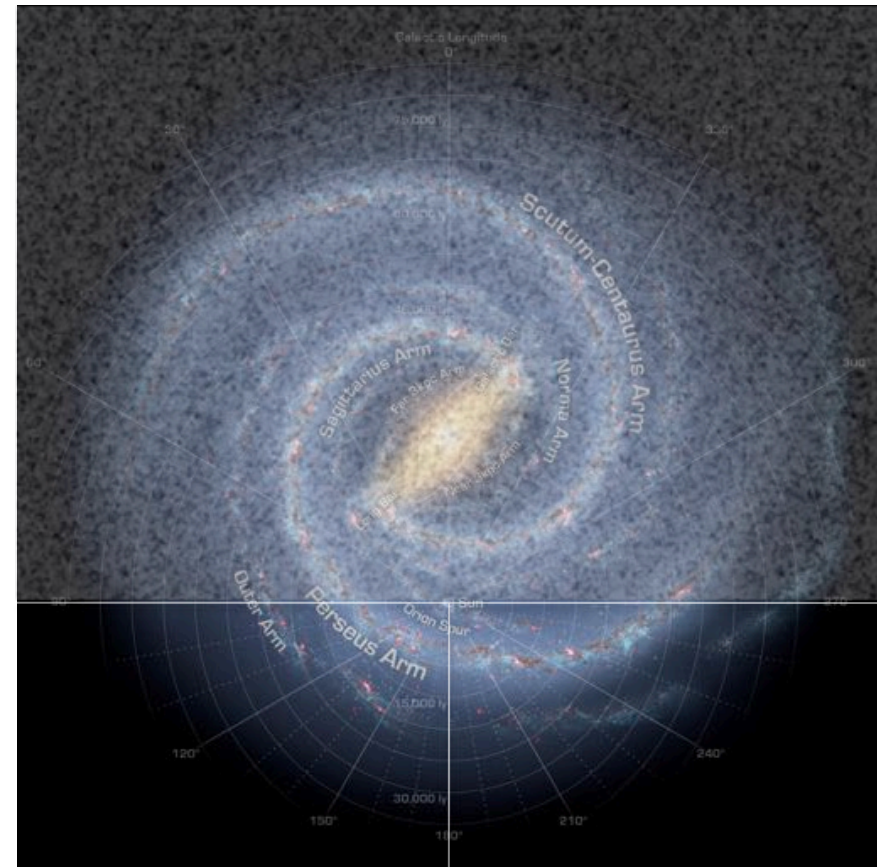
Relative motion of stars

- ▶ Radial velocity of the star
 - ▶ $V_r = V \cos \alpha - V_0 \sin l$
 - ▶ now use law of sines to get...
 - ▶ $V_r = (\omega_* - \omega_0) R_0 \sin l$,
 - ▶ ω is the angular velocity defined as V/R .
 - ▶ l is the Galactic longitude
- ▶ Transverse velocity of the star
 - ▶ $V_T = (\omega_* - \omega_0) R_0 \cos l - \omega_* d$



Longitudinal dependence

- ▶ $90^\circ \leq l \leq 180^\circ$
 - ▶ larger d
 - ▶ $R > R_0$
 - ▶ $\omega_*^* < \omega_0$
 - ▶ this means increasingly negative radial velocities
- ▶ $180^\circ \leq l \leq 270^\circ$
 - ▶ V_R is positive and increases with d

