Astro 500

Techniques of Modern Observational Astrophysics

Matthew Bershady

University of Wisconsin

Lecture Outline

Part I. Course Overview

- Regressions, error models and intrinsic scatter

Part II. Detectors

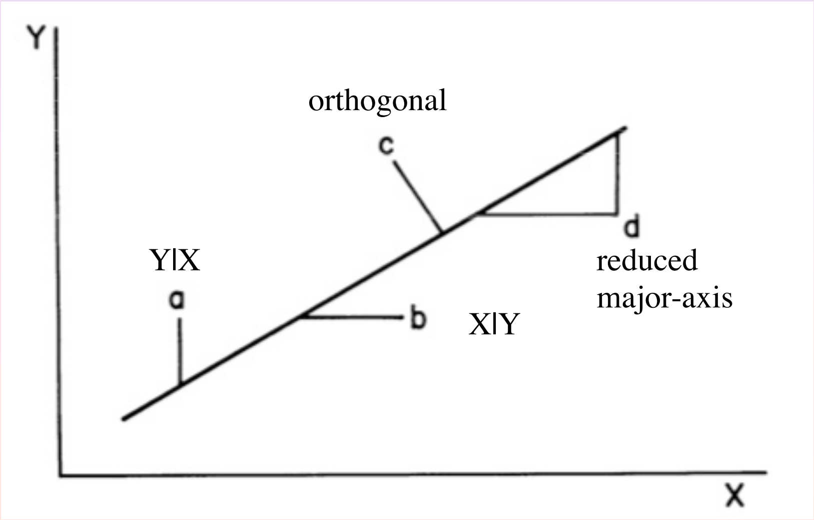
- CCDs: how they work, types, attributes & operation
- The digital unit: sampling, gain, and detector noise
- S/N formulation
- S/N regimes

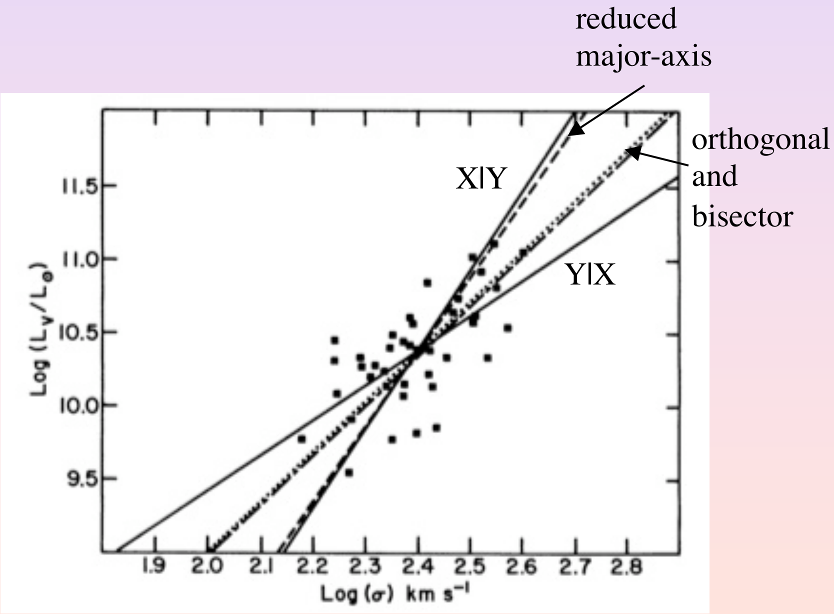
Linear Regressions

- Regressions are based on solving a set of linear equations based on different moments of the data and weighted by errors or priors.
- There are difference kinds of regression models (moments)
  - X|Y, Y|X, bisector, orthogonal
  - There are different assumptions to be made about errors that also lead to different moments and regressions
  - Is there an independent variable? (one variable with no errors)
  - Are the errors heteroscedastic or homoscedastic? (different or the same for all data)
  - Is there intrinsic scatter (usually other dimensions not known)?
- There is *no right regression model* (it depends what you want to learn), but there are correct and incorrect errors models and assumptions.
  - Social science analysis is plagued by systematic errors due to inaccurate models, but we're not free of such pitfalls because the universe is complicated.

Different Regressions

- X|Y, Y|X, bisector, orthogonal regressions
- Isobe et al. (1990, ApJ, 364, 104):
  - ordinary least squares (OLS) – no errors
- Akritas & Bershady (1996, ApJ, 470, 706)
  - bivariate correlated errors (heteroscedastic) and intrinsic scatter (BCES)





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OLS Regression formulae

TABLE 1

LINEAR REGRESSION FORMULAE FOR SLOPES

Method	Expression for Slope	Estimate of the Variance of the Slope $\widehat{\mathrm{Var}}\,(\beta_i)$
OLS($X \mid Y$)	$\beta_1 = \dfrac{S_{xy}}{S_{xx}}$	$\dfrac{1}{S_{xx}^2}\left[\sum_{i=1}^{n}(x_i - \bar{x})^2(y_i - \beta_1 x_i - \bar{y} + \beta_1 \bar{x})^2\right]$
OLS($Y \mid X$)	$\beta_2 = \dfrac{S_{yy}}{S_{xy}}$	$\dfrac{1}{S_{xy}^2}\left[\sum_{i=1}^{n}(y_i - \bar{y})^2(y_i - \beta_2 x_i - \bar{y} + \beta_2 \bar{x})^2\right]$
OLS bisector	$\beta_3 = (\beta_1 + \beta_2)^{-1}[\beta_1\beta_2 - 1 + \sqrt{(1+\beta_1^2)(1+\beta_2^2)}]$	$\dfrac{\beta_3^2}{(\beta_1+\beta_2)^2(1+\beta_1^2)(1+\beta_2^2)}[(1+\beta_2^2)^2\,\widehat{\mathrm{Var}}\,(\beta_1) + 2(1+\beta_1^2)(1+\beta_2^2)\,\widehat{\mathrm{Cov}}\,(\beta_1, \beta_2) + (1+\beta_1^2)^2\,\widehat{\mathrm{Var}}\,(\beta_2)]$
Orthogonal regression	$\beta_4 = \frac{1}{2}[(\beta_2 - \beta_1^{-1}) + \mathrm{Sign}\,(S_{xy})\sqrt{4 + (\beta_2 - \beta_1^{-1})^2}]$	$\dfrac{\beta_4^2}{4\beta_1^2 + (\beta_1\beta_2 - 1)^2}[\beta_1^{-2}\,\widehat{\mathrm{Var}}\,(\beta_1) + 2\,\widehat{\mathrm{Cov}}(\beta_1, \beta_2) + \beta_1^2\,\widehat{\mathrm{Var}}\,(\beta_2)]$
Reduced major-axis	$\beta_5 = \mathrm{Sign}\,(S_{xy})(\beta_1\beta_2)^{1/2}$	$\dfrac{1}{4}\left[\dfrac{\beta_2}{\beta_1}\,\widehat{\mathrm{Var}}\,(\beta_1) + 2\,\widehat{\mathrm{Cov}}(\beta_1, \beta_2) + \dfrac{\beta_1}{\beta_2}\,\widehat{\mathrm{Var}}\,(\beta_2)\right]$

