Astro 500

Techniques of Modern Observational Astrophysics

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Telescopes & Optics

Outline

- Defining the telescope & observatory
- Mounts
- Foci
- Optical designs
- Geometric optics
- Aberrations

Conceptually separate

Critical for understanding telescope and instrument capabilities and preliminary instrument design

Essential for detailed instrument design *and* for evaluating data.

Optics

Where we're headed ↓

- Geometric Optics
  - Reflection
  - Refraction
  - Thin lens
  - Spherical optics
  - Conics
  - Stops
  - Pupils
  - Chief rays
  - Marginal ray
  - System design
  - Telescope summary

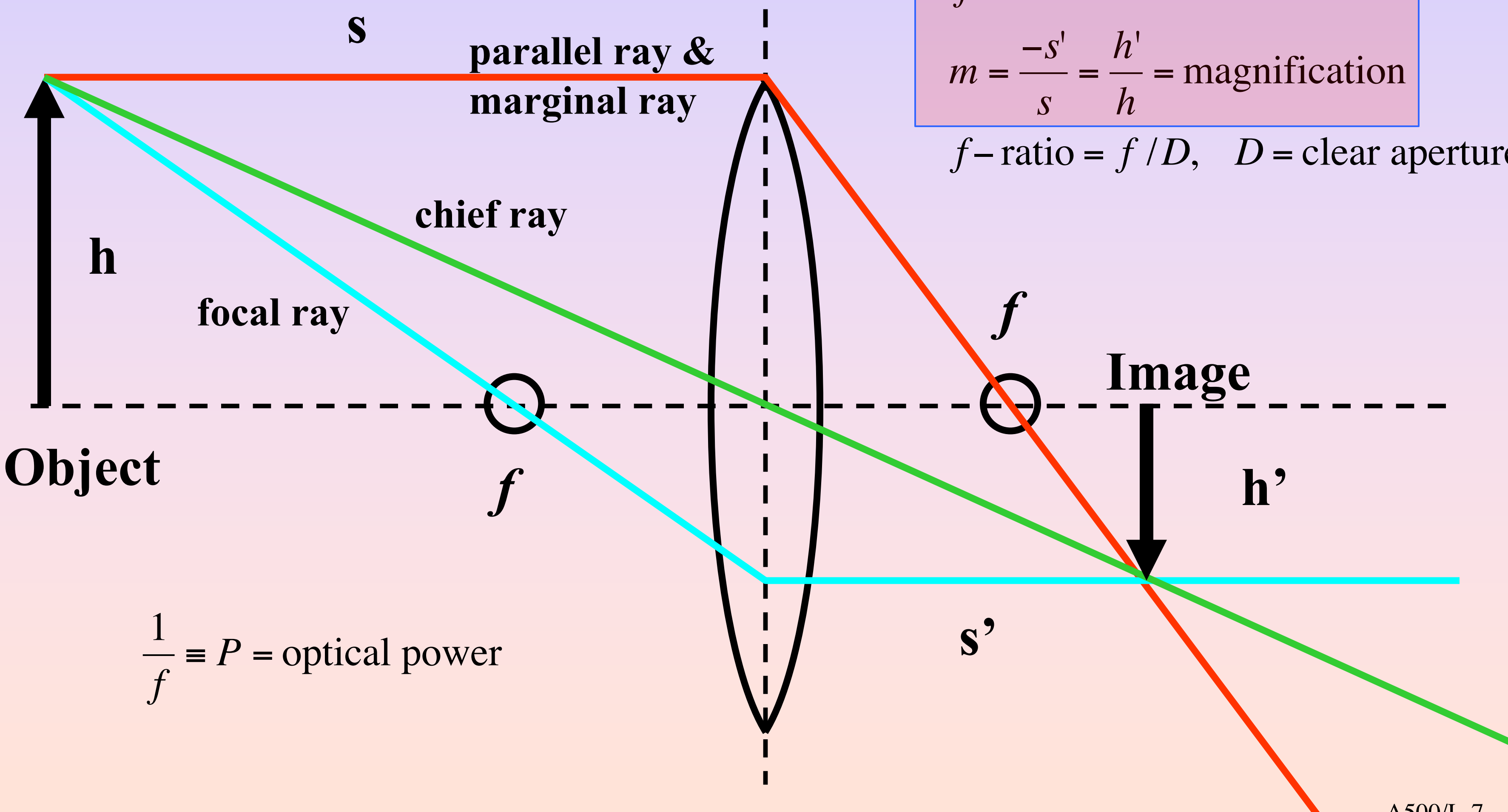
- Aberrations
  - Chromatic
  - Spherical
  - Coma
  - Astigmatism
  - Distortion
  - Field curvature

The Thin Lens

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

$$m = \frac{-s'}{s} = \frac{h'}{h} = \text{magnification}$$

$$f\text{-ratio} = f/D, \quad D = \text{clear aperture}$$



$$\frac{1}{f} \equiv P = \text{optical power}$$

The Thin Lens

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

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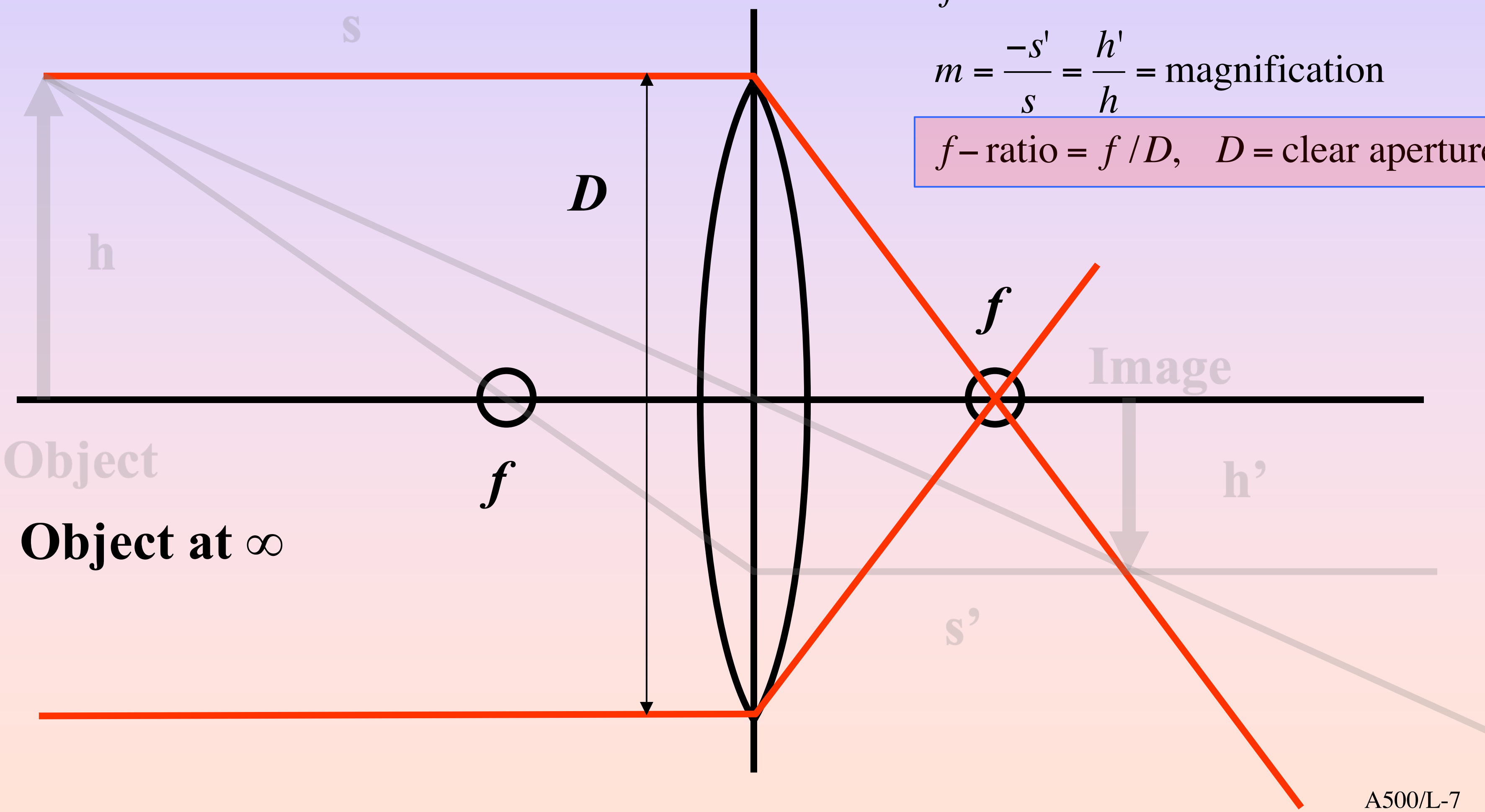
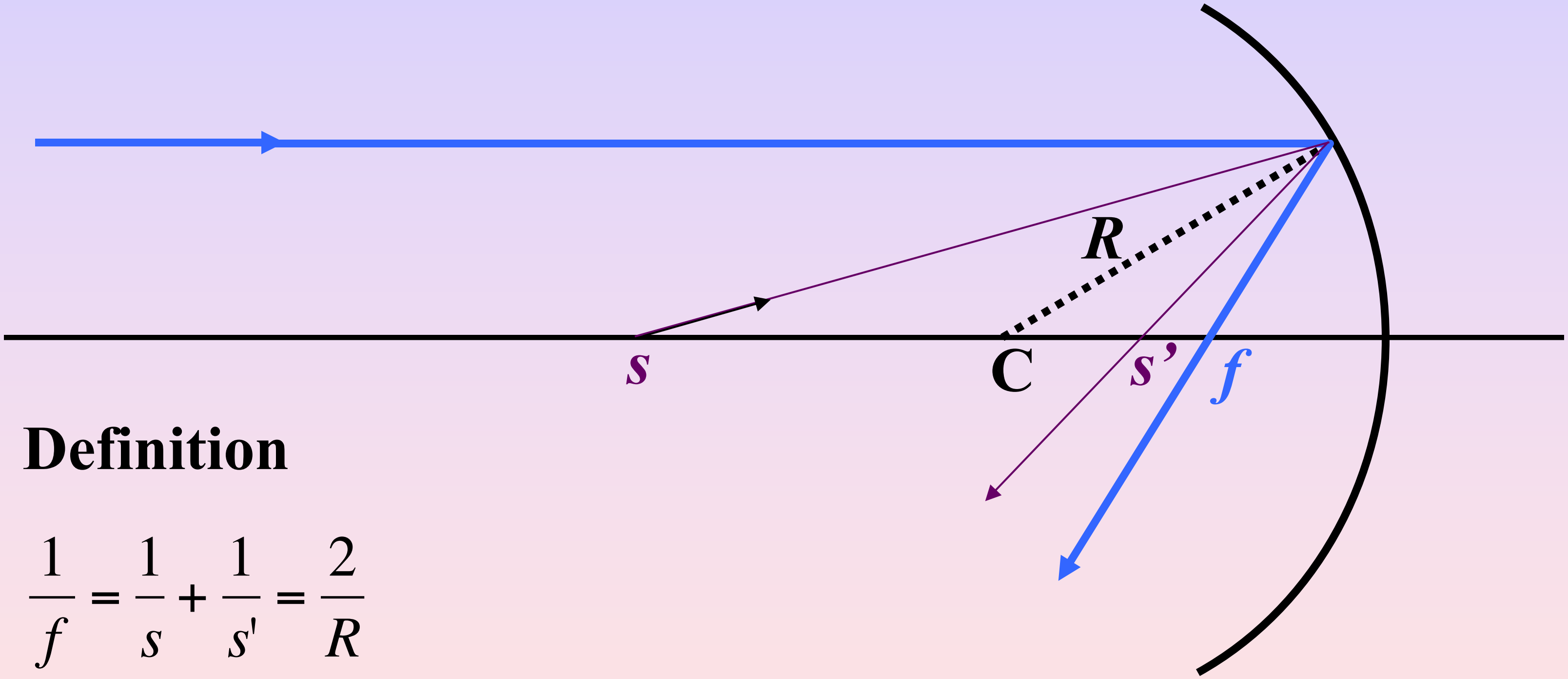