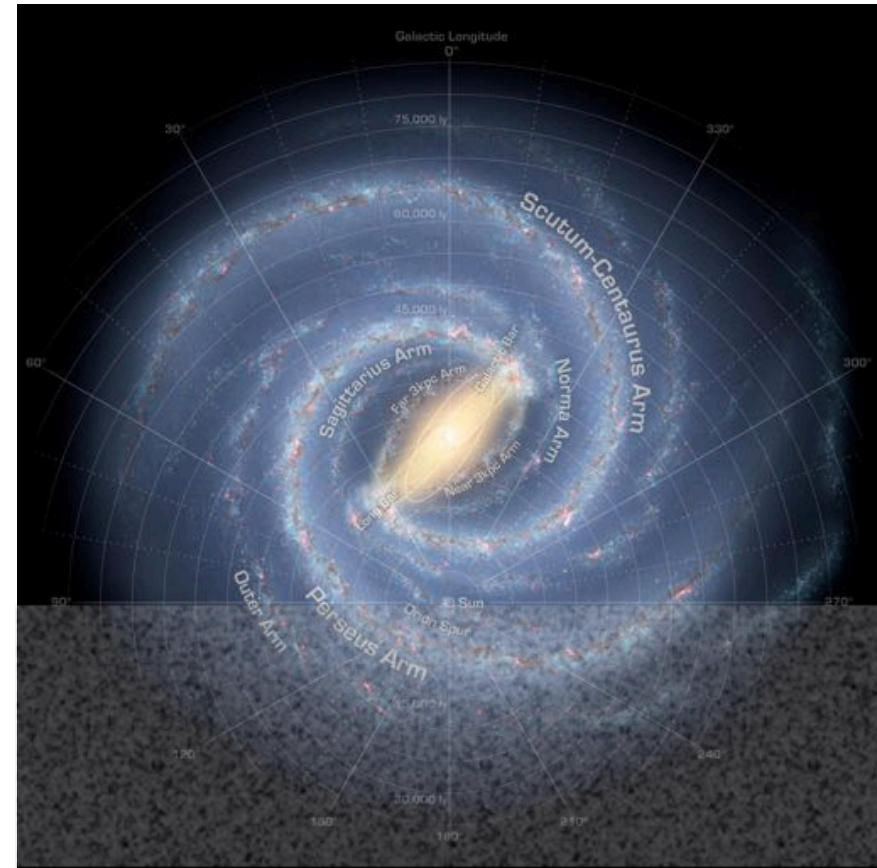


$90^\circ \leq l \leq 180^\circ$

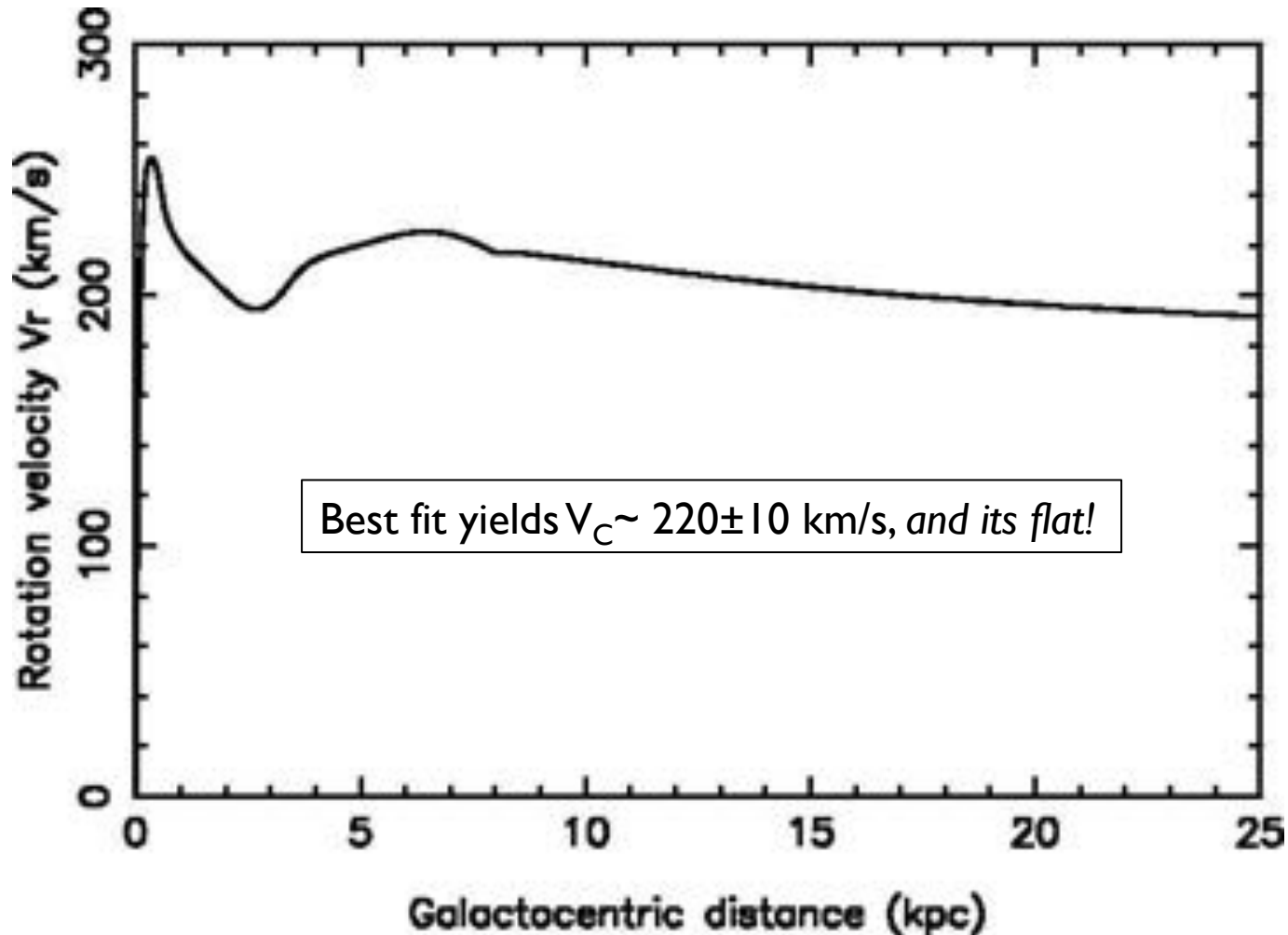
$180^\circ \leq l \leq 270^\circ$

Longitudinal dependence

- ▶ $0^\circ \leq l \leq 90^\circ$
 - ▶ starting with small R , large ω
 - ▶ At some point $R = R_0 \sin(l)$ and $d = R_0 \cos(l)$
 - ▶ Here, V_R is a maximum \rightarrow tangent point.
 - ▶ We can derive $\omega_*(R)$ and thus the *Galactic Rotation Curve*!
- ▶ Breaks down at $l < 20^\circ$ (why?) and $l > 75^\circ$ (why?), but it's pretty good between 4-9 kpc from Galactic center.

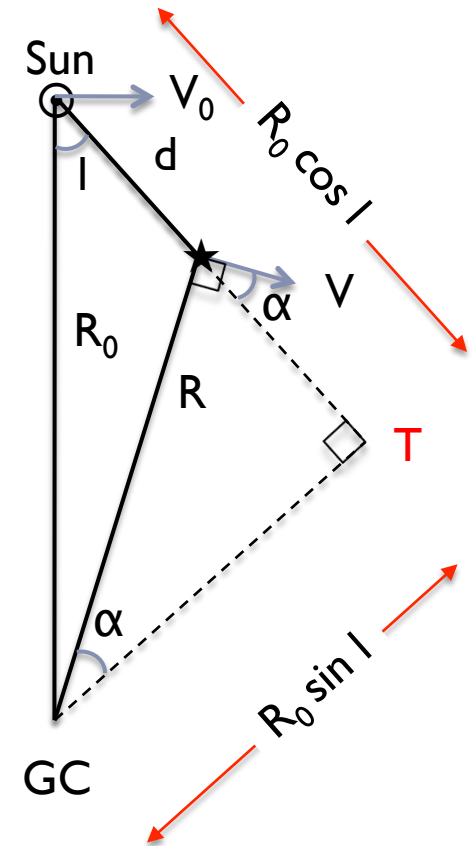


Galactic Rotation Curve



Galactic rotation

- ▶ Inner rotation curve from “tangent point” method
 - ➔ $V_{\text{circ},\odot} = 220 \text{ km s}^{-1}$
 - ▶ Derived from simple geometry based on a nearby star at distance, d , from us.
 - ▶ Tangent point where $R = R_0 \sin l$ and $d = R_0 \cos l$:
Observed V_R is a maximum
- ▶ Outer rotation curve from Cepheids, globular clusters, HII regions ➔ anything you can get a real distance for
- ▶ Best fit: $(220 \pm 10 \text{ km/s})$ depends on R_0 (think back to the geometry)
- ▶ Yields $\omega_0 = V_0/R_0 = 29 \pm 1 \text{ km s}^{-1} \text{ kpc}^{-1}$



Rotation model

- ▶ Observations of local kinematics can constrain the global form of the Galactic rotation curve
- ▶ Components of rotation model:
 - ▶ Oort's constants which constrain local rotation curve.
 - ▶ Measurement of R_0
 - ▶ Global rotation curve shape (e.g., flat)
- ▶ Oort's constants A and B:
 - ▶ $\omega_0 = V_0/R_0 = A - B$
 - ▶ $(dV/dR)_{R_0} = -(A + B)$
 - ▶ $V_{c,\odot} = R_0(A - B)$