Errors on Regressions

- How do you estimate errors on slope and intercept?
- Resample your data:
  - Boot-strap – pick N data points out of sample of N, m times. Each pick is a random selection from N data points with equal probability of selecting i^{th} element.
  - Jack-knife – recalculate leaving out one datum, N times (N data)
- Monte Carlo simulation – artificial data

✧ When in doubt, “Monte Carlo” your data

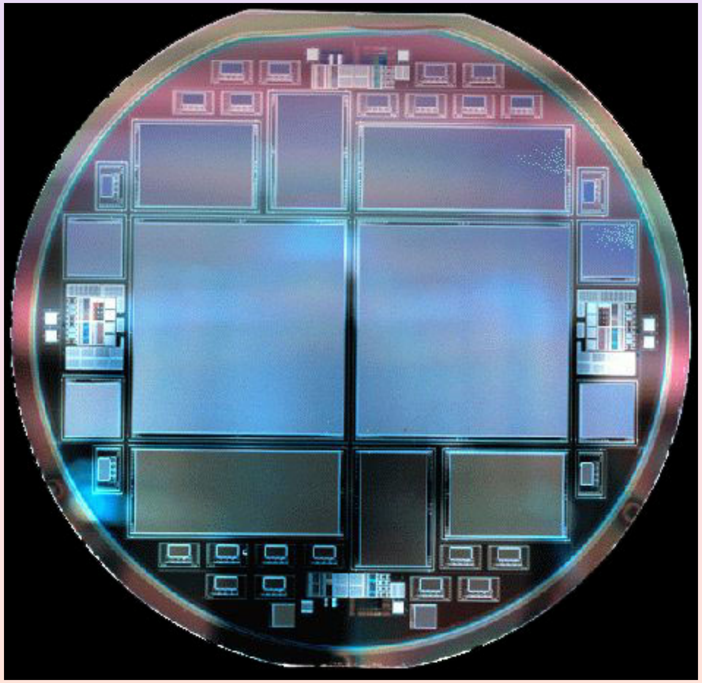
✧ This applies not just to linear regressions but any modeling.

When in doubt….

- “Monte Carlo (MC) your data”
- Monte Carlo: a town in Monaco (country in SE France) famous for gambling casinos
- What you need:
  - Model of data
  - Model of errors
  - Model of data sampling (range, censorship, incompleteness, spurious source (when applicable).
  - A good random-number generator
  - A modicum of computing skill and cpu time.
- How good is it?
- Only as good as your assumptions (i.e., model)
- Test your assumptions by comparing distributions (and their characterization) generated by MC against those from the data.

Digital Detectors

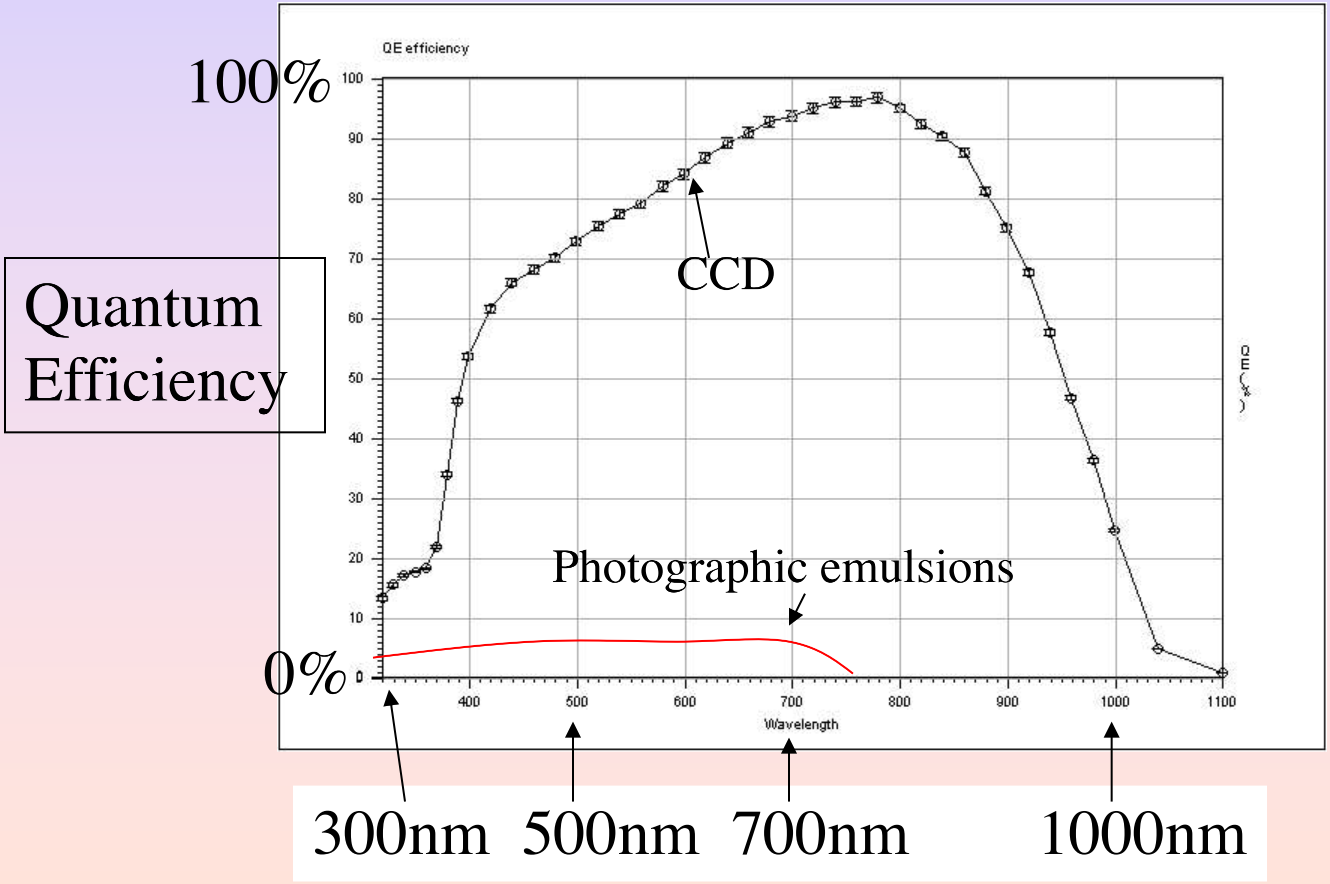
- By far the most common detector for wavelengths 300nm$<\lambda<$1000nm is the CCD.



