A500 / Problem Set #3 / Solutions

1.a. Equating $\beta_{\rm spherical}$ with $\beta_{\rm coma}$ or $\beta_{\rm astigmatism}$, one finds $\theta_{\rm coma} = \frac{1}{24(f/D)}$ or $\theta_{\rm astimagtism} = \frac{1}{8(f/D)}$, where θ is the limiting off-axis angle where the sub-scripted aberration equals spherical aberration. For a given focal-ratio, $\theta_{\rm coma} < \theta_{\rm astimagtism}$, i.e., coma is the limiting aberration. Even though $\beta_{\rm astigmatism}$ grows faster than $\beta_{\rm coma}$ with θ in a relative sense, θ (in radians) is a small number so that for reasonable θ and f/D, $(\beta_{\rm coma}/\beta_{\rm astigmatism}) > 1$ (this conditions holds for $\theta < 21.5 \times (f/D)^\circ$). What this means is that if the desired FoV is $> \theta_{\rm coma} = 2.39 \times (f/D)^\circ$ for the desired f-ratio as given by the above formula, then you are better off starting with a spherical rather than a parabolic mirror.

1.b. For $\theta_{\rm coma} = 2^\circ = 0.0349$ radians, the fastest telescope you would want to build with a parabolic mirror is $\sim$ f/1.2; anthing faster might as well use a spherical mirror.

2. SALT

(a) SALT has a 26.165 m radius of curvature and an 11 m entrance aperture, yielding an f-ratio of 1.189. The circle of least confusion is given by the formula for spherical aberration: $\beta_{\rm spherical} = [128(f/D)^3]^{-1}$ where f/D is the f-number. From this it follows that SALT has spherical aberration of 0.266 deg, about half the size of the full-moon, equivalent to 60.5 mm at the SALT prime focus.

(b) Coma and astigmatism for a parabolic mirror are $\beta_{\rm coma} = \theta/[16(f/D)^2]$ and $\beta_{\rm astig} = \theta^2/[2(f/D)]$, where β and θ are in radians. At $\theta = 8$ arcmin $= 2.33 \times 10^{-3}$ radians, we have $\beta_{\rm coma}/\beta_{\rm spherical} = 0.022$ and $\beta_{\rm astig}/\beta_{\rm spherical} = 4.90 \times 10^{-4}$. ($\beta_{\rm coma} \sim 21.3$ arcsec, $\beta_{\rm astig} \sim 0.47$ arcsec.)

(c) The diffraction spot, in radians, is given by $\beta_{\rm diffraction} = 1.22\lambda/D$. Equating $\beta_{\rm diffraction} = \beta_{\rm spherical}$ where $D = 1$m and $f/D = 1.189$ for the entire primary yields $\lambda = 3.8$ mm.

3. Both conic constants are < -1 which makes this a two-hyperbaloid reflector, or an RC telescope.

4. Referring to figures in Slide 36 of Lecture 7, two variables x and y are defined in red to indicate the distance of the secondary away from the prime focus, and the diameter of the beam at the secondary, respectively. These can be used to solve for the back focal-distance first by noting from inspection that $d + e$ are related to y via the f-ratio: $(d + e)/y = f/D$, where d is the primary-secondary separation, e is the back-focal distance we are looking for, f, is the final telescope focal length, and D is the primary diameter. So f/D is the final telescope f-ratio. This yields $e = f(y/D) - d$. We can also see by inspection that $x/y = f_1/D$, and that $x + d = f_1$. Note that x will be positive or negative depending on whether the secondary is concave (Gregorian) or convex (RC). From these identifies we find $y = (1 - d/f_1)D$, from which it follows that $e = f\ (1 - d/f_1) - d = f - d\ (f/f_1 + 1)$.

5. In case you didn't guess, this is the Sloan 2.5m telecsope.

(a) Inverting the formula for back focal-distance $f_1 = f \times d/(f - e - d)$. From direct inspection of the given table, suming spacings between surfaces 1 and 7 gives $e = 756.7$, while the spacing between surfaces 1 and 2 are by definition $d = 3644.46$. Since the $f/D = 5$ and $D = 2500$mm are given, it follows that $f_1 = 5625$ mm, i.e., the primary mirror is $f/2.25$.

(b) The prime-focus focal-scale $s = 206265/(D(\text{mm}) \times f-\text{ratio})$ arcsec mm^{-1} = 36.7 arcsec mm^{-1}.

(c) Solving for f_2 using the formula for the power of a two-surface optic we have $f_2 = (f_1 - d)/(f_1/f - 1) = -3600.9$ mm. Note the sign, consistent with a hyperbolic focus.