Oort's Constant A: Disk Shear

- ▶ Assume d is small
 - ▶ this is accurate enough for the solar neighborhood
- ▶ Expand $(\omega_* - \omega_0) = (d\omega/dR)_{R_0}(R - R_0)$
- ▶ Do some algebra....
 - ▶ $V_R = [(dV/dR)_{R_0} - (V_0/R_0)] (R - R_0) \sin l$
- ▶ If $d \ll R_0$,
 - ▶ $(R_0 - R) \sim d \cos(l)$
 - ▶ $V_R = A d \sin(2l)$
- ▶ where $A = \frac{1}{2}[(V_0/R_0) - (dV/dR)_{R_0}]$
 - ▶ This is the 1st Oort constant, and it measures the shear (deviation from rigid rotation) in the Galactic disk.
 - ▶ In solid-body rotation $A = 0$
- ▶ If we know V_R and d , then we know A and $(d\omega/dR)_{R_0}$



Oort's Constant B: Local Vorticity

- ▶ Do similar trick with the transverse velocity:
 - ▶ $V_T = d [A \cos(2l) + B]$, and
 - ▶ $\mu_l = [A \cos(2l) + B] / 4.74$ = proper motion of nearby stars
- ▶ B is a measure of angular-momentum gradient in disk (vorticity: tendency of objects to circulate around)
- ▶ $B = -12.4 \pm 0.6$ km/s/kpc
 - ▶ A measure of angular-momentum gradient in disk
- ▶ $\omega_0 = V_0 / R_0 = A - B$
- ▶ $(dV/dR)_{R_0} = -(A + B)$
- ▶ Observations of local kinematics can constrain the global form of the Galactic rotation curve



Measuring Oort's Constants

- ▶ Requires measuring V_R , V_T , and d
- ▶ V_R and d are relatively easy
- ▶ V_T is hard because you need to measure proper motion
 - ▶ μ (arcsec yr⁻¹) = $V_T(\text{km s}^{-1})/d$ (pc) = $V_T/4.74d$
 - ▶ Proper motions + parallaxes
- ▶ $A = 14.82 \text{ km s}^{-1} \text{ kpc}^{-1}$, $B = -12.4 \pm 0.6 \text{ km s}^{-1} \text{ kpc}^{-1}$
- ▶ The interesting thing you also measure is the relative solar motion with respect to the Local Standard of Rest (LSR)



Solar Motion

- ▶ Stellar motion in the disk is basically circular with some modest variations.
- ▶ There is an increase in the velocity dispersion of disk stars with color → age
 - ▶ Seen in vertical, radial, and azimuthal dimensions
 - ▶ Results in v_{\odot} correlation with (B-V)
 - ▶ What about the thickness of the disk?
- ▶ Disk stars come in all different ages, but tend to be metal rich...



Solar Motion

- ▶ LSR \equiv velocity of something moving in a perfectly circular orbit at R_0 and always residing exactly in the mid-plane ($z=0$).
- ▶ Define cylindrical coordinate system:
 - ▶ R (radial)
 - ▶ z (perpendicular to plane)
 - ▶ ϕ (azimuthal)
- ▶ *Residual* motion from the LSR:
 - ▶ u = radial, v = azimuthal, w = perpendicular
- ▶ *Observed* velocity of star w.r.t. Sun:
 - ▶ $U_* = u_* - u_{\odot}$, etc. for v, w
- ▶ Define means:
 - ▶ $\langle u_* \rangle = (1/N) \sum u_*$, summing over $i=1$ to N stars, etc. for v, w
 - ▶ $\langle U_* \rangle = (1/N) \sum U_*$, etc for V, W



Solar Motion

▶ Assumptions you can make

- ▶ Overall stellar density isn't changing
 - ▶ there is no net flow in either u (radial) or w (perpendicular):
 - ▶ $\langle u_* \rangle = \langle w_* \rangle = 0$.
- ▶ If you do detect a non-zero $\langle U_* \rangle$ or $\langle W_* \rangle$, this is the reflection of the Sun's motion:
- ▶ $u_{\odot} = -\langle U_* \rangle$, $w_{\odot} = -\langle W_* \rangle$, $v_{\odot} = -\langle V_* \rangle + \langle v_* \rangle$

▶ Dehnen & Binney 1998 MNRAS 298 387 (DB88)

- ▶ Parallaxes, proper motions, etc for solar neighborhood (disk pop only)
- ▶ $u_{\odot} = -10.00 \pm 0.36 \text{ km s}^{-1}$ (inward; DB88 call this U_0)
- ▶ $v_{\odot} = 5.25 \pm 0.62 \text{ km s}^{-1}$ (in the direction of rotation; DB88 call V_0)
- ▶ $w_{\odot} = 7.17 \pm 0.38 \text{ km s}^{-1}$ (upward; DB88 call this W_0)
- ▶ No color dependency for u and w , but for v



Solar Motion

▶ Leading & Lagging

- ▶ Stars on perfectly circular orbits with $R=R_0$ will have $\langle V \rangle = 0$.
- ▶ Stars on elliptical orbits with $R > R_0$ will have higher than expected velocities at R_0 and will “lead” the Sun
- ▶ Stars on elliptical orbits with $R < R_0$ will have lower than expected velocities at R_0 and will “lag” the Sun

▶ Clear variation in v_\odot with (B-V)!

- ▶ Why?
- ▶ Why only v and not u or w ?

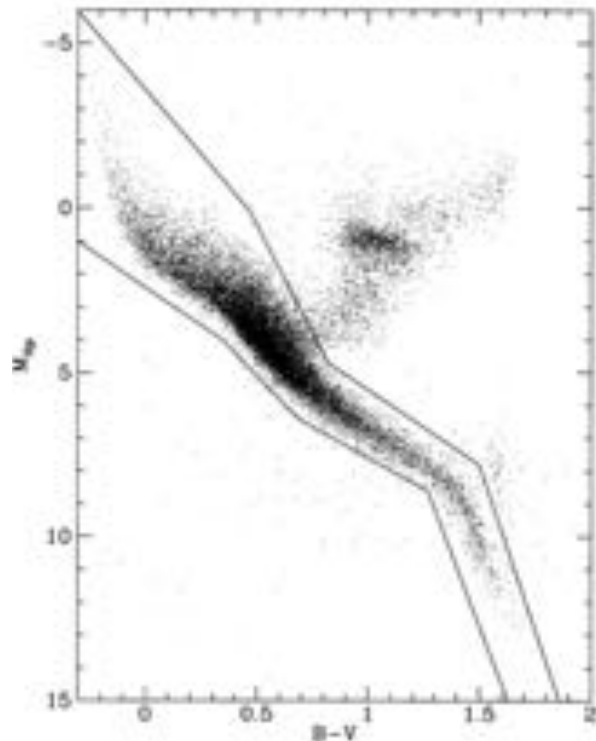
▶ We can also measure the random velocity, S^2 , and relate this to v_\odot . This correlation is actually predicted by theory (as we shall see)!