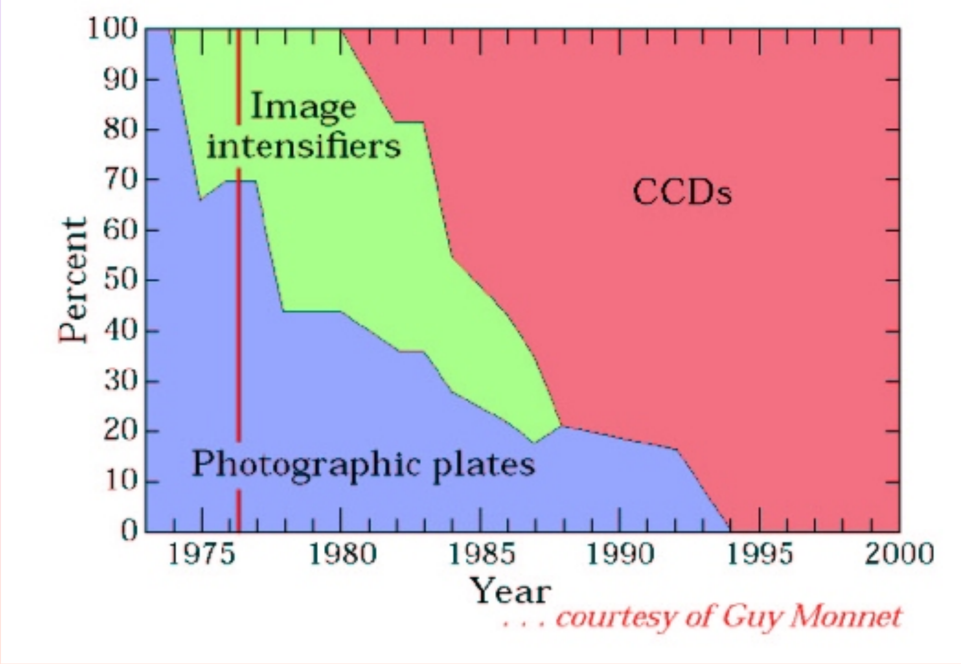
In 10 years they may be a thing of the past…
…replaced by CMOS.

CCDs

1. Quantum efficiency is more than an order of magnitude better than photographic plates.



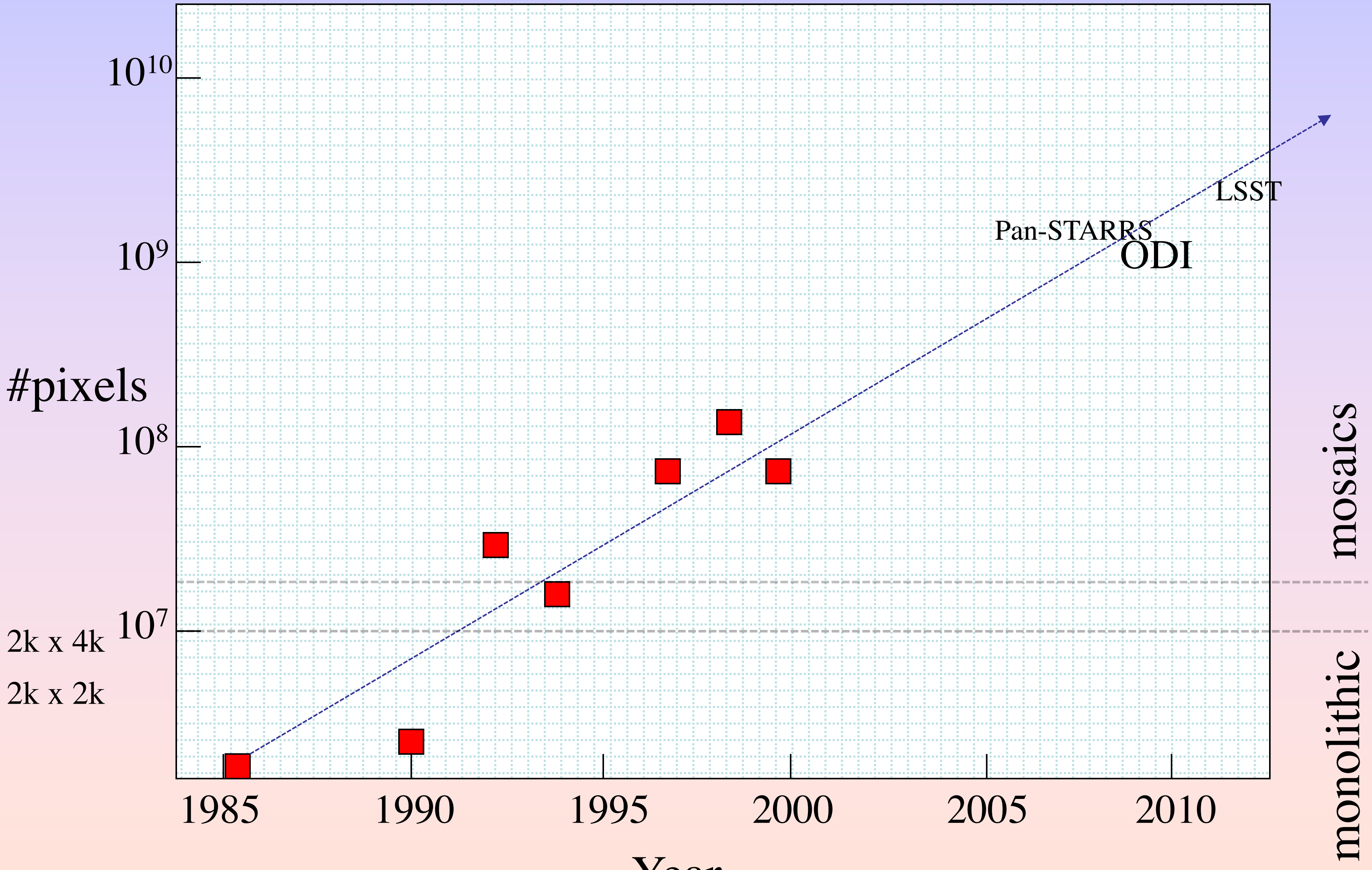
These are silicon fab-line devices and complicated to produce.



digital revolution

CCDs remain physically small compared to photographic plates, but they took over rapidly anyway.

CCD size



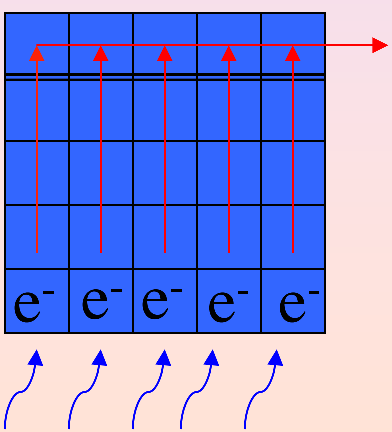
10^10
10^9
10^8
10^7
#pixels
2k x 4k
2k x 2k
LSST
Pan-STARRS
ODI
mosaics
monolithic
1985
1990
1995
2000
2005
2010
Year

CCDs: How do they work?

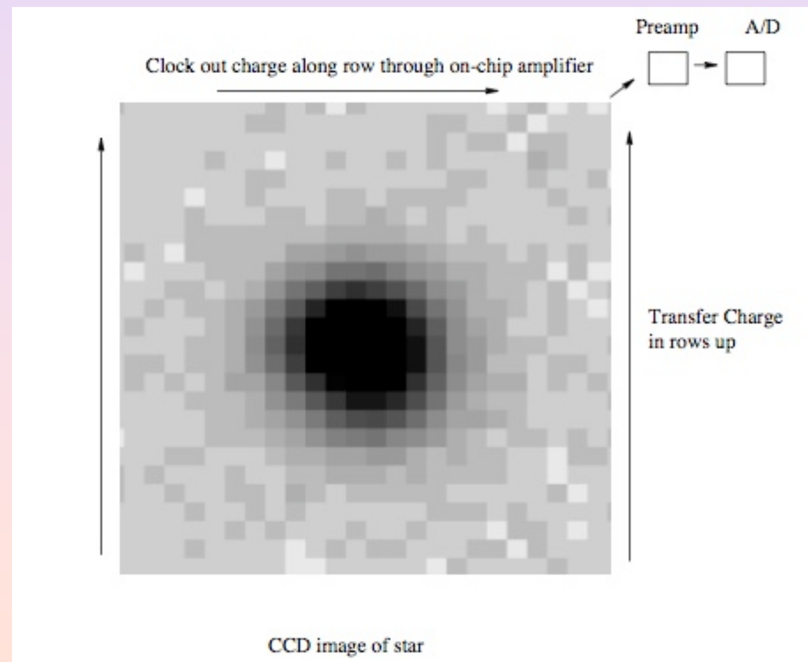
Photons ⇨ Electrons bumped into conduction band ⇩ Read out & amplify charge ⇦ A/D convert to DN