Spherical Optics

Focal Length Defined



Definition

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$$

Object at Infinity $\quad \frac{1}{s'} = \frac{1}{f} \qquad \frac{1}{s'} = \frac{2}{R}$

f = R/2 **is referred to as the *paraxial* focal-length**

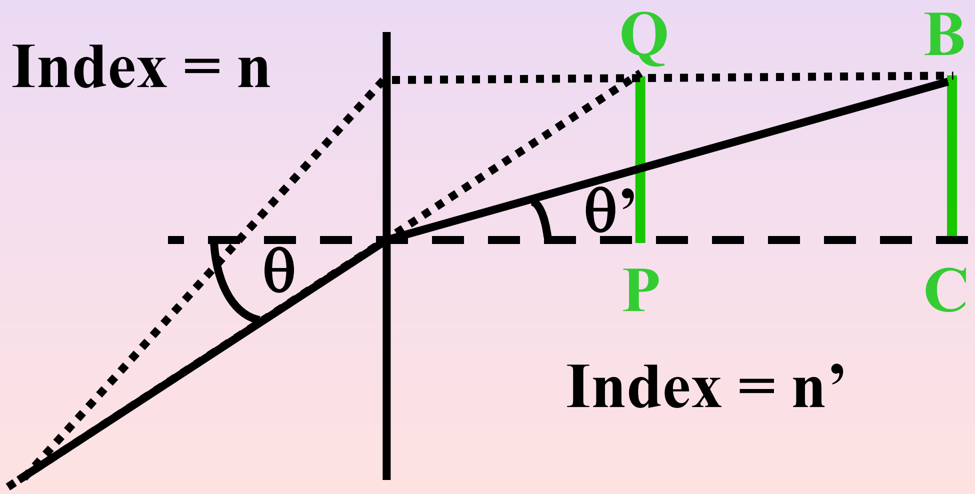
Snell's Law

$$n \sin\theta = n' \sin\theta'$$

***n* and *n'* depend on λ**

$$\frac{1}{V} = (n_F - n_C)/(n_D - 1)$$

= dispersive power

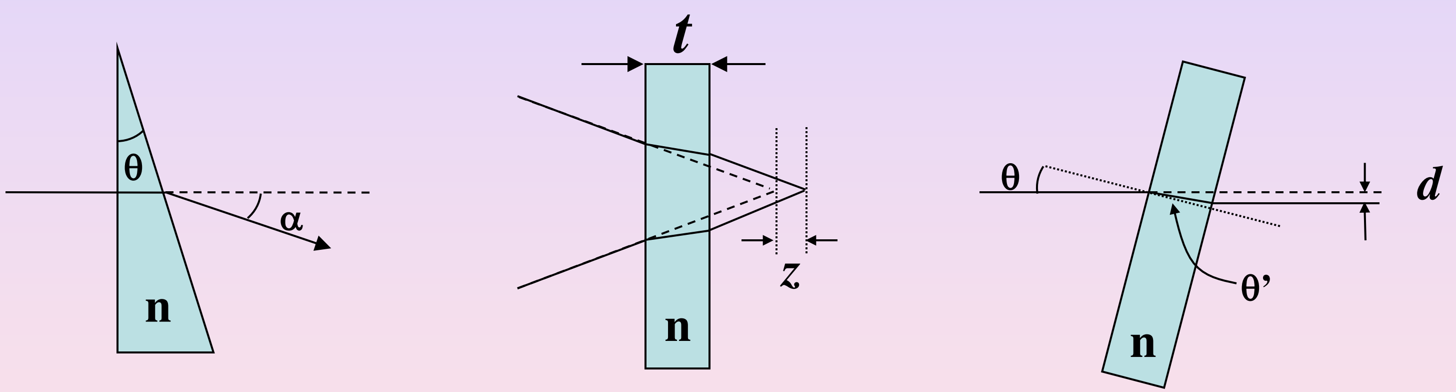


***F,D,C:* solar-spectrum Fraunhofer lines (486, 589, 656 nm).**
***V vs n* is called a glass table.**

Fermat's principle: least time

An Aside: Implications of Snell's Law

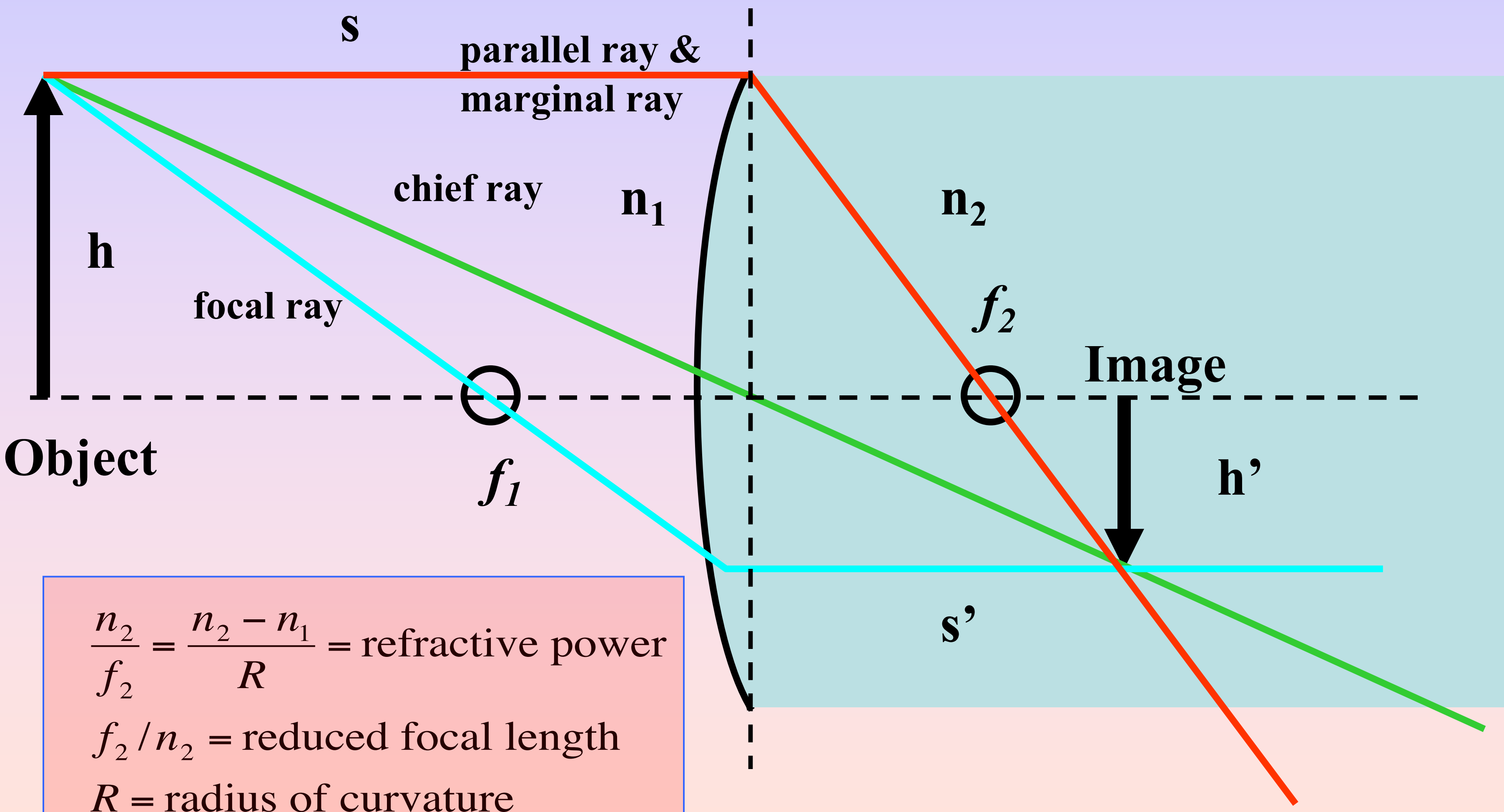
$$\alpha \approx (n-1)\theta \qquad z \approx \frac{(n-1)t}{n} \qquad d \approx t\sin\theta\left(1-\frac{\cos\theta}{n\cos\theta'}\right)$$



Impact of wedges and and plane parallel plates on an optical beam

If beam is converging, astigmatism introduced.
Can be eliminated with wedge.

The Thick Lens

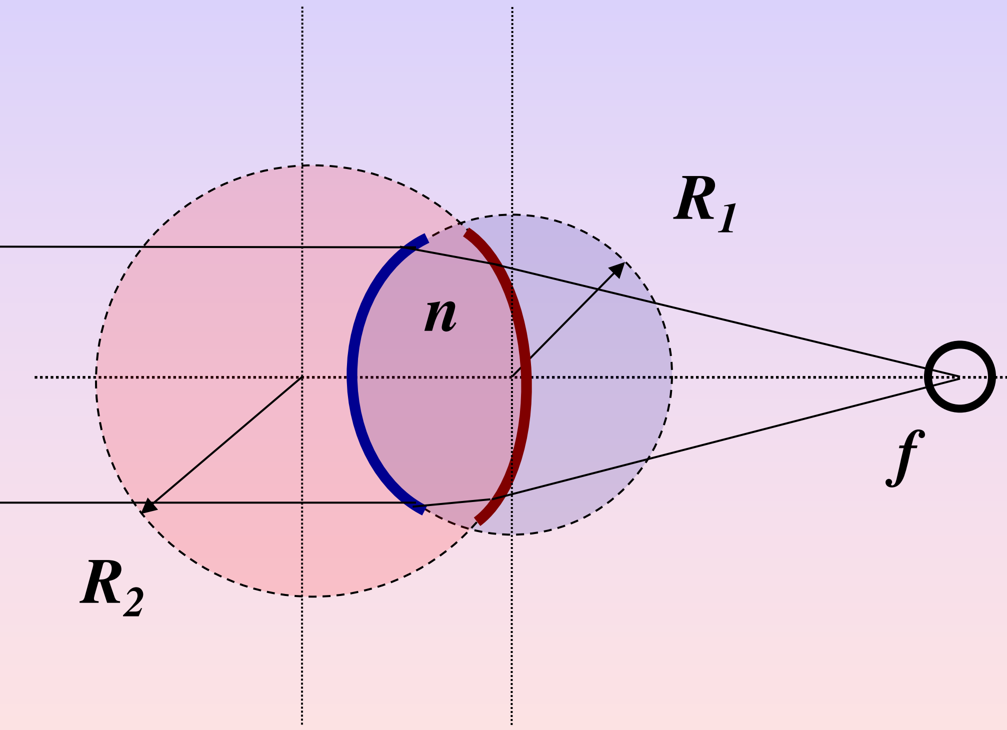


$$\frac{n_2}{f_2} = \frac{n_2 - n_1}{R} = \text{refractive power}$$

f_2 / n_2 = reduced focal length

R = radius of curvature

Optical Power, *P*



Two surfaces

- **separation *d***
- **index *n***

$$P \equiv \frac{1}{f} = P_1 + P_2 - \frac{d}{n} P_1 P_2$$

Two-surface spherical lens in air or vacuum

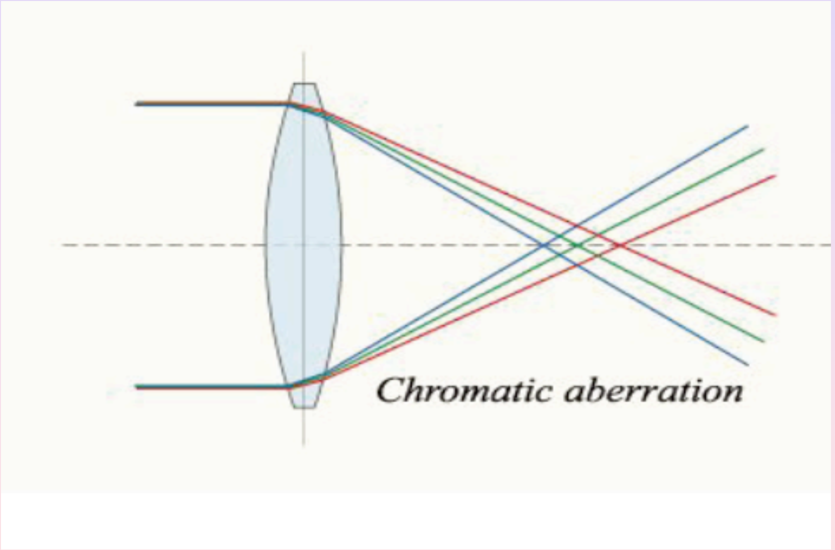
- **R_1, R_2 radii of curvature**
- **R>0: center of curvature behind lens**