- ▶ $S = [\langle u^2 \rangle + \langle v^2 \rangle + \langle w^2 \rangle]^{1/2}$

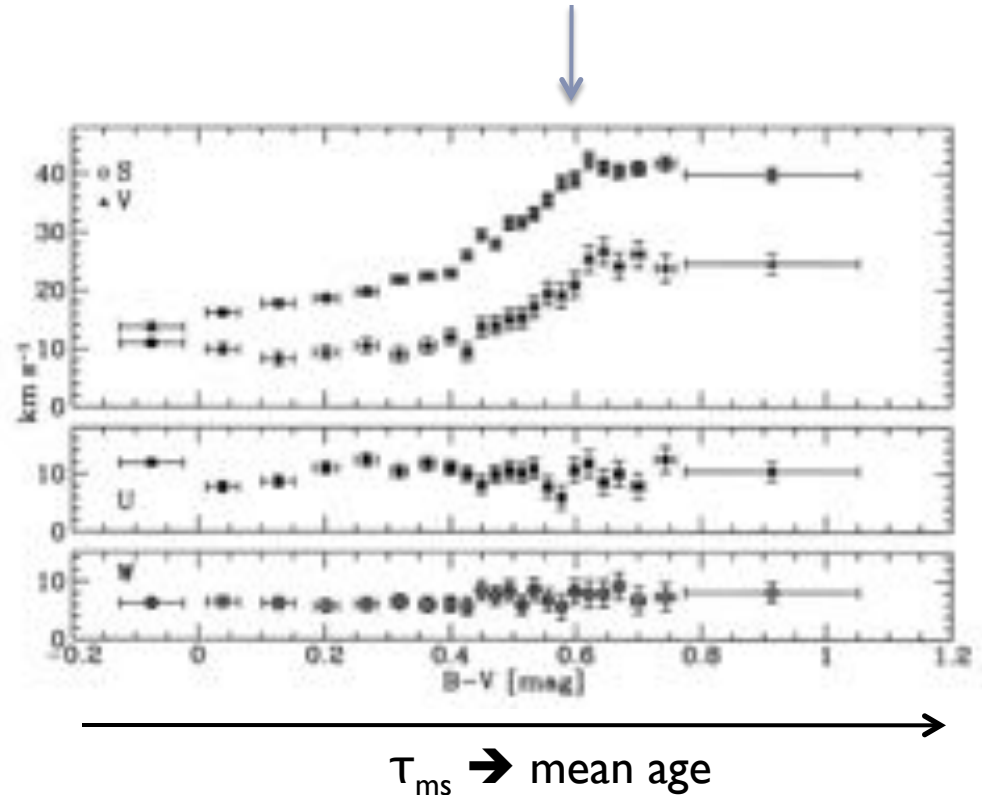


Parenago's Discontinuity

Clues to disk evolution:



Hipparcos catalogue:
geometric parallax and
proper motions



Binney et al. (2000, MNRAS, 318, 658)

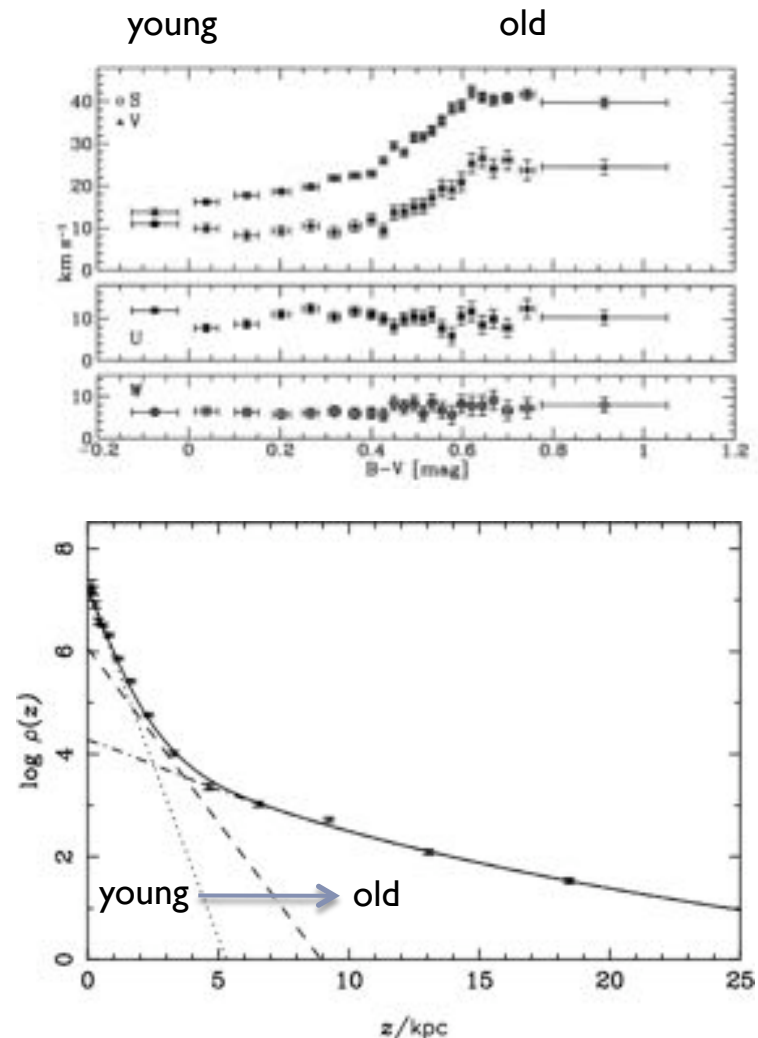
$S = S_0 [1 + (t/\text{Gyr})^{0.33}]$ ← random grav. encounters

$S_0 = 8 \text{ km s}^{-1}$ ← why might this be?