- Silicon semiconductors with "gate" structure to produce little potential corrals or wells.

serial register

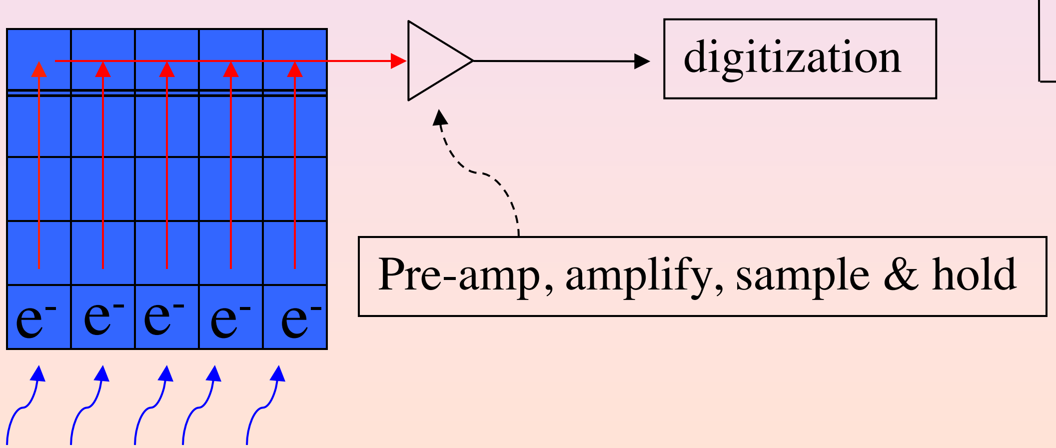


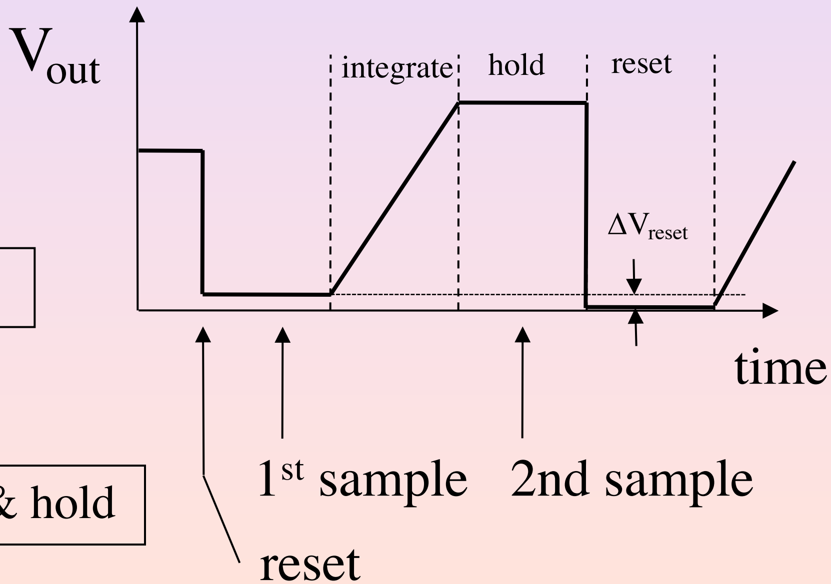
'clock' parallel and serial registers
"CTE" > 0.99999



Correlated double-sampling

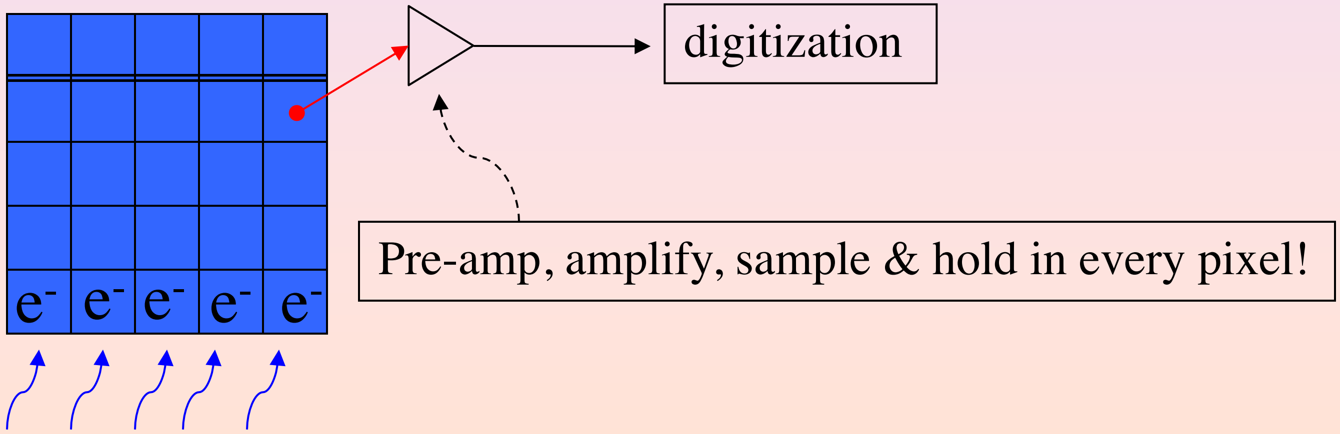
- After charge from each pixel is clocked out, amplified, and sampled, read-out amps are reset to a reference voltage.
- Reset has inherent (kT) noise.
- This is completely eliminated by measuring the voltage difference after reset and after integration (before next reset).