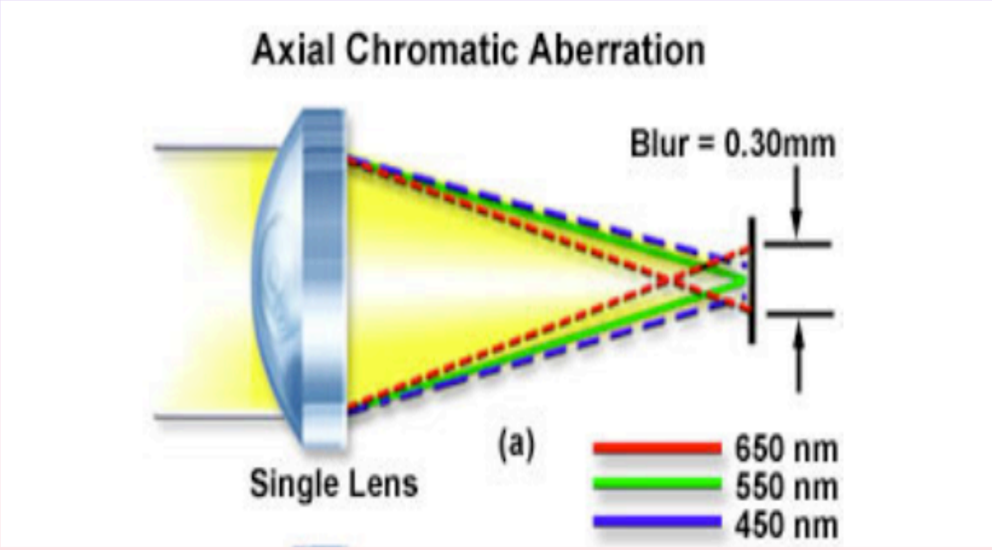
Lensmaker's formula:

$$P = \frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{d(n-1)}{nR_1R_2}\right]$$

$$\text{units: dioptor } (\text{m}^{-1})$$

Refractors: Chromatic Aberration





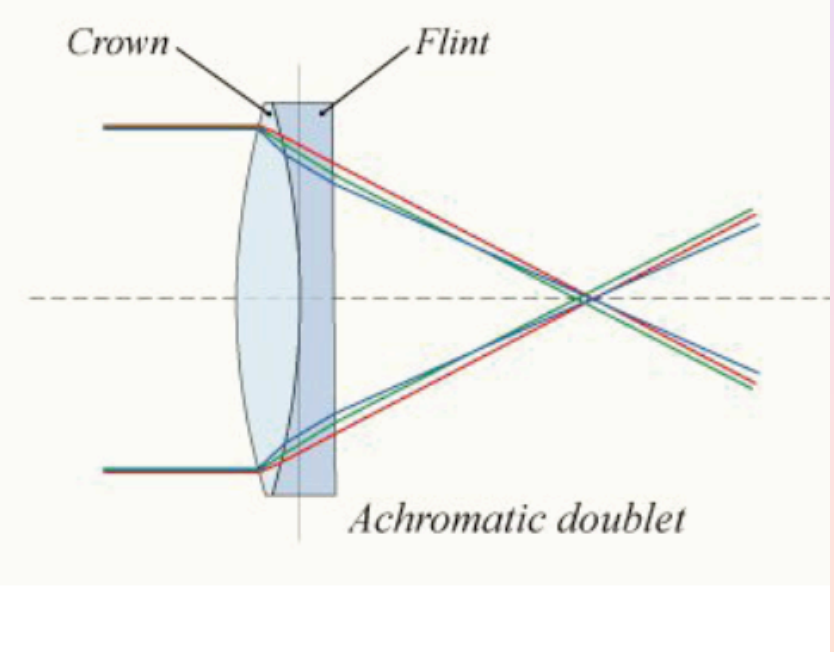
Chromatic Aberration

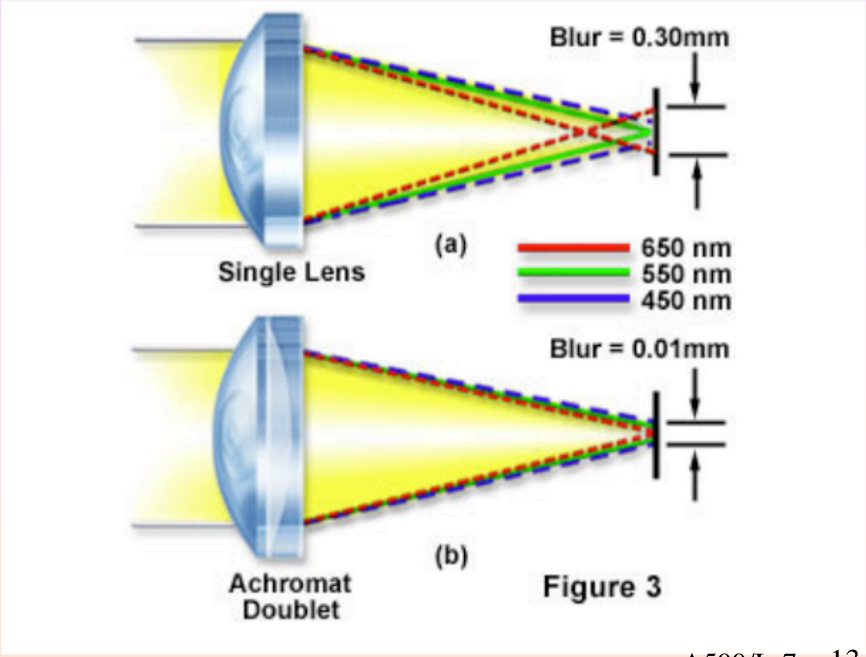
Credit: Wikepedia

Two-lens achromat

- Two refractive indices
- Three radii of curvature
- More lenses, better correction over more wavelengths

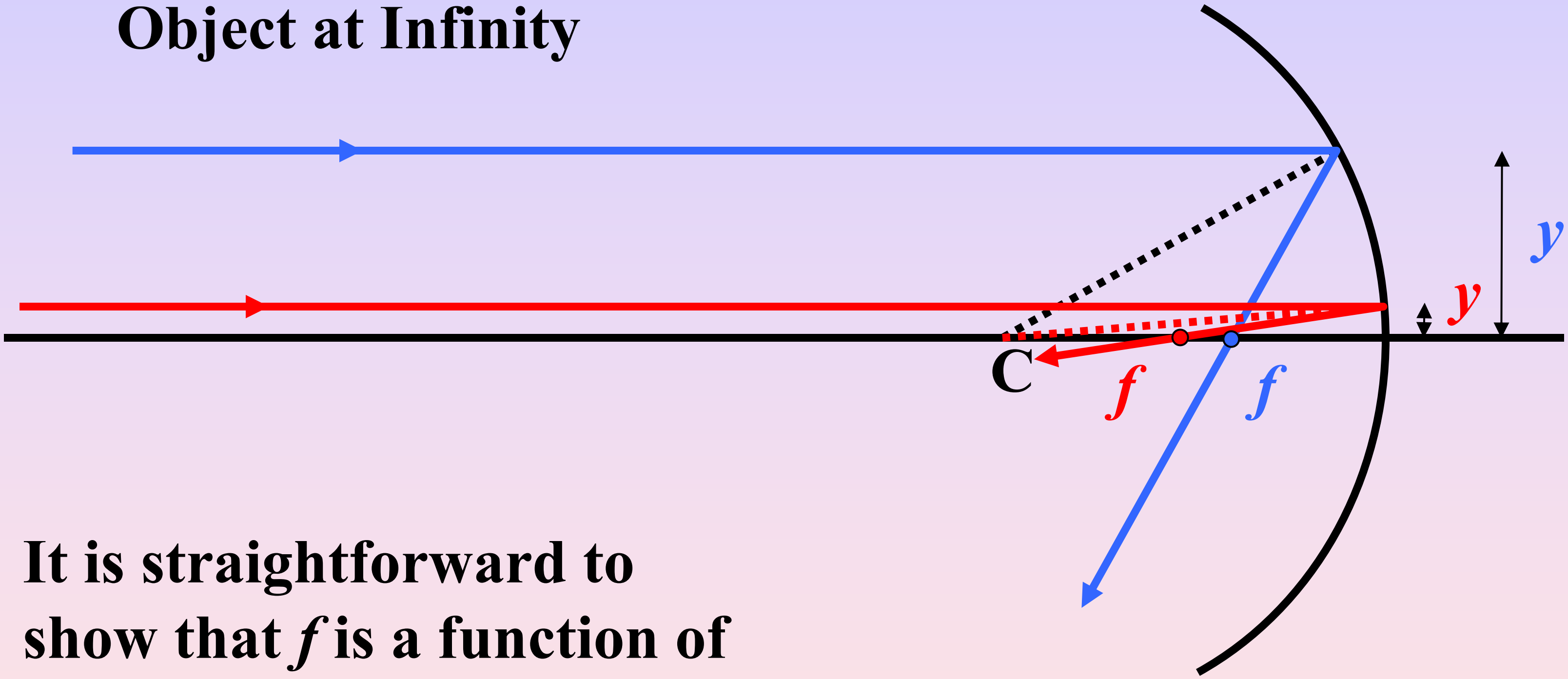
- **Difficult to make large achromats:**
  - **300mm diameter challenging**
  - **Largest refractor: 1m**
- **Solution: *Mirrors***





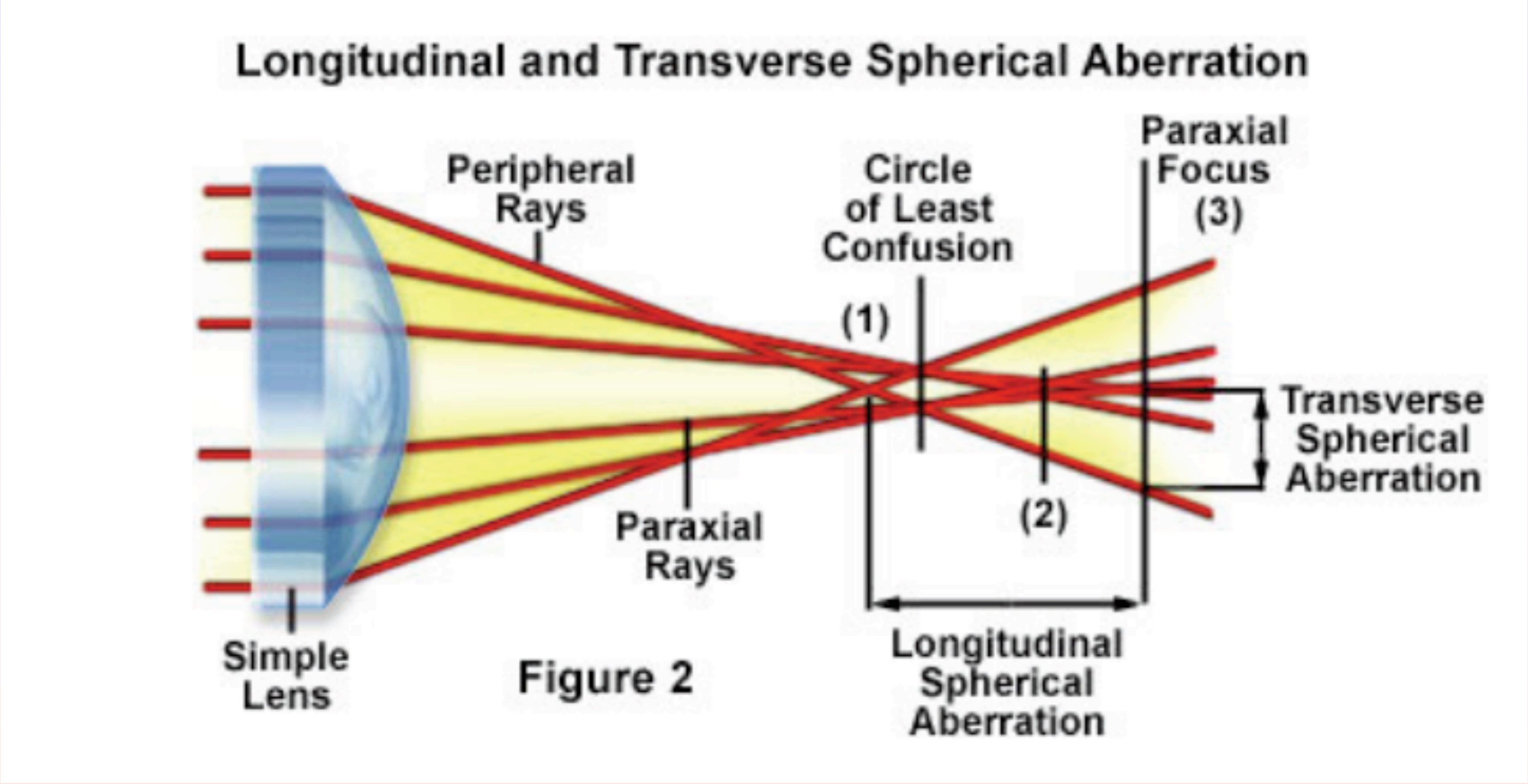
Spherical Aberration-1

Object at Infinity



It is straightforward to show that *f* is a function of height, *y*, proportional to √area.