See also Wielen 1977, A&A, 60, 263

Parenago's Discontinuity: the disk

- ▶ The disk is observed to be well described by a double exponential in radius (R) and vertical height (z)
 - ▶ Revisit nomenclature from lecture 6 to be consistent with S&G:
 - ▶ $\rho(R,z) = \rho_0 \exp(-z/h_z) \exp(-R/h_R)$
 - ▶ ρ is matter density, e.g., in stars $\rho_* = n_* \times m_*$
 - ▶ Integrate $\rho(R,z)$ in z to get $\Sigma(R)$, e.g. $M_\odot \text{ pc}^{-2}$
 - ▶ $\Sigma(R) = \int \rho(R,z) dz$
 - ▶ Multiply by the mass-to-light ratio ($M/L = Y$) to get $I(R)$, the surface-brightness : $I(R) = Y^{-1} \times \Sigma(R)$
 - ▶ $\mu(R)$ often is used to denote surface-brightness in magnitudes arcsec^{-2} .
 - ▶ $\mu(R,\theta)$ would be surface-brightness at location R, θ in the disk (cylindrical coordinates)
 - ▶ Integrate $\Sigma(R)$ in R to get total mass within a given radius $M(R)$, ... or $I(R)$ to get total light
 - ▶ $M(R) = 2\pi \int \Sigma(R) r dr$
- ▶ Why is the distribution exponential in radius?
 - ▶ This is hard to answer definitively, but it is an observed fact.
 - ▶ Why is the distribution exponential in height?
 - ▶ Here we will attempt to get a better physical standing in coming lectures.



The Halo: Clues to formation scenario?

► Layden 1995 AJ 110 2288

- Age of halo RR Lyrae stars > 10 Gyr
- $-2.0 < [\text{Fe}/\text{H}] < -1.5$; $V_{\text{rot}}/\sigma_{\text{los}} \sim 0$; $\sigma_{\text{los}} \sim 100\text{-}200 \text{ km s}^{-1}$
- $-1.0 < [\text{Fe}/\text{H}] < 0$; $V_{\text{rot}}/\sigma_{\text{los}} \sim 4$; $\sigma_{\text{los}} \sim 50 \text{ km s}^{-1}$

► Relative to LSR

- $\langle U \rangle = -13 \text{ km s}^{-1}$
- $\langle W \rangle = -5 \text{ km s}^{-1}$
- $\langle V \rangle_{[\text{Fe}/\text{H}] < -1.0} = 40 \text{ km s}^{-1}$
- $\langle V \rangle_{[\text{Fe}/\text{H}] > -1.0} = 200 \text{ km s}^{-1}$

Velocity dispersion defined:

$$\sigma_{\text{los}}^2 = \int (v_{\text{los}} - \underline{v})^2 F(v_{\text{los}}) dv_{\text{los}}$$

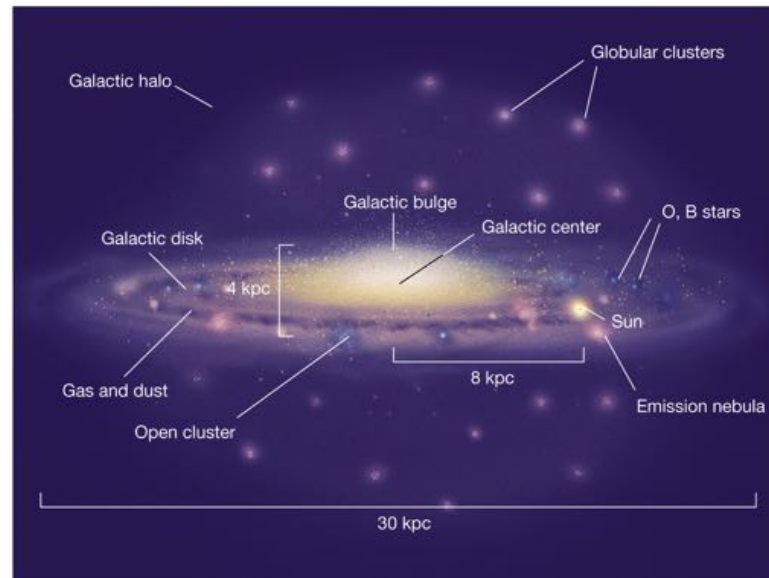
or, $\sigma_{\text{los}} = ((v - \underline{v})^2)^{1/2}$

where $F(v_{\text{los}})$ = velocity distribution function

- Conclusion: there is an extended old, metal poor stellar halo dominated by random motions with very little, if any, net rotation ($0 < V < 50 \text{ km/s}$)

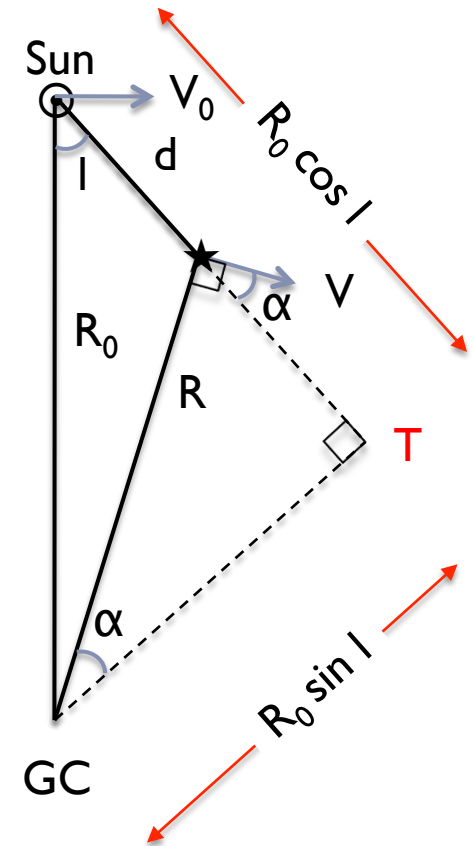
Globular Cluster Population

- ▶ Harris, W.E. 2001 “Star Clusters”
 - ▶ ~150 globular clusters in MWG
 - ▶ Distribution is spherically symmetric, density falls off as $R_{GC}^{-3.5}$
 - ▶ Bimodal metallicity distribution
 - ▶ $[Fe/H] \sim -1.7$ (metal-poor) → found in halo
 - ▶ $[Fe/H] \sim -0.2$ (metal rich) → found in bulge