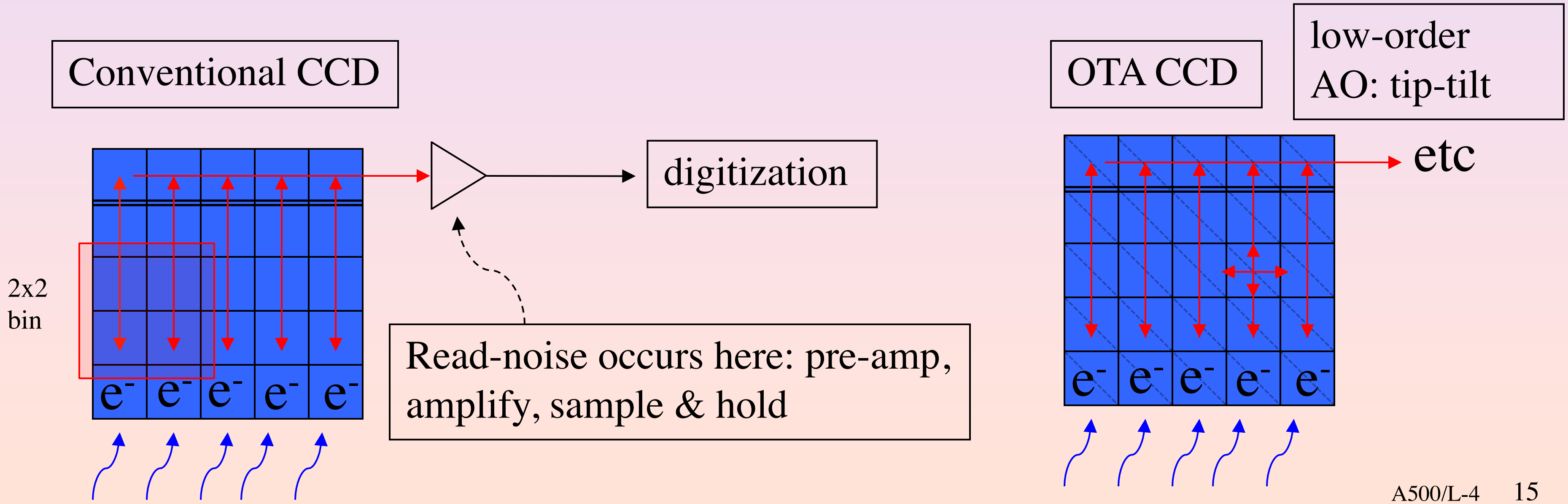
CMOS

- Complementary Metal Oxide Semicondcutors – complementary pairs of p- and n-type MOSFETs.
- Advantages over CCD (with only p- or n-type): *low-power consumption*
- Allows additional circuitry to be placed in each pixel
  - Every pixel has its own R.O.E. and is directly addressable.
- Led to < 100% fill-factor of light-sensitive region in early devices
  - Can be ameliorated somewhat by micro-lenses but these are lossy too, and scatter
  - Solved with back-side illumination devices
- Gain, bias, and noise non-uniformity add additional calibration demands
  - e.g., fixed-pattern noise and more



CCDs: unusual features

- Non-destructive shifting of charge
  - Drift-scanning: optimizes flatness and efficiency (read-time)
  - Nod-and-shuffle: optimizes flatness and sky-subtraction
  - Frame-transfer: optimizes high-speed photometry
  - On-chip binning: optimizes read-noise
  - Orthogonal-transfer (OTA, e.g., ODI): optimizes image quality



CCD types

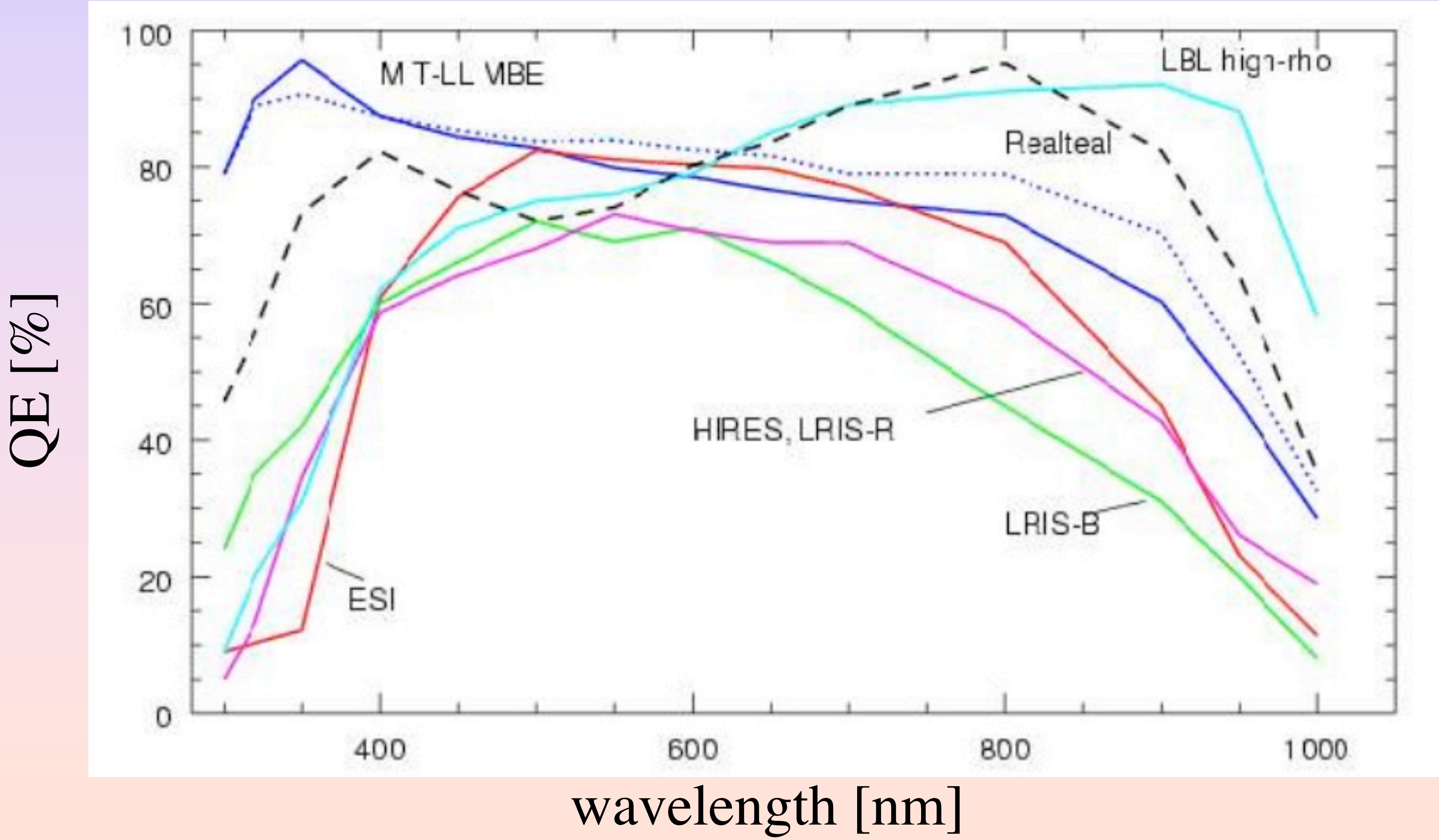
- Front-side vs back-side illuminated
- Thinned (back-side) illuminated
- Coated (UV enhanced)
- Deep-depletion (improved red response; decreased blue response)
- High vs low resistivity (improved red response)

CCD attributes

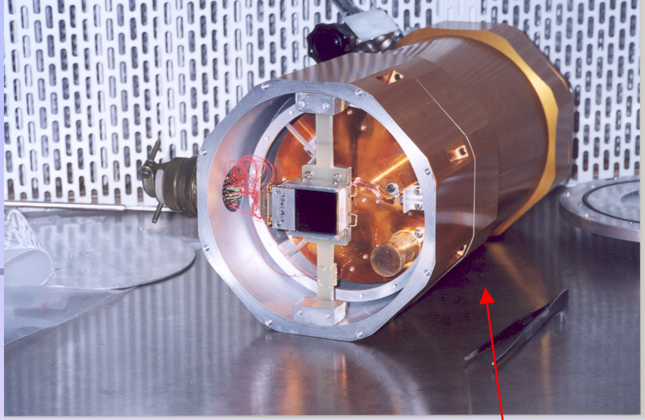
- Pixel size
- Pixel fill-factor
- Array size
- Array flatness
- Quantum efficiency (QE_λ)
- Dark current
- Charge-transfer efficiency (CTE)
- Electron diffusion (MTF)
- Blooming
- Cosmetics / defects
  - Column defects
  - White and black spots
  - traps
- Amplifiers & electronics
  - How many
  - Read-noise
  - Noise uniformity (btwn amps)
  - Hysterisis / latency
  - Cross-talk and ghosting
  - System noise (RF) pickup
  - Stability (bias drift)