Spherical Aberration-2



Spherical Aberration-3

- **Diameter of blur circle (of least confusion):**

$$\beta_s = \frac{1}{128(f/D)^3} \quad \text{(radians)}$$

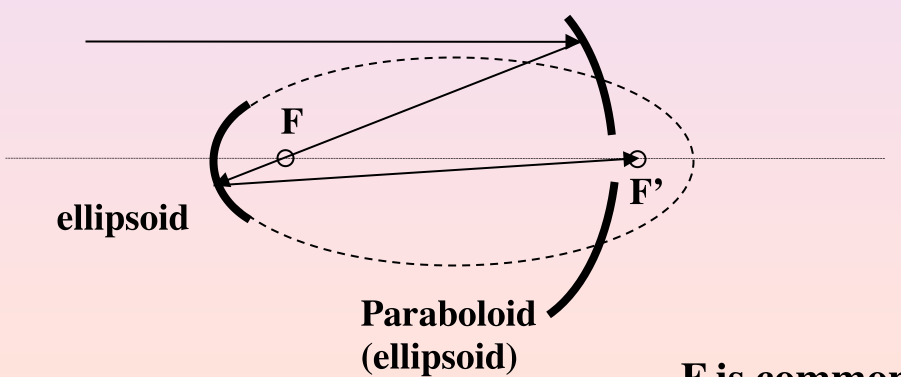
Spherical mirror

- ***So why do refractors work at all?***
- **Paraxial rays (optics):**
  - **Angles small ($\sin\theta \approx \theta$)**
- ***What does this imply about f/D?***

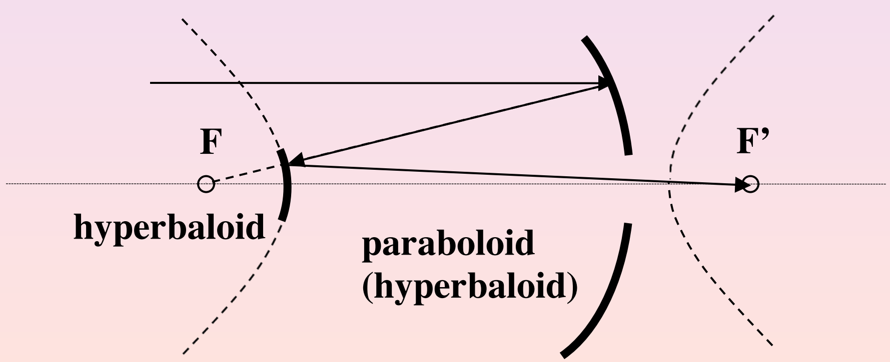
Conical Mirrors

- **Cross-sectional cuts through a cone.**
  - **Ellipse, parabola, hyperbola**
- **All eliminate spherical aberration:**
  - **all rays passing through one focus form perfect image at other focus.**
- **Other aberrations present.**

Gregorian
(Aplonatic Gregorian)



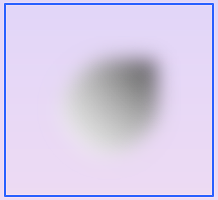
Cassegrain
(Ritchey-Cretien)

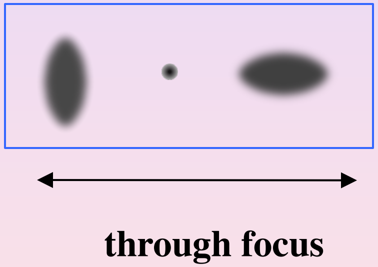


<u>F is common focus</u>

Parabola

- **Parabola:**
  - **one focus at infinity – suitable for astronomy**
  - **Off-axis aberrations:**
    - **Coma (negative)**
    - **Astigmatism (positive)**

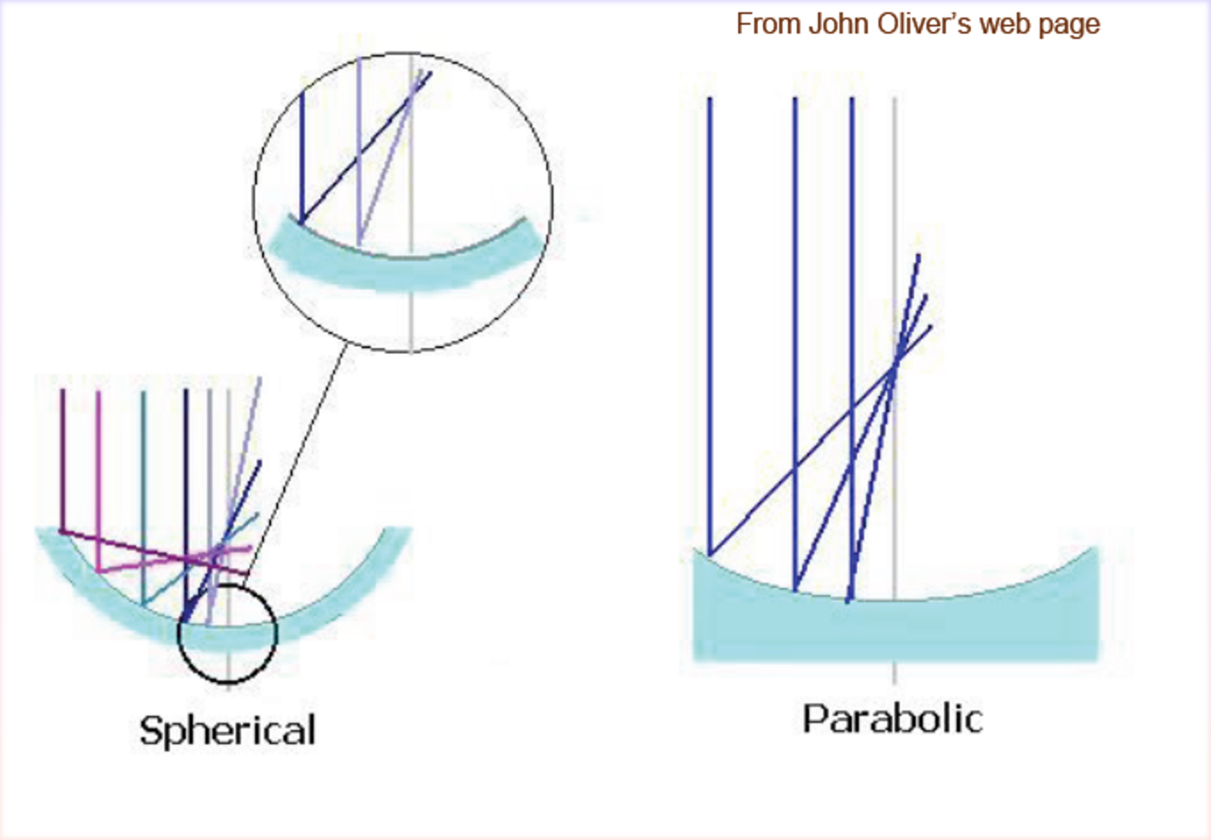




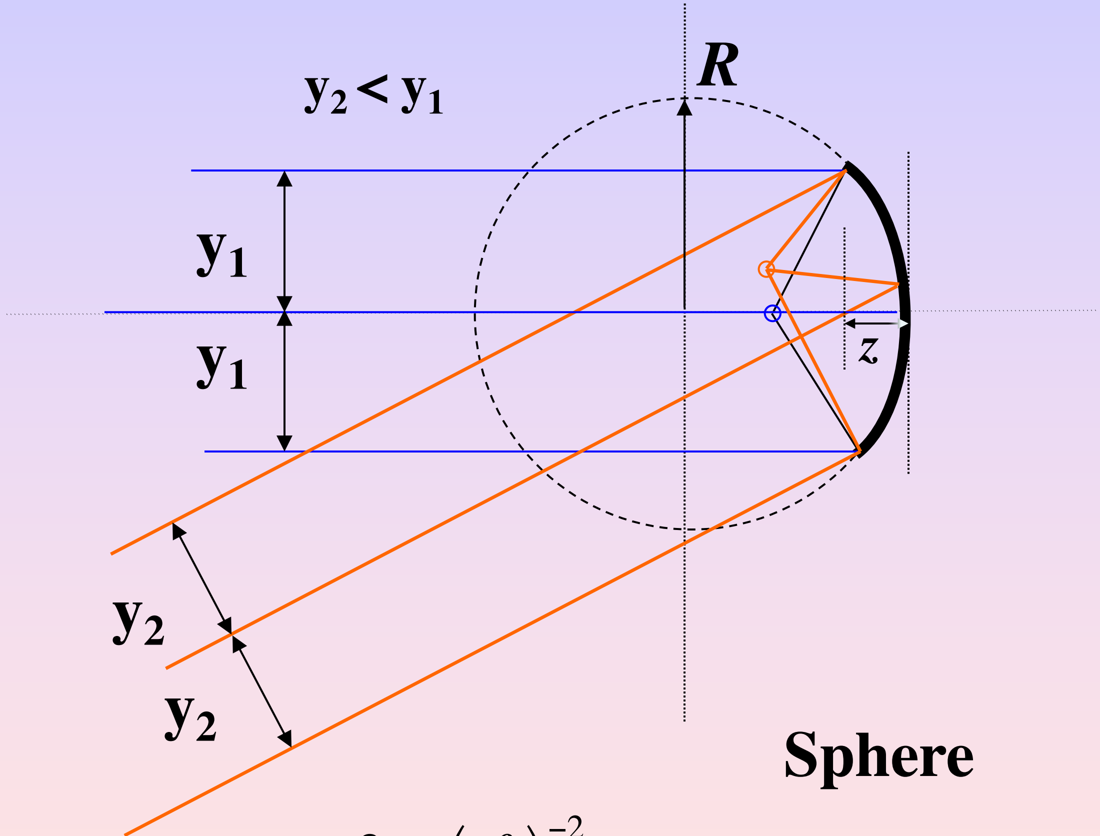
$$\beta_c = 3\theta \,/\, \left[16\left(\frac{f}{D}\right)^2\right] \quad \text{(radians)}$$

$$\beta_a = \theta^2 \,/\, \left(2\frac{f}{D}\right) \quad \text{(radians)}$$

Spherical Aberration: Parabola vs Sphere



Coma: Parabola vs Sphere



Sphere

$$\beta_c = \frac{3}{16}\theta\left(\frac{f}{D}\right)^{-2}\left(1 - z/2f\right) \quad \text{(radians)}$$

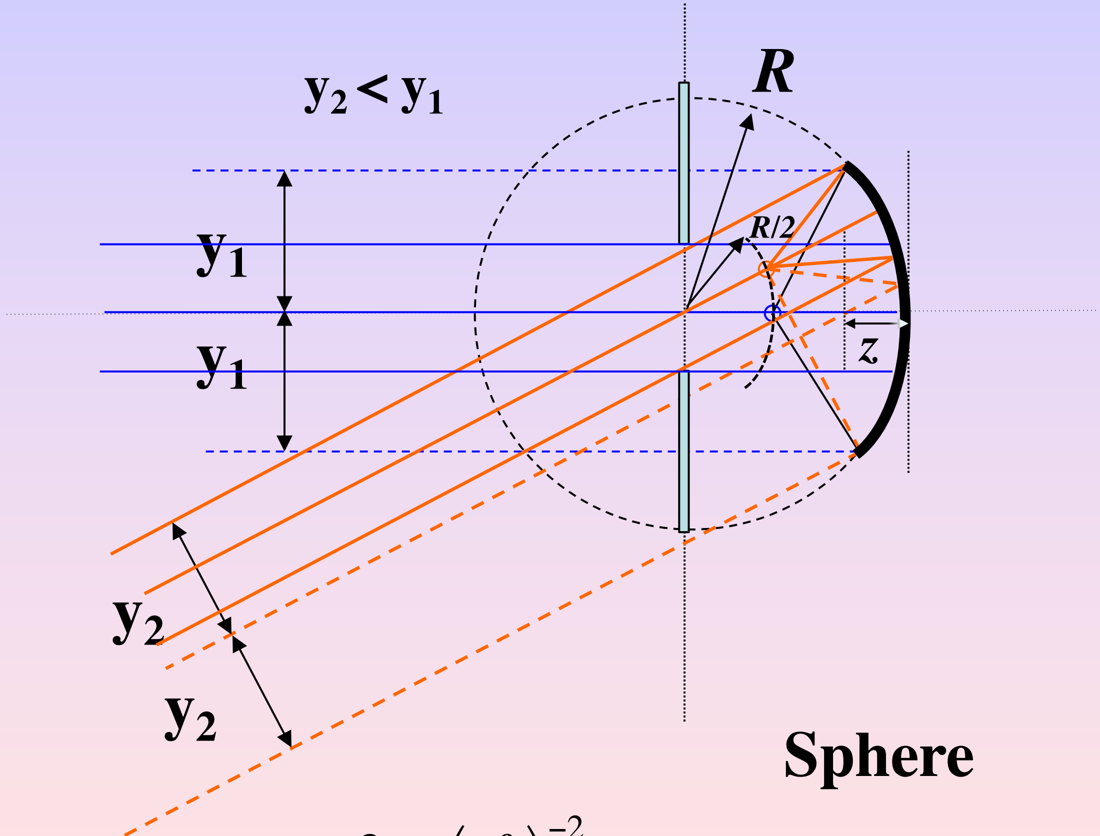
$f = R/2$

Parabola

$$\beta_c = \frac{3}{16}\theta\left(\frac{f}{D}\right)^{-2} \quad \text{(radians)}$$

How can you eliminate coma (an off-axis aberration) from a spherical system?