Measuring Galactic Rotation

- ▶ **Gas:**
 - ▶ Good because the MW is optically thin at CO (mm) and HI (21cm) wavelengths
 - ▶ Bad because you have to use the tangent method –
 - ▶ essentially impossible to measure distances
- ▶ **Stars:**
 - ▶ Good because you can measure distances directly
 - ▶ Bad because it is difficult to measure distances for distant or faint stars
 - ▶ Bad because traditional studies are done in optical, which can't penetrate mid-plane dust
- ▶ ... enter the Sloan Digital Sky Survey (SDSS):



Measuring Galactic Rotation: Example

- ▶ Select stars of a single spectral type....A stars

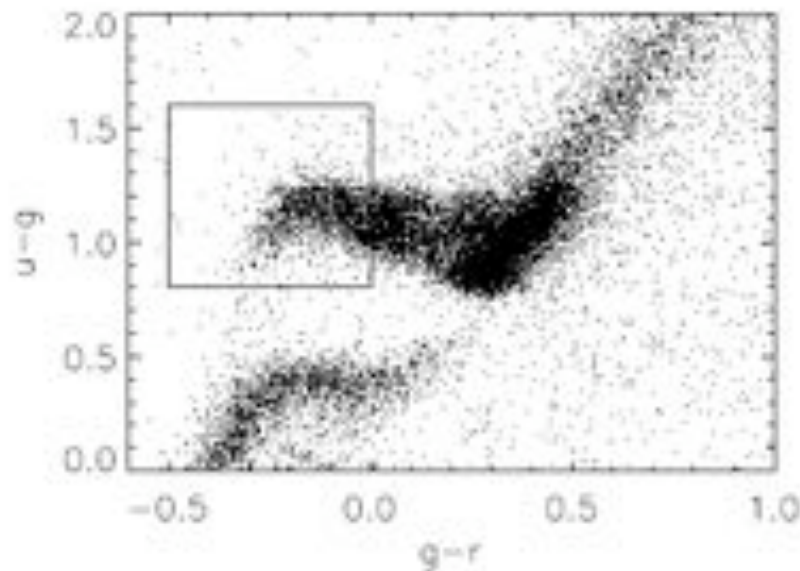
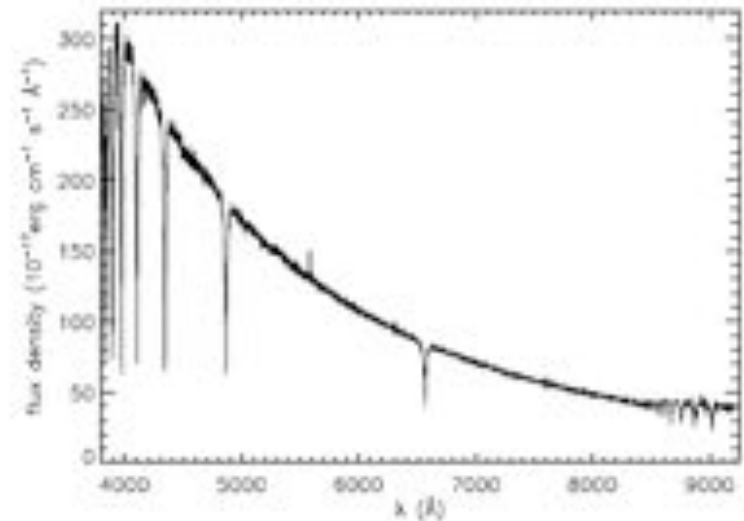


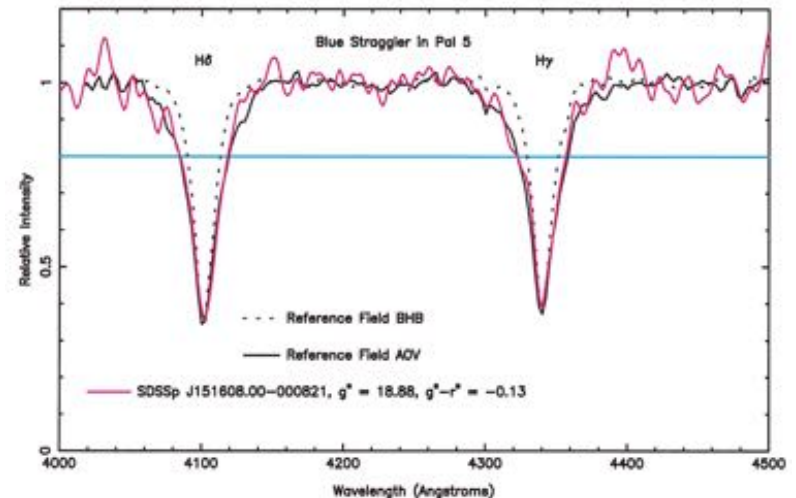
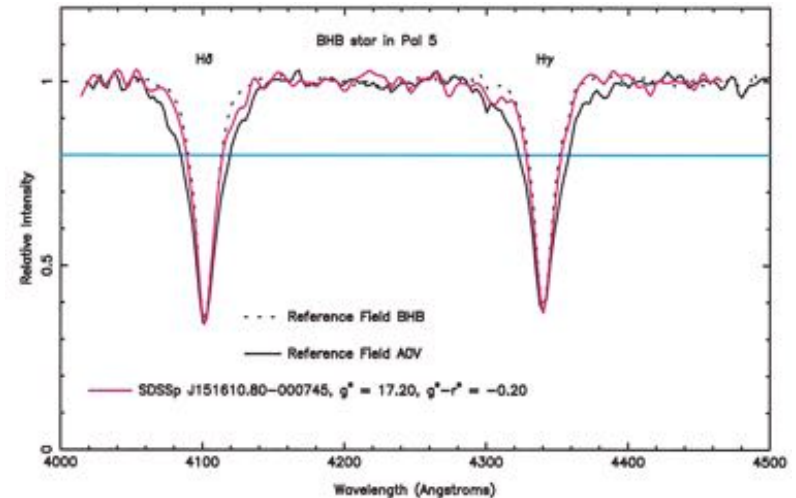
FIG. 1.—SDSS color-color diagram showing all spectroscopically targeted objects that were subsequently confirmed as stars. The large Balmer jump of A-type stars places them in the region where our “color-cut” selection box is drawn. This color selection approach follows Yanny et al. (2000).