CCD QE_λ

CCDs from Lick Observatory: present and future

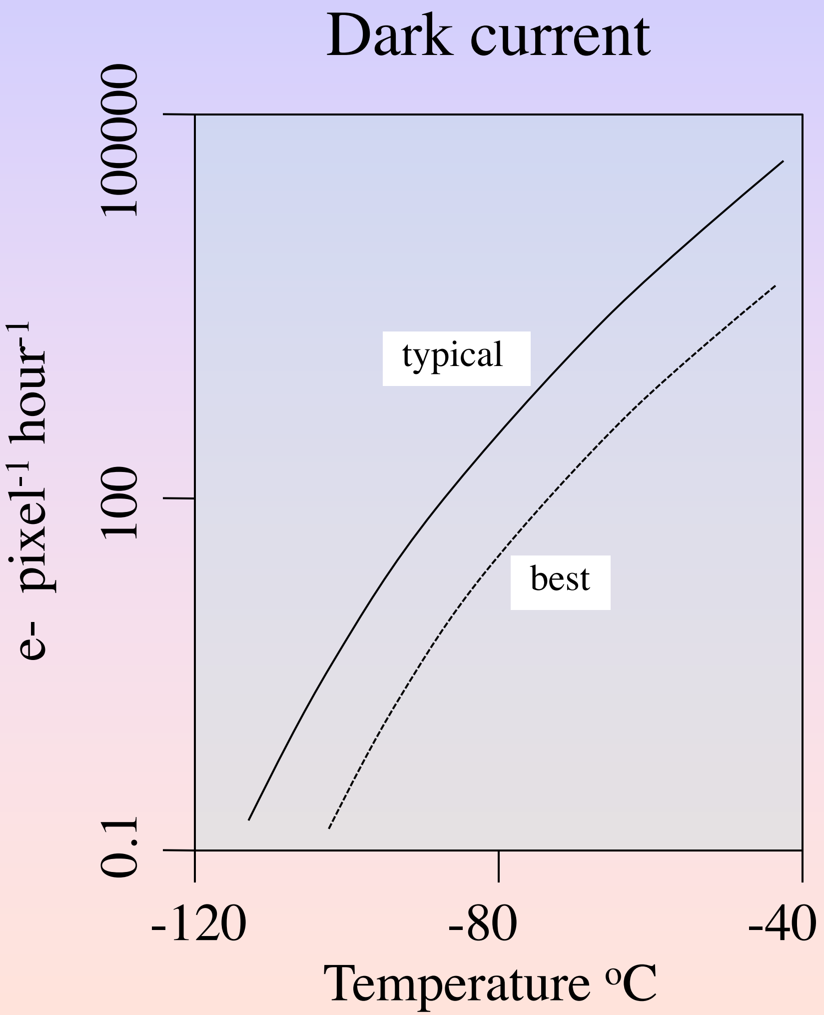


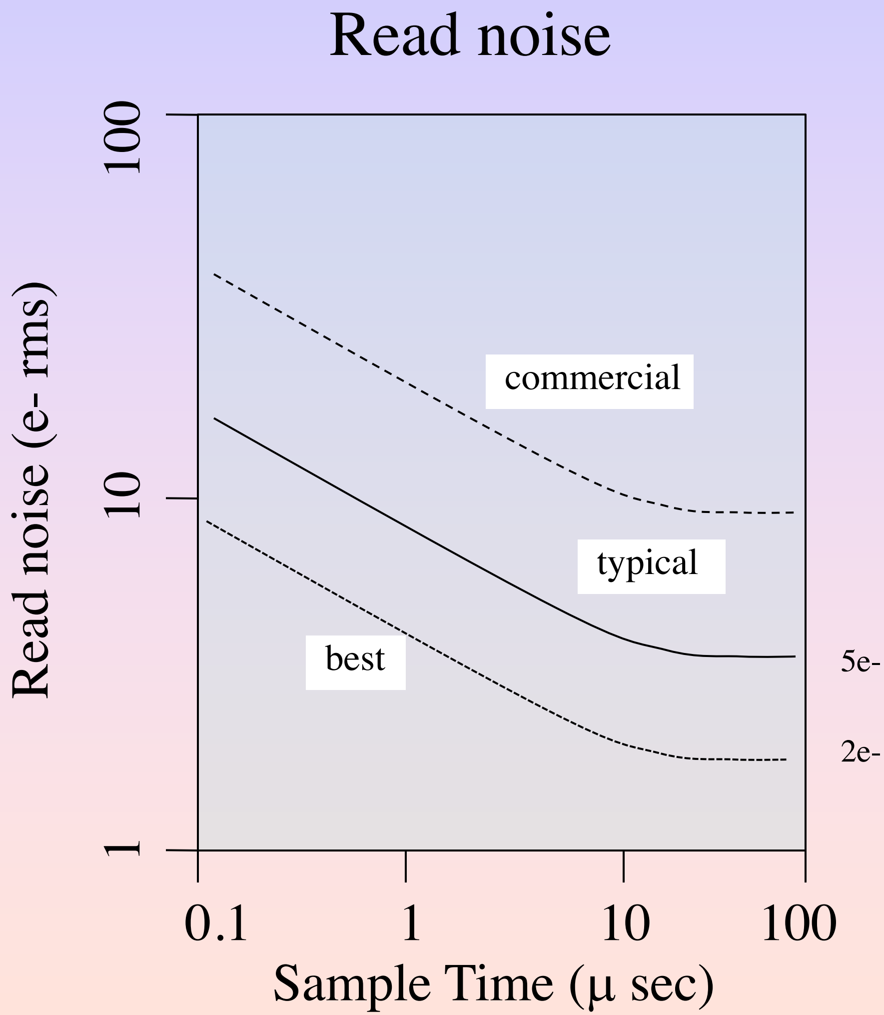
CCD operation



- At room temperature, electrons in high-energy thermal tail of the silicon spontaneously pop up into the conduction band: “dark current.” Cooling the detectors reduced the dark current although at about -120C the quantum efficiency starts to decrease.
- Therefore, CCDs usually are put into dewars with liquid nitrogen cold baths and heaters and the temperature is actively controlled to ~1C.
- Readout speed is typically adjustable--faster readout gives higher readout noise per pixel.

Dark current and Read-noise





Gain, linearity, and bias

- The potential corrals that define the pixels of the CCD start to flatten as e^- collect. This leads first to saturation, then to e^- spilling out along columns.
- The "inverse gain" is the number of e^- per final "count" post the A/D converter.
- One *very* important possibility for CCDs is to tune the response to be linear.
- An electronic pedestal voltage (bias) is introduced into the read-out electronics to ensure no negative data values occur due to noise. This pedestal has nothing to do with well-depth.

Digital Units

- "Counts'' = ADU = DN

Analogue-to-digital unit

DigitalUnit

- DN is not the fundamental unit, the # of detected electrons is. The "Gain" is set by the electronics.