Coma: Parabola vs Sphere



Sphere

$$\beta_c = \frac{3}{16}\theta\left(\frac{f}{D}\right)^{-2}\left(1\text{-}z/2f\right) \quad \text{(radians)}$$

$f = R/2$

Parabola

$$\beta_c = \frac{3}{16}\theta\left(\frac{f}{D}\right)^{-2} \quad \text{(radians)}$$

Place an <u>aperture</u> stop at the mirror pupil (located at the center of curvature).
This is the basic concept behind Schmidt and Maksutov designs, plus additional corrective dioptrics.

Aberrations: 3rd order equations

- Imperfect images caused by geometric factors

$$\sin\theta = \sum_{n=1}^{\infty} \frac{\theta^n}{n!}$$

- Seidel or third-order aberrations consider departures from first-order, or paraxial condition, i.e., going from

$$\sin\theta \approx \theta$$

to

$$\sin\theta \approx \theta + \frac{\theta^3}{6}$$

→

Chromatic
Spherical
Coma
Astigmatism
Distortion
Field curvature

Aberrations: Zernike model

- Zernike polynomials: Orthogonal basis-set over circle of unit radius
- Aberrated wave-front fit with Zernikes
- Coefficients are linearly independent, and can be related to different types of aberrations

There are even and odd Zernike polynomials. The even Zernike polynomials are defined as

$$Z_n^m(\rho,\phi) = R_n^m(\rho)\,\cos(m\,\phi)$$

and the odd Zernike polynomials as

$$Z_n^{-m}(\rho,\phi) = R_n^m(\rho)\,\sin(m\,\phi),$$

where m and n are nonnegative integers with $n \geq m$, φ is the azimuthal angle in radians, and ρ is the normalized radial distance. The radial polynomials R_n^m have no azimuthal dependence, and are defined as

$$R_n^m(\rho) = \sum_{k=0}^{(n-m)/2} \frac{(-1)^k\,(n-k)!}{k!\,((n+m)/2-k)!\,((n-m)/2-k)!}\;\rho^{n-2\,k} \quad \text{if } n-m \text{ is even}$$

and $R_n^m(\rho) = 0$ if $n - m$ is odd.

Meet the Zernike's

The first few Zernike polynomials are:

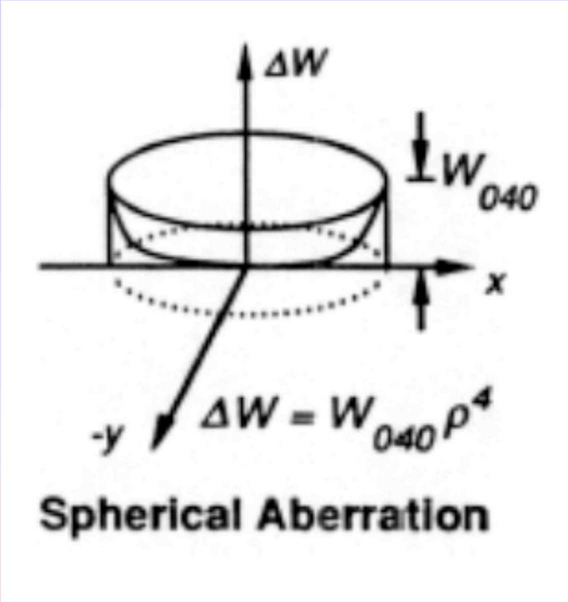
a_0	"Piston", equal to the mean value of the wavefront
$a_1 \times \rho\cos(\theta)$	"X-Tilt", the deviation of the overall beam in the sagittal direction
$a_2 \times \rho\sin(\theta)$	"Y-Tilt", the deviation of the overall beam in the tangential direction
$a_3 \times (2\rho^2 - 1)$	"Defocus", a parabolic wavefront resulting from being out of focus
$a_4 \times \rho^2\cos(2\theta)$	"X-Astigmatism", a horizontally oriented cylindrical shape
$a_5 \times \rho^2\sin(2\theta)$	"Y-Astigmatism", a vertically oriented cylindrical shape
$a_6 \times (3\rho^2 - 2)\rho\cos(\theta)$	"X-Coma", comatic image flaring in the horizontal direction
$a_7 \times (3\rho^2 - 2)\rho\sin(\theta)$	"Y-Coma", comatic image flaring in the vertical direction
$a_8 \times (6\rho^4 - 6\rho^2 + 1)$	"Third order spherical aberration"

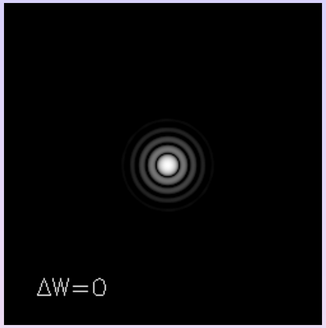
$0 \leq \rho \leq 1$: normalized pupil radius

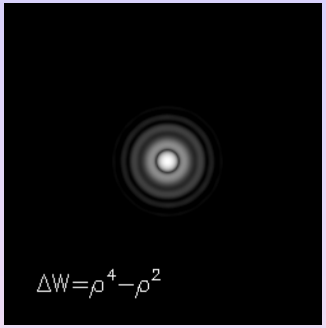
$0 \leq \theta \leq 2\pi$: azimuthal angle around pupil

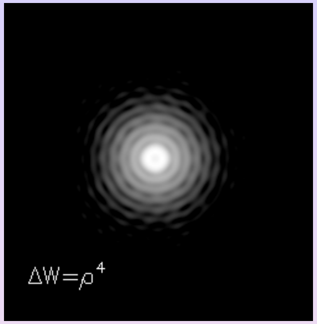
a_0 to a_8: (fitting coefficients) wavefront errors in wavelengths

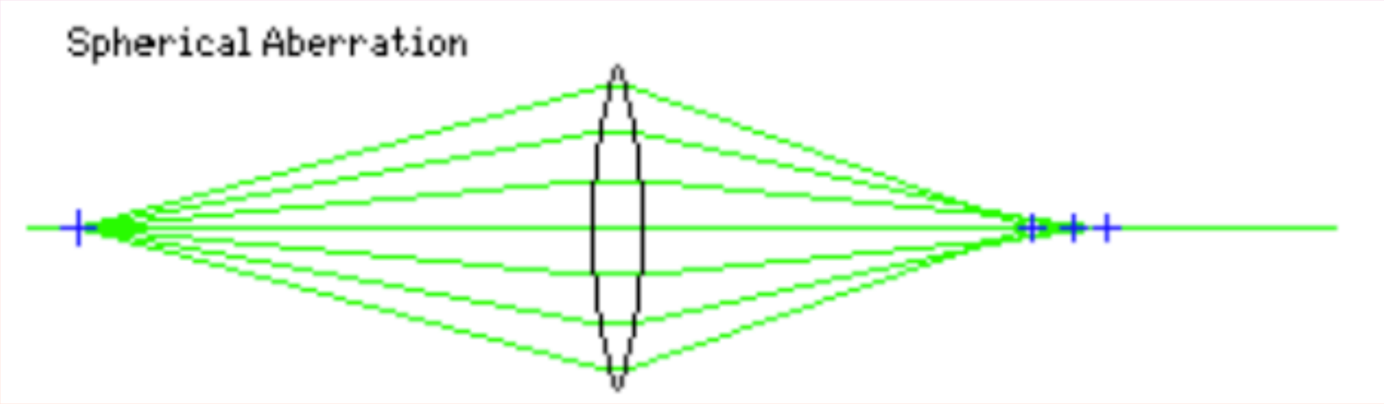
Spherical Aberration (revisited)







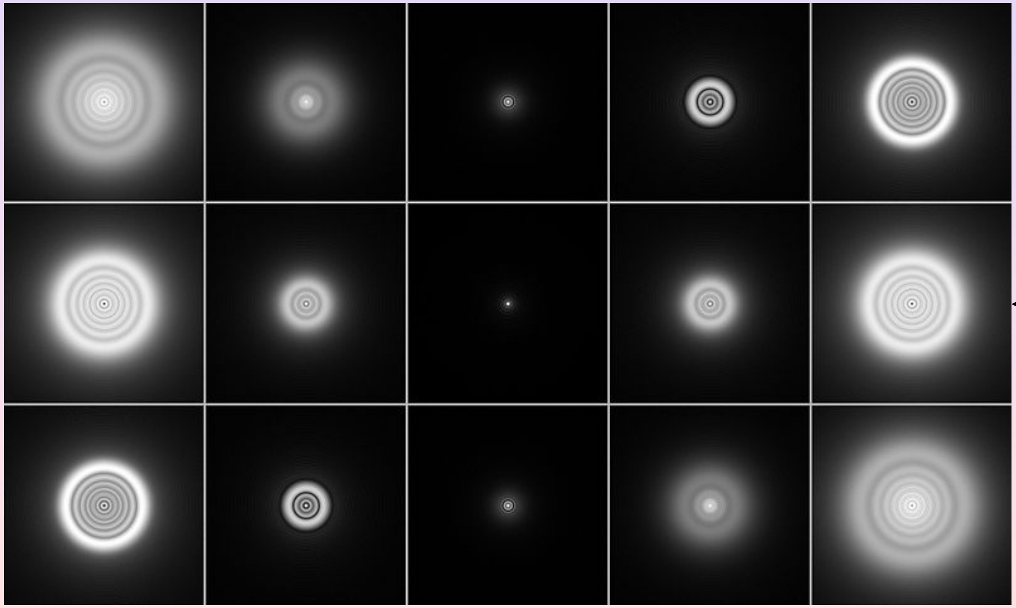


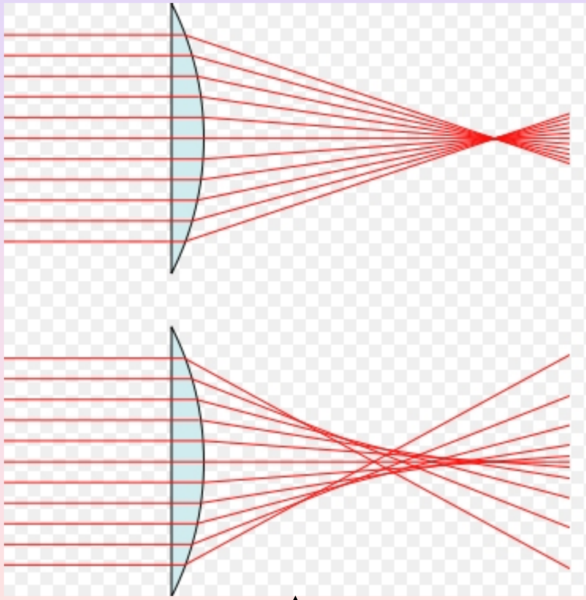


Spherical Aberration (revisited)

