A500 / Problem Set #4 / Solutions

1. Magnification is given by the ratio of focal-lengths, which can be obtained (assuming a constant beam-size) as the ratio of the focal-lengths, or $1.33/3.32 = 0.40$. This is a 2.5 times demagnification from entrance slit to the detector focal-plane.

2. The above demagnification takes $D_f = 300$ microns to $D'_f = 120$ microns, or eight 15-micron pixels.

3. Since the focal-ratio is just the focal-length (f) divided by the beam-diameter (D), and we are given D = 115 mm, we have $f_1 = 382$ mm and $f_2 = 153$ mm.

4. Following the sign conventions from Lecture 13, Slide 11, shown in Figure 1 below, for a Littrow grating transmission grating we have $\alpha = -\beta$, $\theta_{\rm cc} = \beta - \alpha$ and the grating equation becomes $m\lambda = 2\sigma \sin|\alpha|$, where σ is the groove density in l/mm. Hence $\theta_{\rm cc} = 2\beta$ and $\beta = 50$ deg. Make the reasonable assumption that you are working in 1st order (unless told otherwise). For for $\lambda = 653$ nm, we find $\sigma = 426.2$ nm is the groove separation, so the groove density is 2346 l/mm.

5. The angular dispersion is given by $\gamma = m/\sigma \cos\beta$. In first order for Littrow configuration this yields $\gamma = 3.65 \times 10^{-3}$ radians per nm.

6. The linear dispersion is just $f_2\gamma = 558$ microns per nm, or 37.2 pixels per nm. The inverse of this is 0.269 Å pix^{-1}.

7. None; the grating is used in Littrow ($\alpha = \beta$).

8. The spectral resolution $R = \lambda/\Delta\lambda$, where $\lambda = 653$ nm, and $\Delta\lambda$ is given by the equivalent wavelength range covered by the number of pixels that are in a monochromatic image. The monochromatic image size is given by roughly the reimaged fiber diameter of $D'_f = 120$ microns. (A better estimate for the monochromatic image size is $\cos(30) \times D'_f$, where the trigonometric factor accounts for the effective half-width of a circular beam.) This gives us $0.87 \times 120/558 = 0.186$ nm, or $R = 653/0.186 \sim 3505$. You can also derive this from $R = 2(f_1/w)\tan\alpha$, with $w = \cos(30) \times D_f$.

9. (a) There are about 7 pixels per resolution element. (b) Since this number is substantially larger than 2 it does not have a significant impact on the spectral resolution. (c) You probably should bin the CCD 3x3 and stay nearly critically sampled (>2 pixels each sampling the FWHM in the spatial and spectral axes).

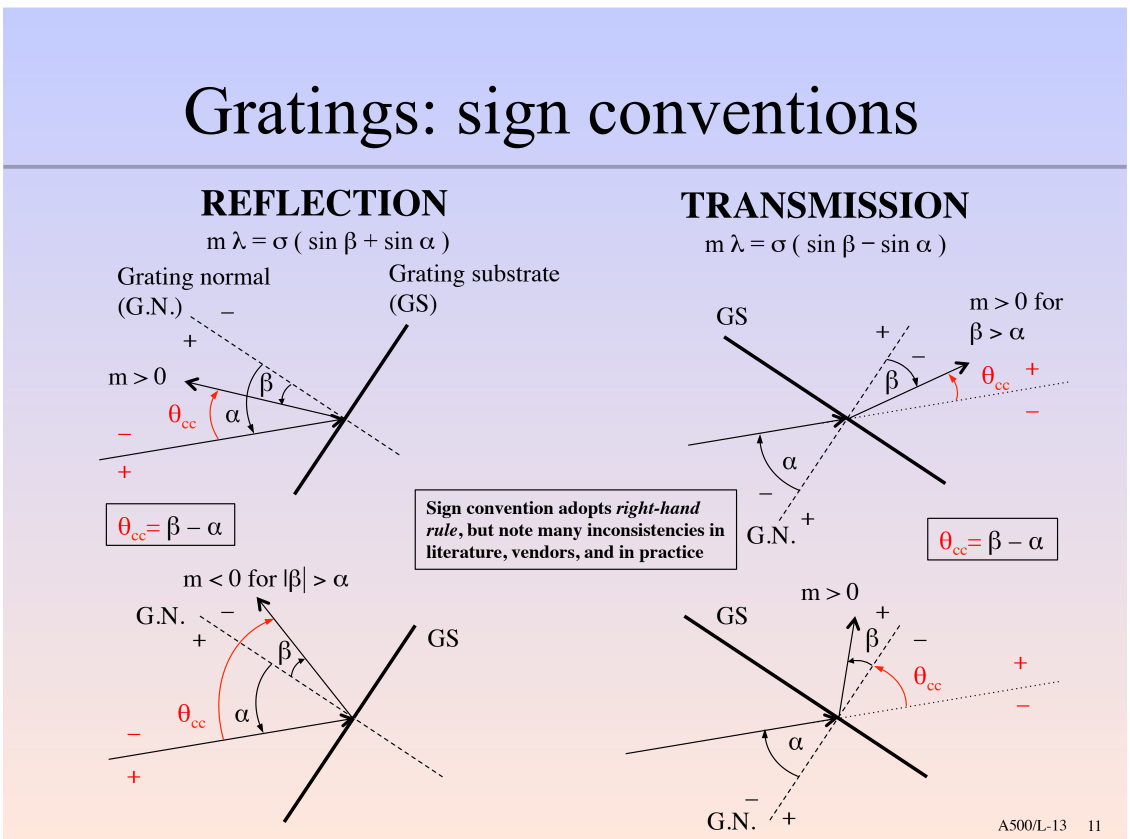

Fig. 1.— Lecture 13, Slide 11.