(d) The exit pupil with respect to the secondary is given as $l = d/(1 - d/f_2) = 1811.3$ mm (again, see Lecture 7), paying attention to the sign of d and f_2. As stated in the notes, this is a distance *behind* the secondary. Therefore the total distance from the Cassegrain focus is $l+d+e = 6212.4$mm.

6. SALT and Dragonfly

(a) From problem 1 of HW-2, we have the area of a single, 6 inch Dragonfly telescope is 0.0182 m^2, and the total efficiency is 48% in the V band (ignoring atmosphere). Hence $(A\Omega\epsilon)_{\text{Dfly}} = 8 \times 0.0182 \times (2.6 \times 1.9) \times 0.48) = 0.345$ m^2 deg^2.

From problem 3 of HW-2 for SALT, we have the 69.19% filled 11m aperture diameter is equivalent to $A = 65.9$ m^2, and the telescope efficieny is $\epsilon_T = 0.31$. Here, $\epsilon_{\text{SCAM}} = 0.99^8 \times 0.9 \times 0.85 = 0.71$, which is the product of the losses from the optics, the filter, and the CCD respectively. So $\epsilon_{\text{SALT}} = \epsilon_T \times \epsilon_{\text{SCAM}} = 0.22$. SALTICAM has an 8 arcmin diameter field of view. Hence $(A\Omega\epsilon)_{\text{SALT}} = 65.9 \times \pi\ (8/(2 \times 60))^2 \times 0.22 = 0.20$ m^2 deg^2.

Conclude: Dragonfly 70% faster: 0.34/0.020 = 1.70.

(b) The prime-focus focal-scale $s = 206265/(D(\text{mm}) \times f-\text{ratio})$ arcsec mm^{-1} = 4.46 arcsec mm^{-1}. The effect of SALTICAM's reimaging optics is to reduce (demagnify) the focal scale by the ratio of focal-lengths, which is the same as the ratio of input to output f-ratios, or $s = 4.46 \times (4.2/1.9) = 9.87$ arcsec mm^{-1}. Dragonfly pixels are 2.8 arcsec across, corresponding to 0.28 mm at SALTICAM's detector focus. This corresponds to $19^2 = 361$ pixels.

(c) From the problem 1b from Problem Set-2 we have 600 sec for Dragonfly to become sky-limited in the V band.

Ignoring differences in read-noise, the exposure time will scale inversely with $A\Omega\epsilon$ where Ω here is the solid angle of unbinned pixels, and A is the suitable aperture that feeds each pixel.

Break out $A\epsilon$ terms first: $(A\epsilon)_{\text{SALT}}/(A\epsilon)_{\text{Dfly}} = (65.9 \times 0.31 \times 0.71)/(0.0182 \times 0.48) = 1660$ where you will note that the factor of 8 in the Dragonfly area must be dropped since we are considering the area feeding each pixel.

Next, the ratio of pixel solid angles is: $\Omega_{\text{pix,SALT}}/\Omega_{\text{pix,Dfly}} = (9.87 \times 0.015\ /\ 2.8)^2 = 2.80 \times 10^{-3}$. Consequently the exposure time for SALTICAM should be $1/(1660 \times\ 2.80)^{-3}) = 4.65^{-1} = 0.22$ that of Dragonfly, or 129 sec.

Now consider the fact that the read-noise (RN) for SALTICAM is 2.5 e- (rms) while for Dragonfly it is 10 e- rms. Since the criteria for being photon-limited scales as RN^2, the exposure times should scale as $A\Omega\epsilon/\mathrm{RN}^2$. This brings the ratio from 0.22 to $0.22 \times (2.5/10)^2 = 0.013$, equivalent to roughly an 8 sec exposure with SALTICAM.

(d) Here $\Omega_{\mathrm{superpix,SALT}}/\Omega_{\mathrm{pix,Dfly}} = 1$ by definition. Including read-noise, SALTICAM will become sky-limited $1660 \times (10/2.5)^2 = 2.66 \times 10^4$ times more quickly than Dragonfly. In an hour we estimated that Dragonfly would be sky-limited for a band-pass with 0.033 nepers. Since the number of photons will scale with the band-width, for SALTICAM this would be 1.24×10^{-6}, or about 0.16 km s^{-1} in equivalent velocity resolution (σ)*!*

Take-away: If you want to survey large areas of sky, Dragonfly is the way to go, particularly for broad-band work. Narrow-band work is much more easily done on SALT.