Astro 500 Problem Set #5

1. Consider a spectrograph with collimator and camera focal lengths of 1000 mm and 250 mm, respectively. The spectrograph is set up with a 1000 l/mm reflection grating **used in first order** with a grating incidence angle of $\alpha = 30$ deg. In this problem both α and diffracted angle β obey the sign convention of the right-hand rule with respect to the grating normal. The camera-collimator angle is defined as $\theta_{cc} = \beta - \alpha$ and has a value of $\theta_{cc} = -30$ deg. For a reflection grating with this sign convention, the grating equation is $m\,\lambda = \sigma\,(\sin\alpha + \sin\beta)$.

(1a) What is the central wavelength?

(1b) What is the *inverse* linear dispersion ($d\lambda/dl$ in, e.g. nm/mm) at the camera focus at this central wavelength? Specify your units.

(1c) With a 231 μm wide entrance slit (note 231 = 200 / cos(30)), what is the effective reimaged monochromatic slit-width, considering both geometric and anamorphic factors at the *center* of the slit? The anamorphic factor $r = |\,d\beta/d\alpha\,|$ is found from the grating equation setting $d\lambda/d\beta = 0$.

(1d) What is the spectral resolution at this central wavelength at the center of the slit?

(1e) Compute the number of detected electrons per spectral resolution element per second for a y' = 20 mag (Vega) source assuming the spectrograph is attached to a telescope with 10 m^2 of collecting area and a total system efficiency (from the top of the atmosphere to detected photo-electrons) of 20%. (y' is a fictional medium-band filter centered near 500 nm.)

2. A spectrograph has a 500 mm focal-length f/2 *parabolic* reflecting collimator and a 100 mm length slit *centered* about the optical axis of the parabola. For practical purposes assume that the size of the collimator and collimated beam is many orders of magnitude larger than the wavelength of light of interest.

For two slit locations, you are asked to identify and quantify sources of collimator aberrations in angular units. These angles can be translated to a physical width at a focus created by a perfect optic. If this optic has the same focal length as the collimator then you can directly compare this width to the physical width of the slit entrance aperture.

In answering the following questions you may wish to start by drawing a diagram and consider how this might be similar to discussions we have had about telescopes and basic optics.

(2a) What are the angular aberrations in the collimated light for the location at the center of the slit?

(2b) What are the angular aberrations **in radians** in the collimated light for the location at either end of the slit? Use the small-angle approximation. Estimate the total angular aberrations (δ_{total} in radians) by summing terms in quadrature.

(2c) What slit-width (in mm) is equivalent to the total collimator aberrations at the *center* of the slit.

(2d) What slit width (in mm) is equivalent to the total collimator aberrations at either *end* of the slit.

3. Compare your results in (2d) to the fiber diameters used on the WIYN Bench Spectrograph and conclude on the following questions. Again, the camera does nothing to correct for, or add to, the aberrations introduced by the collimator. Also, anamorphism is not an aberration.

(3a) What dominates the monochromatic spot at the ends of the slit: the physical size of the fibers (their diameter) or the aberrations? Provide relevant numbers.

(3b) Does your conclusion in (3a) depend on the camera focal length (why or why not)?

(3c) Does your conclusion in (3a) depend on any anamorphism introduced by the grating (why or why not)?

4. An extended source has a surface-brightness of 25 mag arcsec^{-2} (AB) in the *V* band. The telescope has a 10m diameter, and images onto a detector with 14.5 μm pixels with a *f*-ratio of 3. The total system throughput (given as a fraction), including the atmosphere, obstructions in the telescope, all optical surfaces, filter and the detector QE is 1/ π in the *V* band. How many **photons per sec per pixel** in the *V* band are collected from the source? Assume no binning.

5. Now the same extended source is reimaged through a spectrograph on the same telescope and with the same detector. The spectrograph collimator is *f*/3, and the camera is *f*/1.5. The spectrograph entrance aperture is a slit with a 1 arcsec width. A grating is used in first order that delivers a spectral resolution of $R = \lambda / \delta \lambda = 5500$ at 550 nm in a Littrow configuration with this slit width. Assume the source has a spectral energy distribution that is constant per unit neper ($\delta\lambda/\lambda$).

(5a) How many photons are collected **per sec per resolution element** *along each 1 arcsec of slit length* from the source?

(5b) How many photons are collected **per sec per pixel** from the source?

6. Compute the line-density of the grating assuming a 1 m collimator focal length.

Formulae:

Aberrations:

$\delta_{diffraction} = 1.22\,\lambda / D$
$\delta_{spherical} = [128\,(f/D)^3\,]^{-1}$
$\delta_{coma} = 3\,\theta\ /\ [\,16 \backslash (f/D)^2\,]$ (parabola)
$\delta_{astigmatism} = \theta^2\ /\ [2\ (f/D)\,]$ (parabola)

where δ is the aberration angular **diameter**, θ is the angle w.r.t. the optical axis, f is the focal-length, and D is the optic diameter. All angles in these formulae are expressed in **radians** where 1 radian = 206265 arcsec ~ 57.3 deg.

y′ Band-pass:
$\lambda = 0.50\ \mu m$
zeropoint = 2500 Jy (Vega system)

V Band-pass:
$\lambda = 550$ nm
$\delta\lambda / \lambda = 0.16$
zeropoint = 3631 Jy (AB system)

Flux conversion:
1 μJy = 15.1 photons $sec^{-1}\ m^{-2}\ neper^{-1}$