- Most science-grade A/D converters use 16 bits.

  DN from: 0 to $(2^{16} - 1) = 65535$

  for unsigned, long integers

- Signed integers are nuts: -32735 to +32735

  $+/-(2^{15} - 1)$

What gain do you want?

Example: LRIS-R has a SITe 24μ-pixel CCD with pixel "wells'' that hold ~350,000 e-

- 16-bit unsigned integer A/D saturates at 65525 DN
- Would efficiently maximize dynamic range by matching these saturation levels:

$$\frac{350{,}000}{65{,}535} = 5.3\,\frac{e-}{DN}$$

- Note, this under-samples the readout noise and leads to "digitization" noise.

Fundamental Performance trades

- Read-time vs Read-noise
- Dark-current vs QE
- Dynamic range vs Well-sampled noise

high-signal limit (Dynamic range)

low-signal limit (Well-sampled noise)

Signal

- Point source
  - We are measuring photon flux
  - $E(\gamma) = f(\gamma) A\ t$
- Resolved source
  - We are measuring surface brightness
  - $E(\gamma) = I(\gamma)\ A\ \Omega\ t$

A is telescope collecting area; *t* is exposure time

$S(\gamma)$

This ignores any inefficiencies in the measurement process

Signal-to-Noise (S/N)

total system efficiency

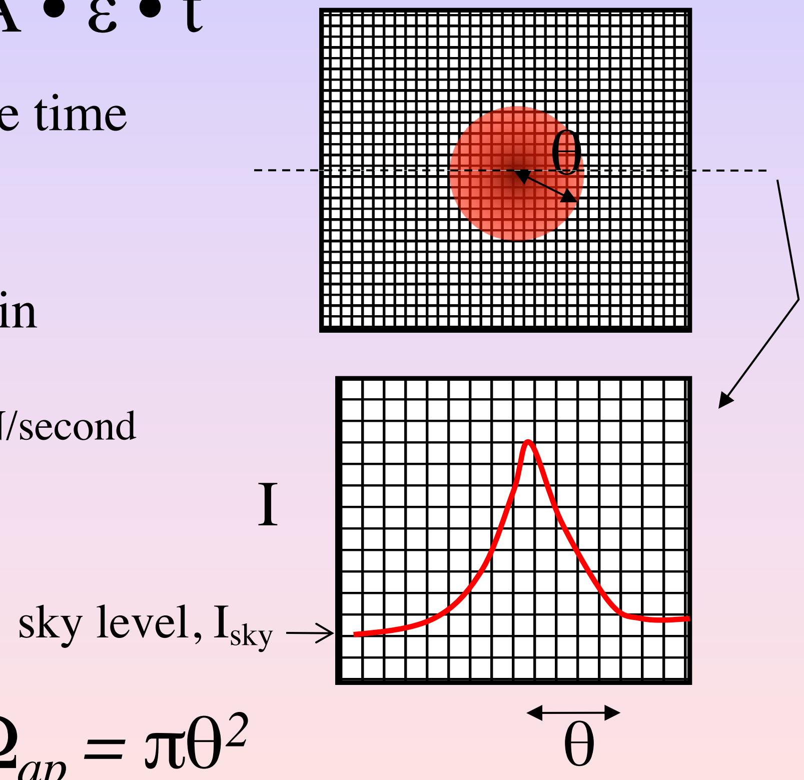
- Signal = $S_{obj} \bullet \varepsilon \bullet t = f_{\rm obj} \bullet A \bullet \varepsilon \bullet t$

exposure time

detected e-/second: $S_{obj} = S_{DET} \bullet$ gain

DN/second

- Consider the case where we count all the detected e- in a circular aperture with radius θ. In this case $\Omega_{ap} = \pi\theta^2$



Aside: how big an area do we want to integrate over?

Noise Sources

$\sqrt{S_{obj} \cdot \varepsilon \cdot t} = \sqrt{f_{obj} \cdot A \cdot \varepsilon \cdot t}$ $\Rightarrow$ shot noise from source

$\sqrt{I_{sky} \cdot A \cdot \pi\theta^2 \cdot \varepsilon \cdot t}$ $\Rightarrow$ shot noise from sky in aperture of circular radius r

$\sqrt{RN^2 \cdot \pi\theta^2}$ $\Rightarrow$ readout noise in aperture of circular radius r

$\sqrt{\left[RN^2 + (0.5 \times \text{gain})^2\right]} \cdot \sqrt{\pi\theta^2}$ $\Rightarrow$ more general RN ?

$\sqrt{\text{Dark} \cdot \pi\theta^2 \cdot t}$ $\Rightarrow$ shot noise in dark current in aperture of circular radius r

$S_{obj} \cdot \varepsilon = \text{e}^- / \text{sec}$ from the source

If I_{sky} is in units of photon flux per pixel, then $I_{sky} \cdot \varepsilon = \text{e}^- / \text{sec} / \text{pixel}$

from the sky and $S_{sky} = I_{sky} \cdot A \cdot \pi\theta^2 = I_{sky} \cdot n_{pix}$

If I_{sky} is in units of photon flux per unit solid angle $d\Omega$ (e.g., arcsec^{-2}),

then $S_{sky} = I_{sky} \cdot \pi\theta^2$ but express $\pi\theta^2$ in the same units of solid angle.

RN = read noise (as if RN^2 e^- had been detected)

$\text{Dark} = \text{e}^- / \text{sec} / \text{pixel}$

Noise Sources

$\pi\theta^2$ must be in units that match I_{sky} (e.g., flux per unit solid angle) →

$\pi\theta^2$ must be in pixel units

$$\sqrt{S_{obj}\cdot\varepsilon\cdot t} = \sqrt{f_{obj}\cdot A\cdot\varepsilon\cdot t} \Rightarrow \text{shot noise from source}$$

$$\sqrt{I_{sky}\cdot A\cdot\pi\theta^2\cdot\varepsilon\cdot t} \Rightarrow \text{shot noise from sky in aperture of circular radius } r$$

$$\sqrt{RN^2\cdot\pi\theta^2} \Rightarrow \text{readout noise in aperture of circular radius } r$$

$$\sqrt{\left[RN^2+(0.5\times\text{gain})^2\right]}\cdot\sqrt{\pi\theta^2} \Rightarrow \text{more general RN}$$

?

$$\sqrt{\text{Dark}\cdot\pi\theta^2\cdot t} \Rightarrow \text{shot noise in dark current in aperture of circular radius } r$$

In general: replace $\pi\theta^2$ with Ω_{ap} where Ω_{ap} is the solid angle of your measurement aperture.

$S_{obj}\cdot\varepsilon = \text{e}^-/\sec$ from the source

If I_{sky} is in units of photon flux per pixel, then $I_{sky}\cdot\varepsilon = \text{e}^-/\sec/\text{pixel}$ from the sky and $S_{sky} = I_{sky}\cdot A\cdot\pi\theta^2 = I_{sky}\cdot n_{pix}$

If I_{sky} is in units of photon flux per unit solid angle dΩ (e.g., $\text{arc}\sec^{-2}$), then $S_{sky} = I_{sky}\cdot\pi\theta^2$ but express $\pi\theta^2$ in the same units of solid angle.