Negative spherical aberration

Paraxial lens

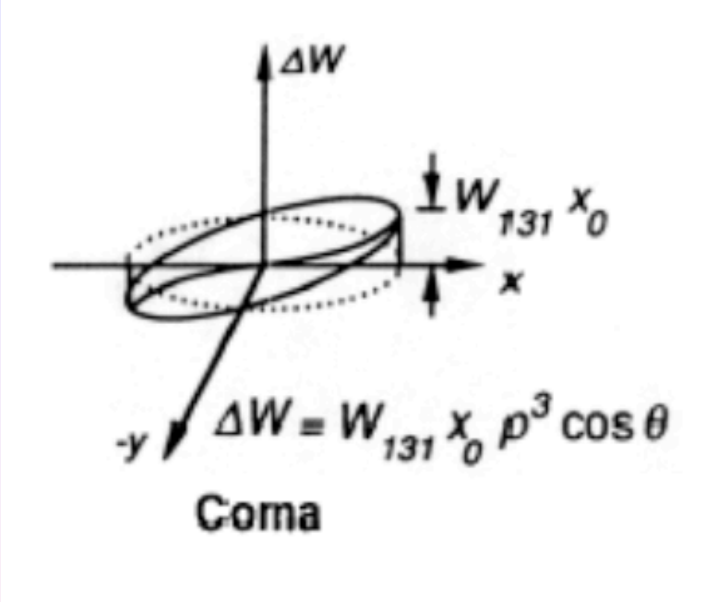




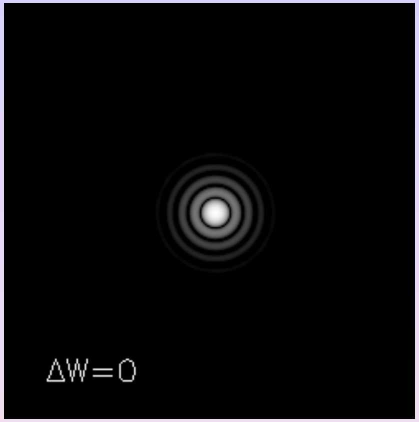
Positive spherical aberration

going through focus

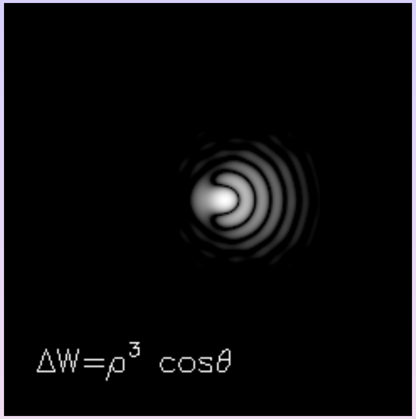
Coma



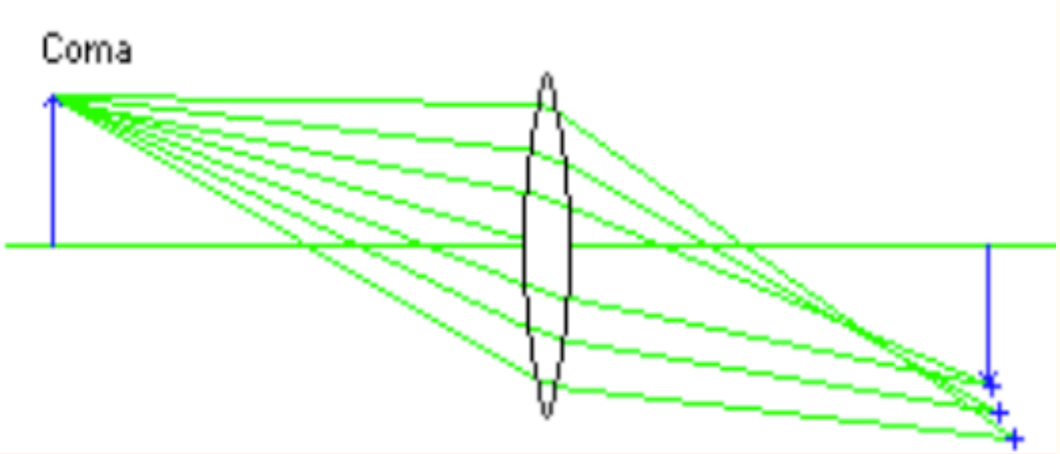
ΔW
$W_{131}\ x_0$
x
-y
$\Delta W = W_{131}\ x_0\ \rho^3 \cos\theta$
Coma



ΔW=0



$\Delta W = \rho^3\ \cos\theta$



Coma

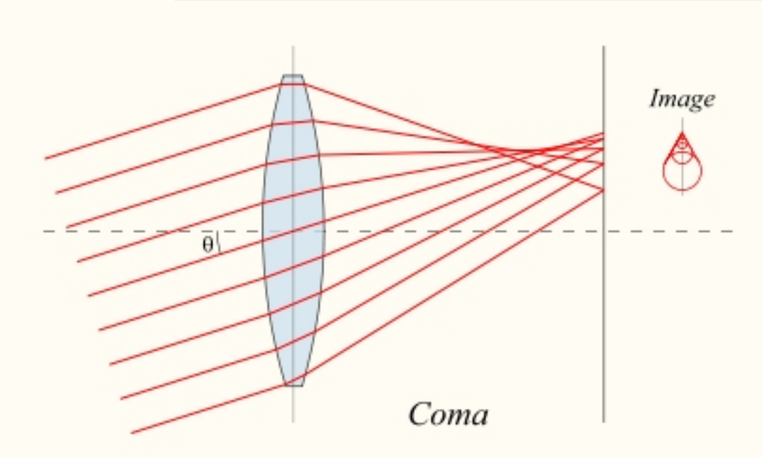
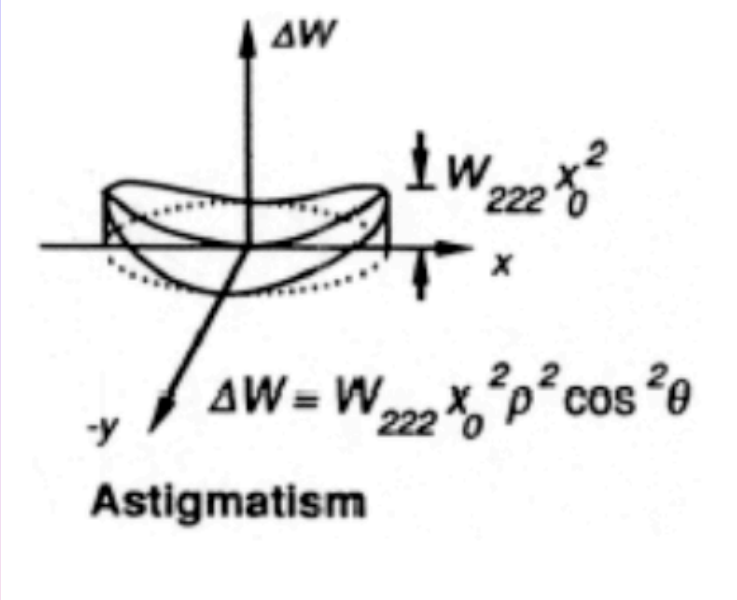
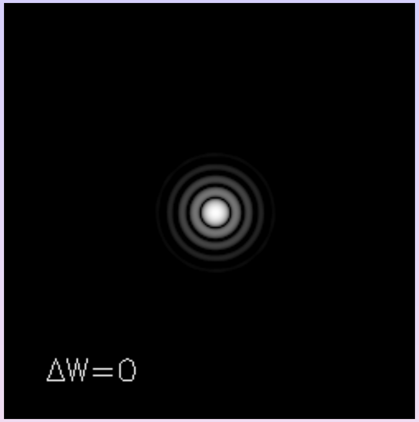
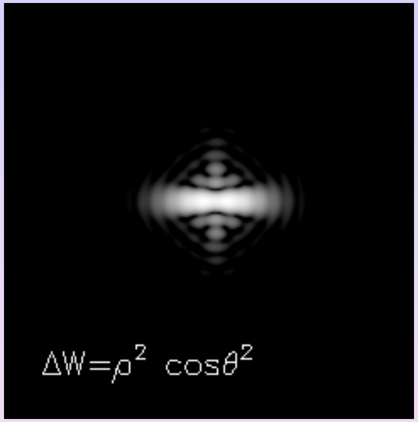