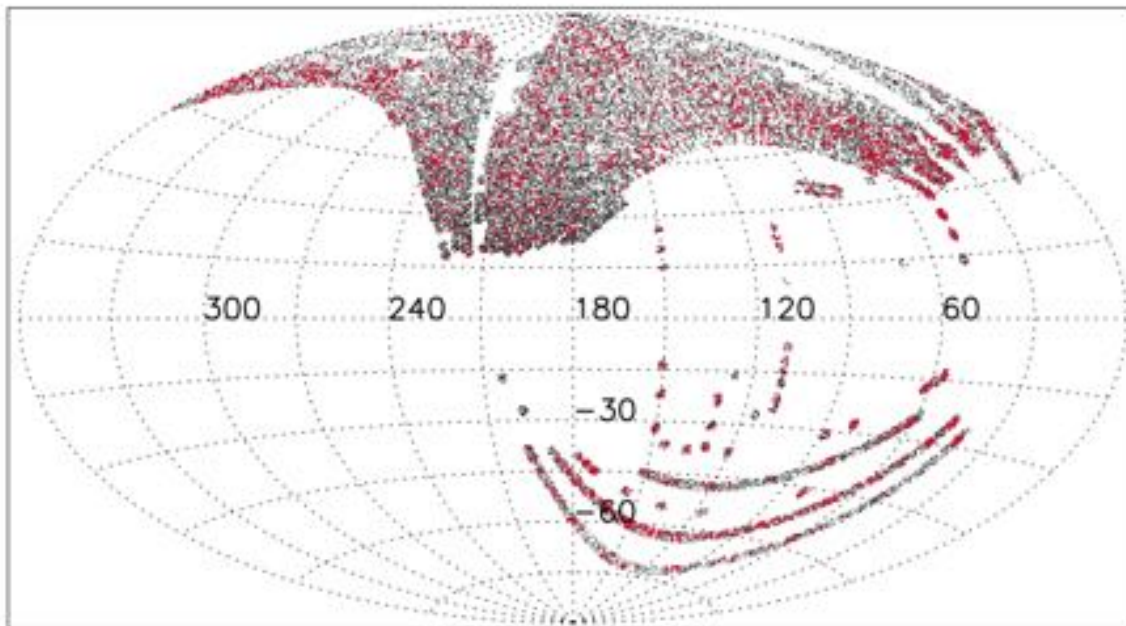
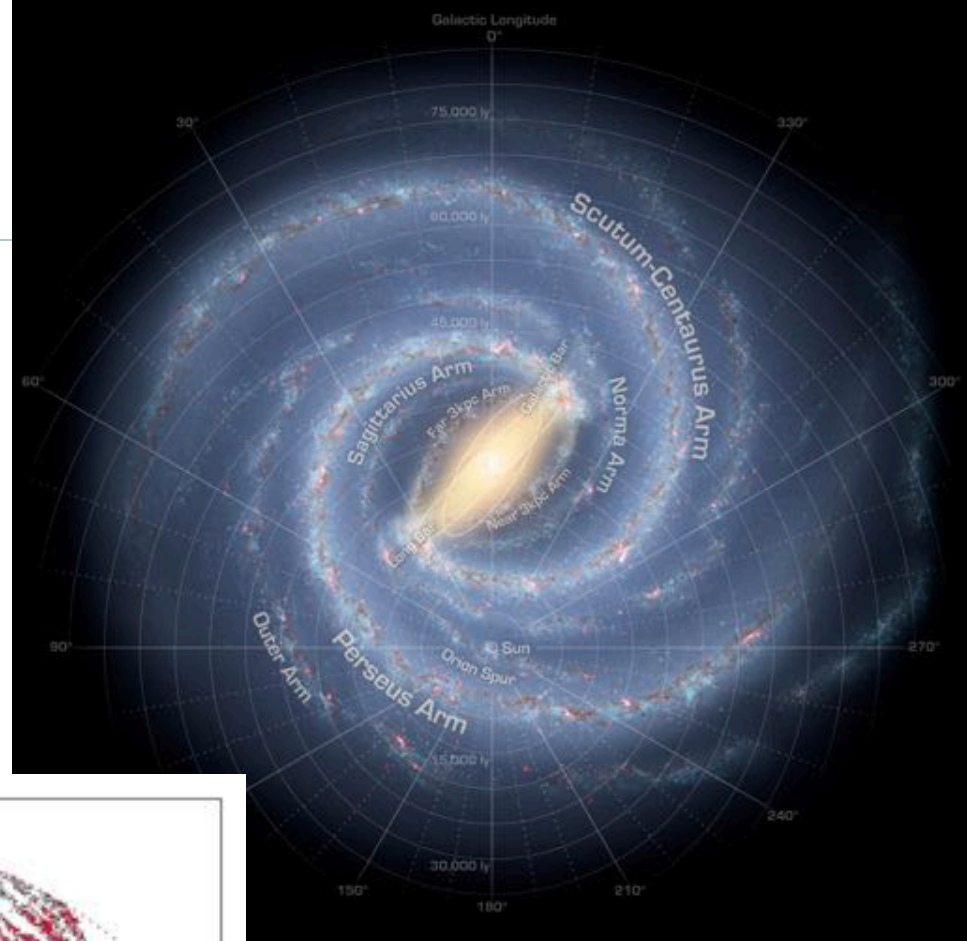
Xue et al. 2008

Measuring Galactic Rotation: Example

- ▶ Distinguish between blue horizontal branch stars and blue stragglers (MS) so the luminosity is known
- ▶ Infer distances