RN = read noise (as if RN^2 e$^-$ had been detected)

Dark = e$^-$ / sec / pixel

NB → Ω_{ap} must have suitable units (pixels for RN and Dark and typically arcsec^2 for I_{sky}).

S/N for object measured in aperture $\mathbf{\Omega}_{\mathrm{ap}}$:

n_{pix} = # of pixels in the aperture = $\pi\theta^2$ for circular aperture

$$\frac{\text{Signal}}{\text{Noise}} = \frac{f_{obj} \cdot A \cdot \varepsilon \cdot t}{\left[f_{obj} \cdot A \cdot \varepsilon \cdot t + I_{\mathrm{sky}} \cdot A \cdot \Omega_{ap} \cdot \varepsilon \cdot t + \left(RN + \frac{\mathrm{gain}}{2} \right)^2 \cdot n_{pix} + \mathrm{Dark} \cdot t \cdot n_{\mathrm{pix}} \right]^{\frac{1}{2}}}$$

All the noise terms added in quadrature

Note: always calculate in *e*- *why?*

S/N for object measured in aperture $\mathbf{\Omega}_{ap}$:

n_{pix}= # of pixels in the aperture = $\pi\theta^2$ for circular aperture

S_{obj}

S_{sky}

Noise from the dark current in aperture with angle Ω_{ap}solid expressed in pixels

$$\frac{\text{Signal}}{\text{Noise}} = \frac{f_{obj} \cdot A \cdot \varepsilon \cdot t}{\left[f_{obj} \cdot A \cdot \varepsilon \cdot t + I_{\text{sky}} \cdot A \cdot \Omega_{ap} \cdot \varepsilon \cdot t + \left(RN + \frac{\text{gain}}{2} \right)^2 \cdot n_{pix} + \text{Dark} \cdot t \cdot n_{\text{pix}} \right]^{\frac{1}{2}}}$$

Noise from object *e*- in aperture

Noise from sky *e*- in aperture with solid angle Ω_{ap}

Readnoise in aperture with angle Ω_{ap}solid expressed in pixels

All the noise terms added in quadrature

Note: always calculate in *e*- *why?*

What is ignored in this S/N eqn?

Telescope diameter

- Explicit inclusion of collecting aperture (i.e., $A \propto D_{tel}^2$)
- Break-out of terms that go into total system efficiency (starting from the top of the atmosphere)

o Bias level/structure correction and errors

o Flat-fielding correction and errors

o Charge Transfer Efficiency (CTE) 0.99999/pixel transfer

o Non-linearity when approaching full well

o Scale changes in focal plane

o Interpolation errors and correlation

S/N regimes

- Two basic regimes:
  1. Photon-limited (shot-noise from source + sky photons)
  2. Detector-limited (read-noise)
- In photon-limited case, two important sub-regimes
  a. Source-limited
  b. Sky-limited

S/N regimes: limiting cases

Let's assume CCD with Dark=0, well sampled read noise.

$$S/N = \frac{f_{obj} \cdot A \cdot \varepsilon \cdot t}{\left[f_{obj} \cdot A \cdot \varepsilon \cdot t + I_{\mathrm{sky}} \cdot A \cdot \Omega_{ap} \cdot \varepsilon \cdot t + (RN)^2 \cdot n_{pix} \right]^{\frac{1}{2}}}$$

Note: seeing or source-size comes in with Ω_{ap} and $\mathrm{n_{pix}}$ terms

<u>1a. Bright Sources:</u> $(S_{obj}\, \varepsilon\, t)^{1/2}$ dominates noise term

$$\mathrm{S/N} \approx \frac{S_{obj}\, \varepsilon\, t}{\sqrt{S_{obj}\, \varepsilon\, t}} = \sqrt{S_{obj}\, \varepsilon\, t} \propto t^{\frac{1}{2}}$$

S/N limiting cases (*contd*)

$$S/N = \frac{f_{obj}\cdot A\cdot\varepsilon\cdot t}{\left[f_{obj}\cdot A\cdot\varepsilon\cdot t + I_{\rm sky}\cdot A\cdot\Omega_{ap}\cdot\varepsilon\cdot t + (RN)^2\cdot n_{pix}\right]^{\frac{1}{2}}}$$

$I_{sky,pix} = I_{sky}\,\Omega_{ap}/n_{pix}$, i.e., this is the sky flux per pixel

1b. Sky Limited: $(\sqrt{I_{sky,pix}\,\varepsilon t} > 3 \times RN)$ *

$$\mathrm{S/N} \propto \frac{S_{obj}\,\varepsilon t}{\sqrt{I_{sky}\,A\Omega_{ap}\varepsilon t}} \propto t^{\frac{1}{2}}$$

Note: seeing comes in with Ω_{ap} or n_{pix} term

2. Read-noise Limited: $(\sqrt{I_{sky,pix}\,\varepsilon t} < 3 \times RN)$ *

$$\mathrm{S/N} \propto \frac{S_{obj}\,\varepsilon t}{RN\sqrt{n_{pix}}} \propto t$$

What does this imply about exposure time?

*Why 3? How about 1?

S/N regimes: limiting cases

Again, let's assume CCD with Dark=0, well sampled read noise.

$$S/N = \frac{f_{obj} \cdot A \cdot \varepsilon \cdot t}{\left[f_{obj} \cdot A \cdot \varepsilon \cdot t + I_{\text{sky}} \cdot A \cdot \Omega_{ap} \cdot \varepsilon \cdot t + (RN)^2 \cdot n_{pix} \right]^{\frac{1}{2}}}$$

But now let's take into account the explicit dependencies not just on time but on collecting area A and measurement aperture Ω_{ap}.

S/N limiting cases (*contd*)

1a. Bright Sources:

$$\mathrm{S/N} \approx \frac{S_{obj}\,\varepsilon\, t}{\sqrt{S_{obj}\,\varepsilon\, t}} \propto (A\ \varepsilon\ t)^{\frac{1}{2}} \propto (\varepsilon\ t)^{\frac{1}{2}} D_{\mathrm{tel}}$$

1b. Sky Limited:

$$\mathrm{S/N} \propto \frac{S_{obj}\,\varepsilon\, t}{\sqrt{I_{sky}\,\Omega_{ap}\varepsilon\, t}} \propto (A\ \varepsilon\ t / \Omega_{ap})^{\frac{1}{2}} \propto (\varepsilon\ t)^{\frac{1}{2}} D_{\mathrm{tel}} / \theta_{ap}$$

Could choose 1

2. Read-noise Limited: $(\sqrt{I_{sky}\,\varepsilon\, t} < 3 \times RN)$

$$\mathrm{S/N} \propto \frac{S_{obj}\,\varepsilon\, t}{\sqrt{n_{pix} RN^2}} \propto \varepsilon\ t \left(\mathrm{D}_{\mathrm{tel}} / \theta_{ap}\right)^2$$

D_{tel}: telescope diameter
θ_{ap}: measurement aperture radius

DQE

- DQE is often defined as the *effective quantum efficiency* of a CCD relative to an ideal detector with no read-noise. In the source-limited regime, ignoring dark-current:

$$DQE = QE \,/\, \left[1 + \frac{RN^2}{QE \cdot S_{obj} \cdot t} \right]$$

where QE is the CCD quantum efficency.

- This can be generalized for any noise-regime, and including dark-current.
- A related concept is the *effective system efficiency*, DQE_{sys}, of which CCD QE is only one part.