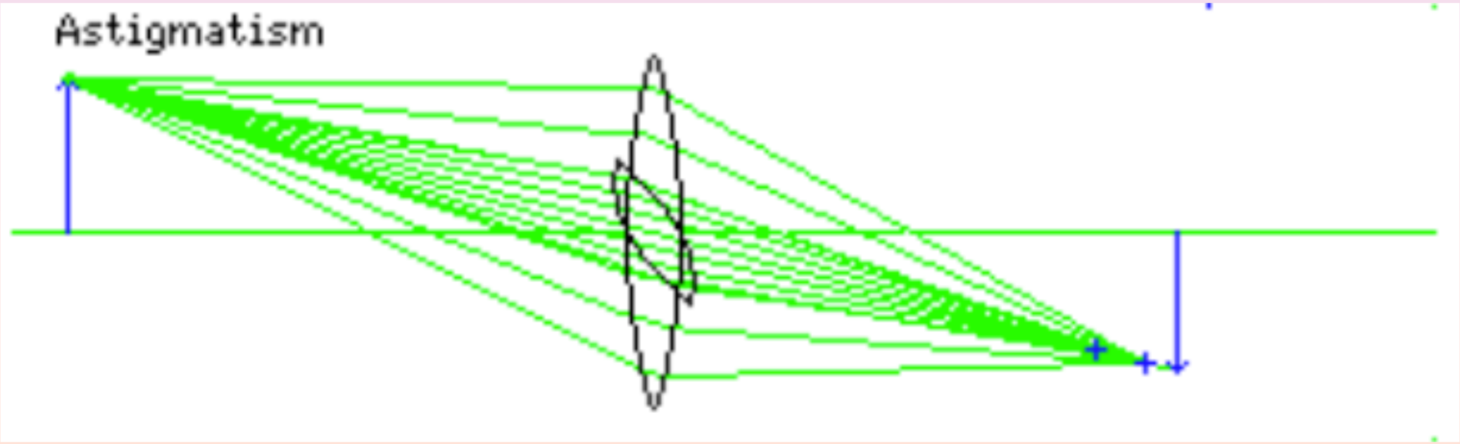
Image
θ
Coma

Astigmatism

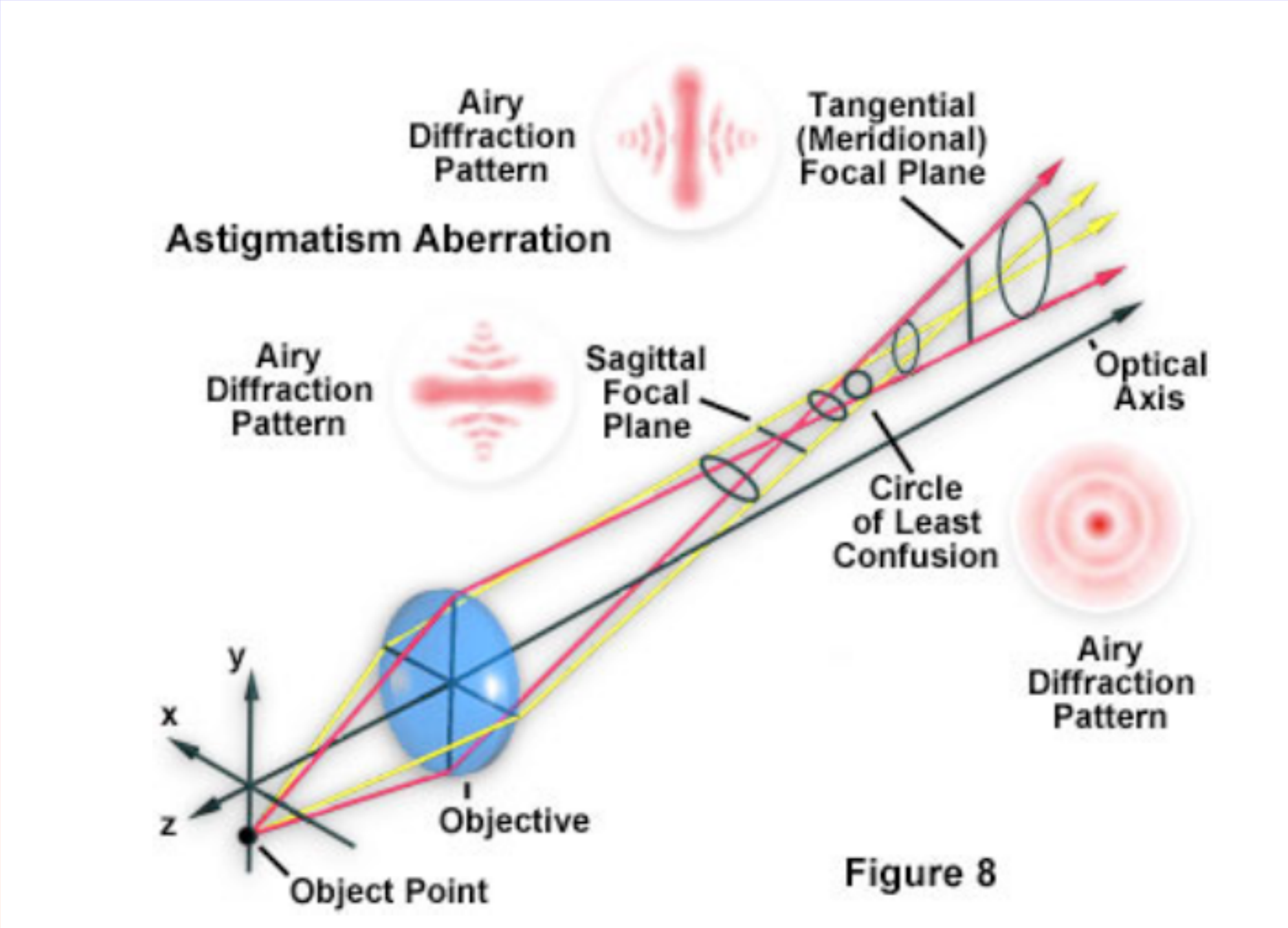




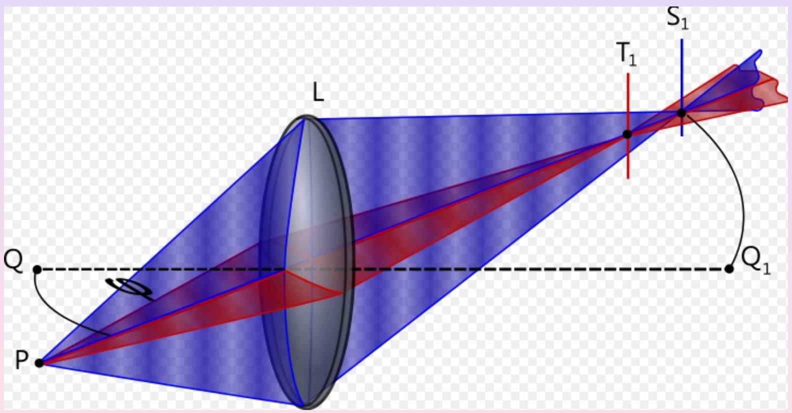


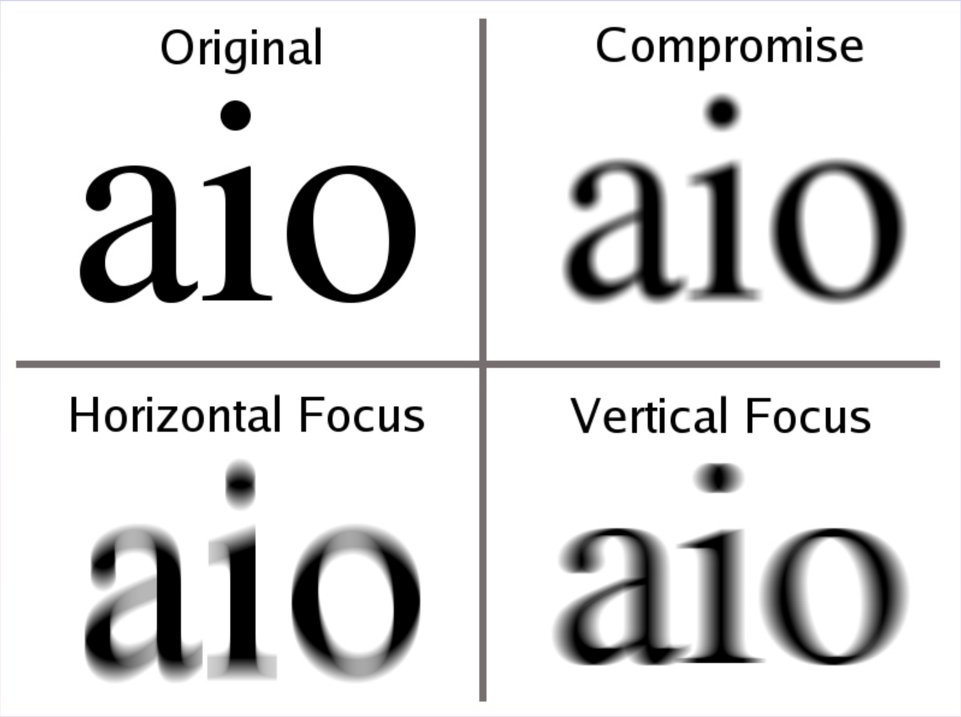


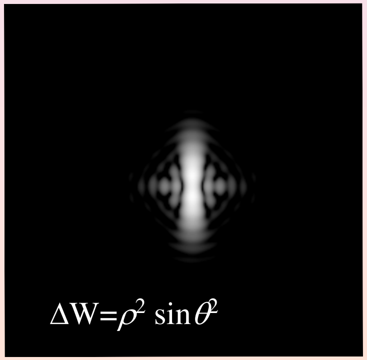
Astigmatism

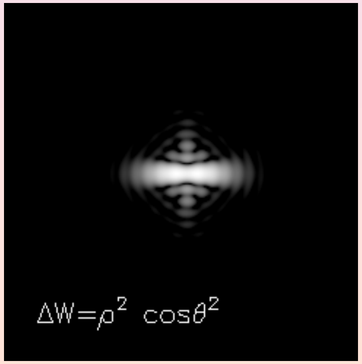


Astigmatism

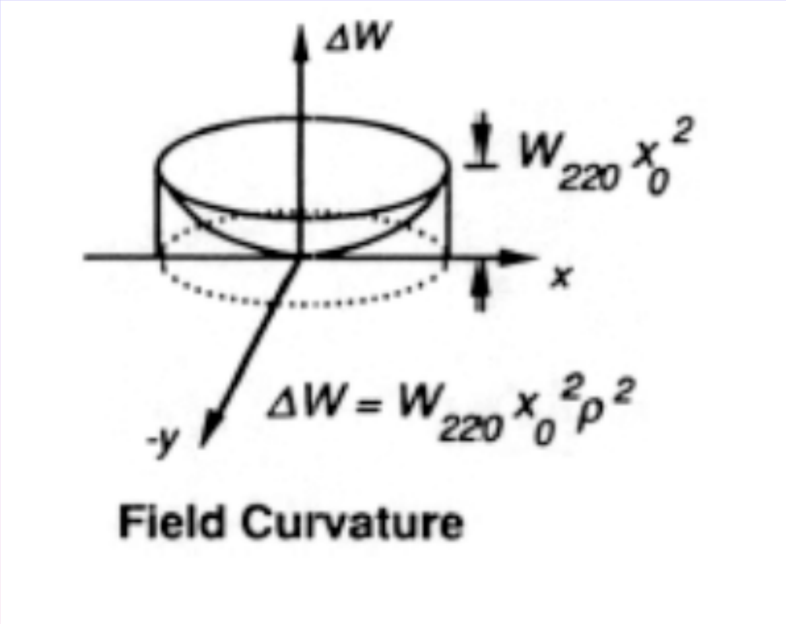


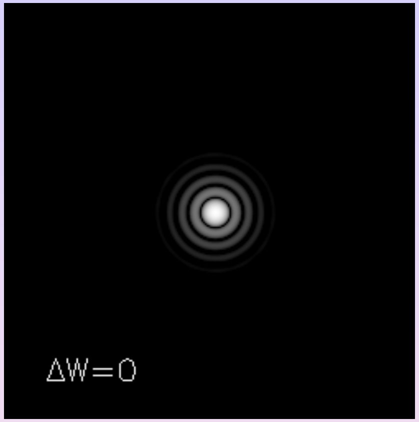


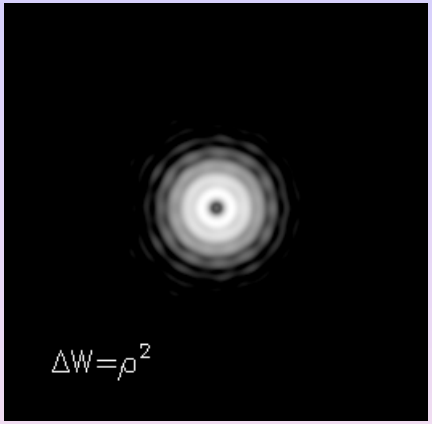


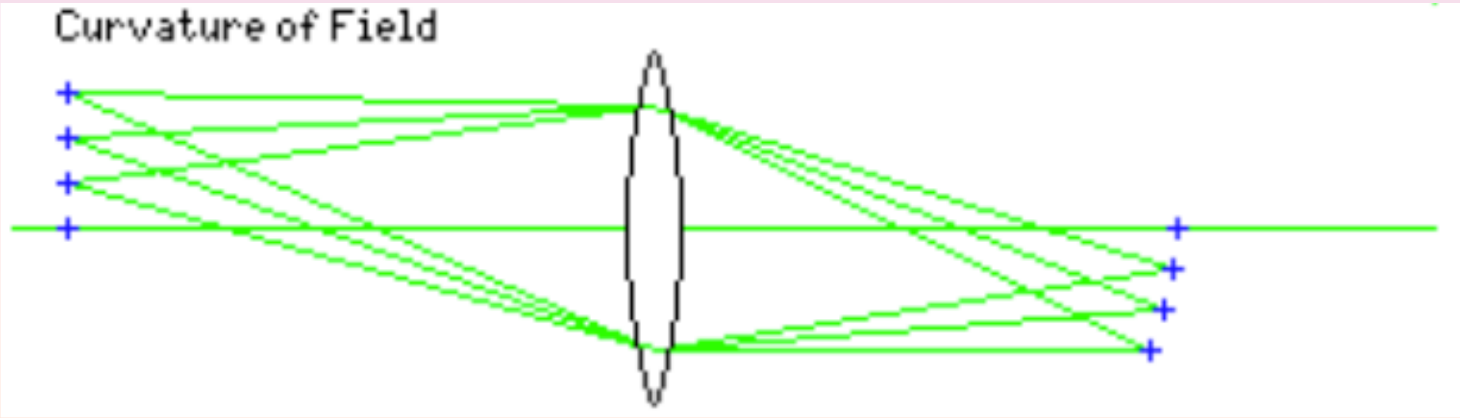


Field curvature

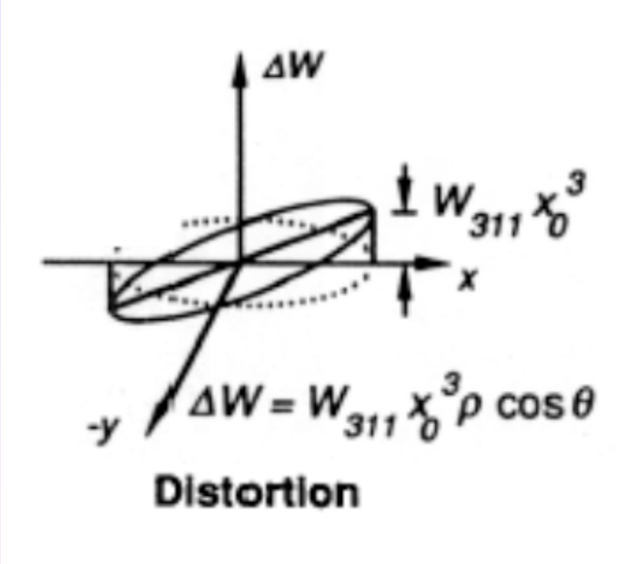




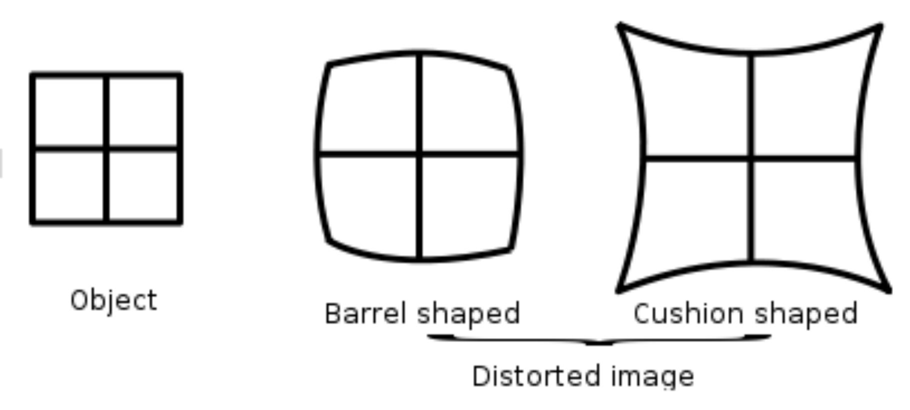




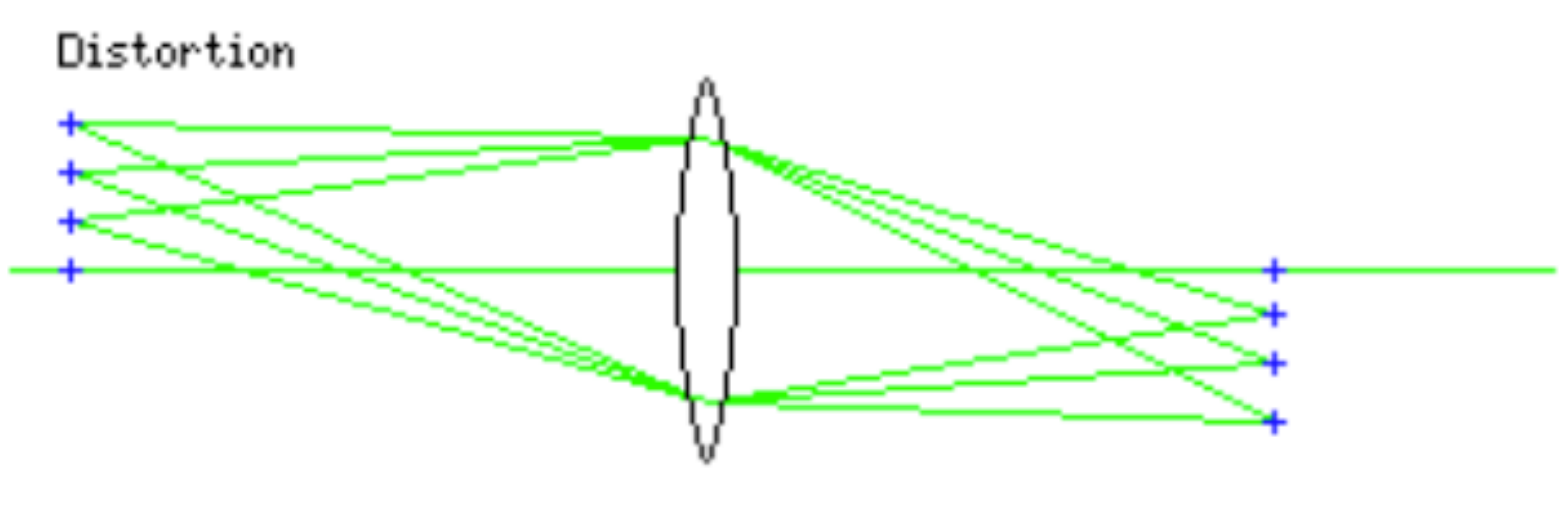
Distortion



ΔW
$W_{311} x_0^3$
x
-y
$\Delta W = W_{311} x_0^3 \rho \cos\theta$
Distortion



Object
Barrel shaped
Cushion shaped
Distorted image



Distortion

Why different designs?