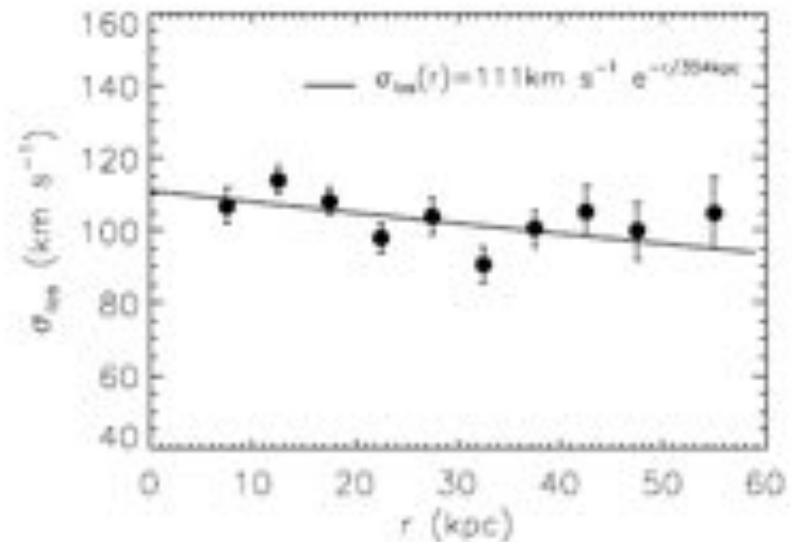
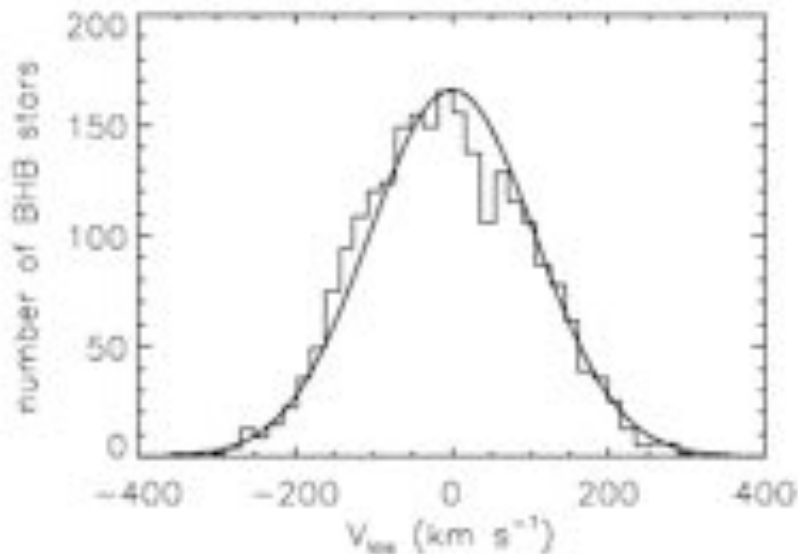
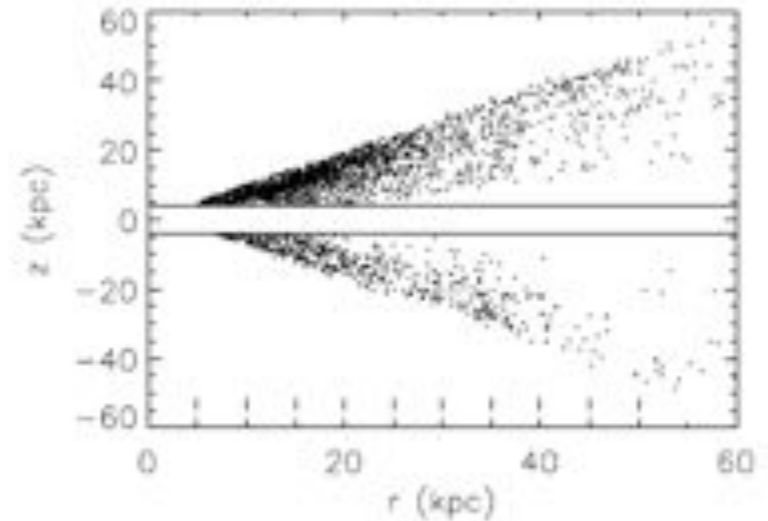


Sight Lines



Measuring Galactic Rotation: Example

- ▶ Determine the spatial distribution w.r.t. the GC →
- ▶ Measure the observed distribution of line-of-sight velocities ($\downarrow V_{\text{los}}$), and the dispersion of these velocities, σ_{los} , as a function of Galactic radius \downarrow



Measuring Galactic Rotation: Example

- And now the trick: Estimate circular velocity (the rotation curve) from the velocity-dispersion data.

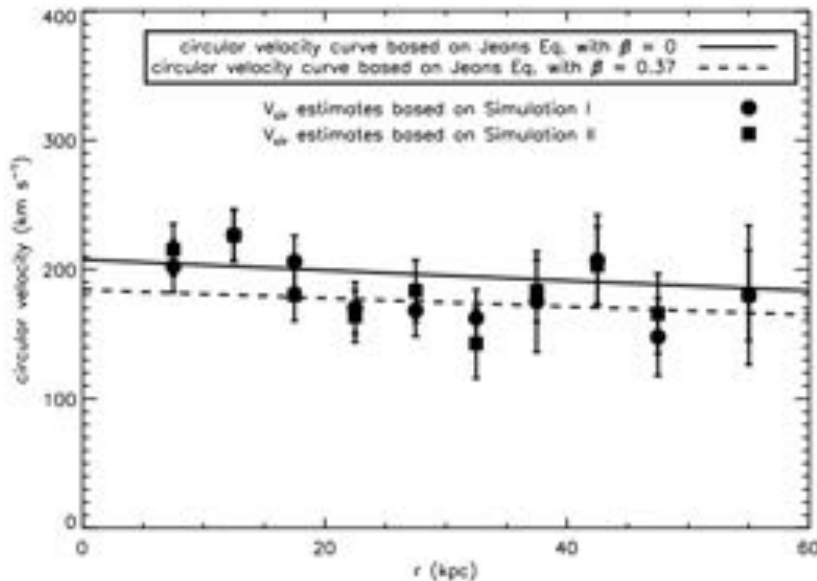


FIG. 15.— Distribution of circular velocity estimates, V_{cir} , for two different simulated galaxies. The circles represent the V_{cir} estimates for the observed halo BHB stars based on simulation I, and the squares represent the V_{cir} estimates based on simulation II. The two lines show the circular velocity curve estimates derived from the velocity dispersion profile (Fig. 10) and the Jeans equation with $\beta = 0.37$ and $\beta = 0$.

For reference, we show how these estimates of $V_{\text{cir}}(r)$ compare to those derived from the Jeans equation and the fit to $\sigma_{\text{los}}(r)$ shown in Figure 10. From the Jeans equation, $V_{\text{cir}}(r)$ can be estimated from the velocity dispersion, σ_r (Binney & Tremaine 1987), as follows:

$$-\frac{r}{\rho} \frac{d(\sigma_r^2 \rho)}{dr} - 2\beta \sigma_r^2 = V_{\text{cir}}^2(r), \quad (8)$$

with

$$\beta = 1 - \frac{\sigma_t^2}{\sigma_r^2}, \quad (9)$$

where $\sigma_r(r)$ and $\sigma_t(r)$ are the radial and tangential velocity dispersions, respectively, in spherical coordinates and $\rho(r)$ is the stellar density.

- So we need to learn some dynamics

Why Dynamics?

- ▶ We can then also interpret the data in terms of a physical model:

Mass decomposition
of the rotation curve
into bulge, disk and
halo components :

- ➔ Dark Matter
- ➔ Stellar M/L $\equiv \Upsilon_*$
- ➔ The IMF
- ➔ Missing physics