- For a classical cassegrain focus or prime focus with a parabolic primary you need a *corrector*.
- The Richey-Chretien (RC) design has a hyperbolic primary and secondary designed to balance out coma and spherical in the focal plane.
- Gregorian and Aplonatic Gregorian (ApG) is another variant attempting to improve on the RC design.

  - In general, correctors are required to achieve good image quality over a wide field

Correctors

- Refractive (catadioptrics)
    - Singlet asphere + spherical mirror:
      can’t correct spherical & astigmatism simultaneously
    - Achromatic spherical doublet + spherical mirror:
      can’t correct spherical, coma and astigmatism simultaneously.
    - Triplet spherical meniscus + parabolic mirror: Wynne
      corrects coma, astigmatism, and eliminates field curvature
    - Multiplet: $$$
      7-lens design for 8.2m Subaru telescope: 0.5m diameter lenses, 30’ FoV
- Reflective secondary: e.g., RC or ApG designs
- Reflective multiplet: HET and SALT

Ingredients in Telescope Design

1. Primary diameter - collecting area
2. Primary *f*-ratio - dome-size
3. Back focal-length - instrument volume
4. Final *f*-ratio - focal scale
5. Field of view - secondary diameter & distance

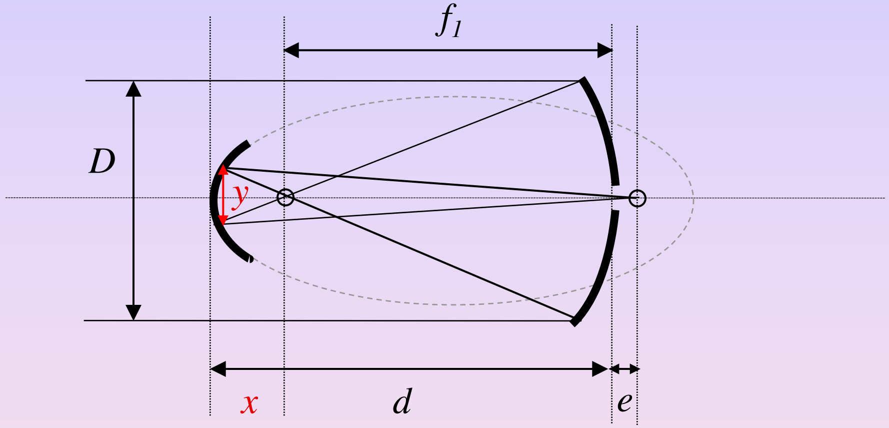
(2)-(5) determine primary+secondary figures

Types of Telescopes

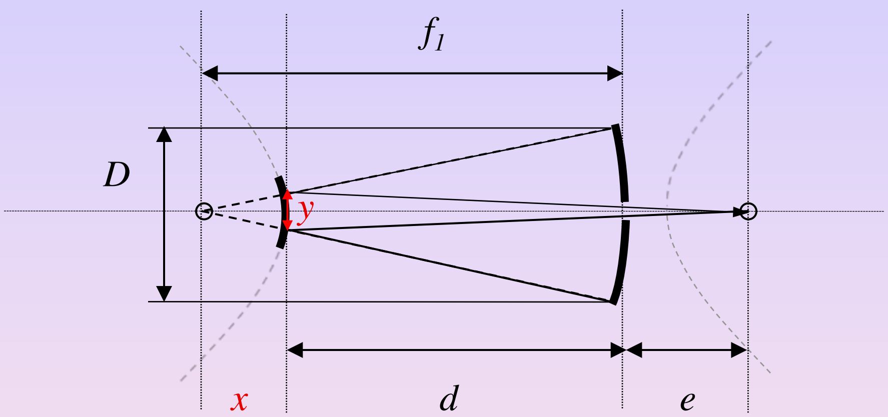
Category	**Primary**	**Secondary**	**Corrector**	**Name**	**Principal Aberrations**
Singlet lens	Spherical			refractor	spherical + chromatic
Singlet mirror	Paraboloid mirror			Newtonian	coma + astigmatism
Doublet mirrors	Paraboloid	Hyperbaloid		Cassegrain	coma
	Hyperbaloid	Hyperbaloid		Ritchey-Chertien (RC)	astigmatism (twice Cassegrain field)
	Parabaloid	Ellipsoid		Gregorian	field curvature
	Ellipsoid	Spherical		Dall-Kirkham	
	Ellipsoid	Ellipsoid		Aplonatic Gregorian (AG)	best images but large obstruction
Multiplets	Spherical		Aspheric lens or achromate doublet	Schmidt	v. wide field
	Spherical	Hyperbaloid	Aspheric lens	Schmidt-Cassegrain	"
	Spherical	Spherical	Spherical meniscus lens	Maksutov	"
	Spherical		4-mirror asphere	HET, SALT	Low cost

Two-mirror telescopes: Cassegrain focus

Gregorian (Aplonatic Gregorian)



Cassegrain (Ritchey-Cretien)



$$f\text{-}ratio \equiv f / D$$

Power: $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$

Know THIS:

Focal scale: $s = 206265 / (D \bullet f\text{-ratio})$ [arcsec/mm]

Back focal distance: $e = f - d\left(\frac{f}{f_1} + 1\right)$

Focal amplitude: $\Delta e = (1 + m^2)\Delta d, \quad m = f_1 / f_2$

Exit pupil distance: $l = f_2 d / (f_2 - d)$

= distance behind secondary

Exit pupil diameter: $D_{pupil} = Dl / d$

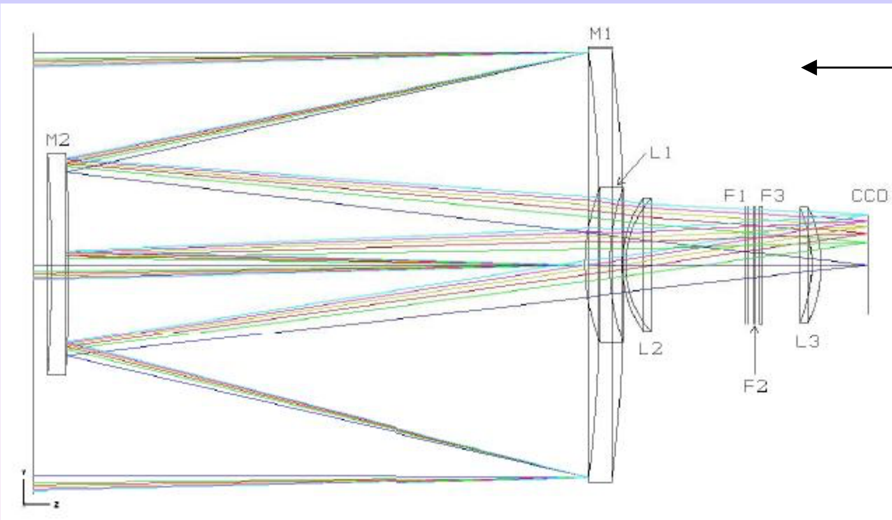
Focal-plan curvature: $\frac{1}{R_{Cass}} = \frac{1}{f_1} + \frac{1}{f_2}$

Information Gathering Power

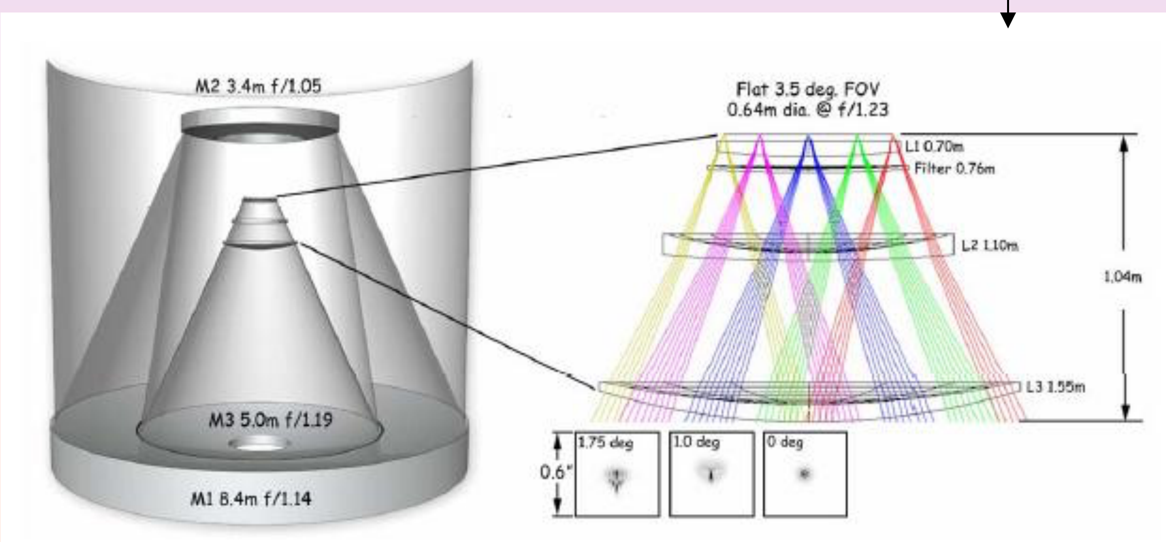
A Ω or etendue			
name	diameter	FoV	A Ω
	m	deg	m^2 deg^2
SDSS	2.5	3	35
CFHT	3.6	1	8.0
WIYN	3.5	1	7.6
PanStarrs	4x1.8	3	72
Subaru	8.1	0.2	1.6
LSST	8.4	3.5	533
SALT	10	0.13	1.0

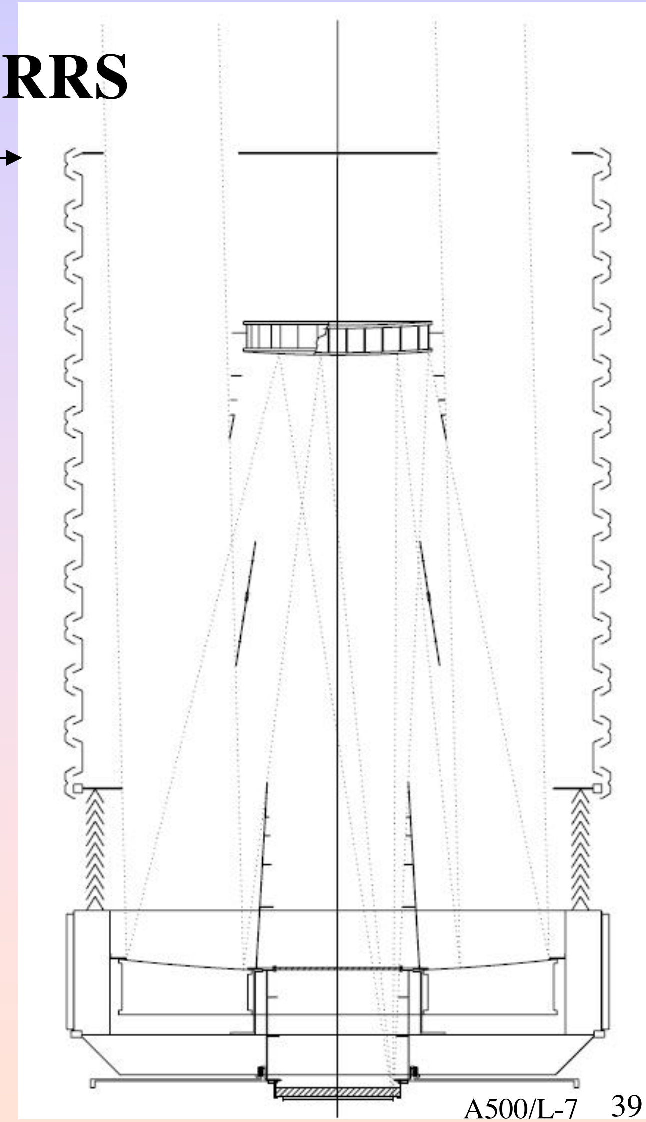
What's missing?

Wide-Field Telescopes: A vs Ω



Pan-STARRS
SDSS
LSST





Information Gathering Power

A Ω or etendue			
name	Effective diameter	Used FoV	Survey A Ω
	m	deg	m^2 deg^2
SDSS	2.1	2.2	13
CFHT	3.4	1	7.1
WIYN	3.2	1	6.3
PanStarrs	3.0	3	50
Subaru	7.7	0.2	1.5
LSST	6.7	3.5	319
SALT	9.2	0.13	0.9

What's missing?

- **Obstructions**
- **Vignetting**
- **Instrument FoV**

…and what else?