A500 / Problem Set #2 / Solutions

1. Dragonfly

(a) $\epsilon = T_{\rm atmos} \times \epsilon_{\rm telescope} \times \epsilon_{\rm filter} \times \epsilon_{\rm CCD} = {\rm dexp}(-0.4\ A_{V,{\rm atmos}}) \times 0.995^8 \times 0.9 \times 0.55 = 0.48\ T_{\rm atmos}$, where $T_{\rm atmos} = {\rm dexp}(-0.4\ A_{V,{\rm atmos}})$ is the transparency of the atmosphere in the V band. You may have found that $A_{V,{\rm atmos}} \sim 0.2X$ mag where X is the number of airmasses of observation, or about 17% loss per airmass.

(b) We will use the criteria from Lecture 4 (slide 33), namely that shot-noise from sky-photons should be 3 times larger than detector noise. This means that $\sigma_{\rm photons} \propto \sqrt{N_{\rm photons}} \geq 3\ \sigma_{\rm RN}$. In this case the photon shot-noise all comes from the sky. (If you chose some other definition of "sky-limited," that's fine within reason, as long as you defined it. The definition here leads to a $<$5% increase in noise over the pure sky shot-noise case.) Assuming we do not bin the CCD pixels, with $\sigma_{\rm RN} = 10e^-$ the sky must contribute 900 e^- pix^{-1} per exposure, t.

Dark sky conditions yield $\mu(V) = 21.5$ mag arcsec^{-2}, which corresponds to

$N_\gamma\ =\ \epsilon\ \times$ (15.1 photons/μJy) $\times$ 0.16 $\times$ 3640 Jy $\times$ dexp(-0.4×21.5) photons m^{-2} arcsec^{-2} s^{-1}

$=\ \epsilon\ \times$ 22.1 photons m^{-2} arcsec^{-2} s^{-1}, where $\Delta\lambda/\lambda = 0.16$. Taking $\epsilon = 0.48$ from (a), a pixel size of 7.84 arcsec^2, and a telescope area (per detector) of 0.0182 m^2 yields $N_\gamma = 1.5$ pix^{-1}s^{-1}. This requires an exposure time of roughly 600 sec, or 10 min. Refering to, e.g., arXiv:1401.5473v1, this is just the exposure time Abraham & can Dokkum use in g and r bands, which both have significantly larger band-widths ($\Delta\lambda/\lambda$).

(c) An hour yields 6 times more counts than the above calculation, so the band width can be reduced by the same factor: $\Delta\lambda/\lambda = 0.027$.

2. SALT focal-plane scales

(a) An 11m effective entrance pupil and corrected f-ratio of 4.3 yields a plate scale of $47.3 \times 10^3/206265 = 0.2293$ mm arcsec^{-1} (4.36 arcsec mm^{-1}). So 1 arcsec corresponds to 0.0526 mm^2.

(b) For a given beam size (telescope or instrument optical diameter) a shorter f-ratio yields a smaller magnification. The reasoning, which you can demonstrate with a simple sketch, is that a change in angle of a collimated beam will result in a larger displacement of the focused image for a longer focal length. In the case of RSS the collimator takes the $f/4.3$ beam from the SALT aberration-corrected focus, collimates it, and then refocuses it at a "faster" speed, i.e., a smaller f-ratio. This means the camera focal length is *shorter* than the collimator focal length, so the magnification is less. In other words, the camera de-magnifies the SALT corrected focal-plane scale by the ratio of the f-ratios, or $1.9/4.3 = 0.442$. Now 1 arcsec corresponds to 0.101 mm, which is 6.75 pixel lengths at 15 microns per pixel. There are 45.6 pixels per arcsec^{-2}.

3. SALT effective area

The equivalent circular aperture diameter of a 69.19% filled 11m aperture diameter is just $\sqrt{0.6919} \times 11 = 9.15$ m. The collecting area is 65.8 m^2 (compared to 95.0 m^2 of an completely filled system).

Taking into account the telescope efficiency $\epsilon_T = 0.75 \times (0.80)^4 = 0.31$, the total effective efficiency of the telescope (surface-losses and obstructions) $0.6919 \times 0.31 = 0.21$. The equivalent circular aperture diameter of a perfect telescope ($\epsilon_T = 1$, 100% filling factor) is 5.1 m.

Next taking into account the spectrograph efficiency, from inspection we have $\epsilon_i = (0.96)^{23} \times 0.90 \times 0.97^4 \times 0.85 = 0.23$ for the total RSS efficiency assuming a single etalon pair and order blocking filter. Often two pairs of etalons are used, plus an order blocking filter. If you assumed this your answer here would be 0.21 rather than 0.23.

Combining the factors from the telescope (pupil fill factor and reflectivity losses) and the spectrograph losses the total effective system efficiency is $0.21 \times 0.23 = 0.044$. This corresponds to an equivalent circular aperture diameter of a perfect telescope and instrument of 2.4m (2.3m with two etalon pairs).

Factors we have not considered include losses from the atmosphere.

4. SALT RSS Observations in FP mode

4a. Since R = $\lambda/\Delta\lambda$, R has units of neper^{-1}. The resolution and the band-pass are inversely related.

4b. The number of detected photo-electrons from the sky (N_e) will depend on the product of the sky brightness (expressed in units of photons sec^{-1} m^{-2}), the total collecting area of the telescope (A_T, in m^2), and the product of the efficiencies of the telescope (ϵ_T), the instrument (ϵ_i), and the detector (ϵ_d). The latter is the quantum efficiency. Since the sky brightness is a surface brightness, N_e will have units of electrons sec^{-1} arcsec^{-2}. The sky brightness, N_{sky} can be expressed as a function of the m_{sky} (the surface brightness in mag), the magnitude zeropoint of the system, m_0, the flux zeropoint of the system, f_0, in μJy, and the band-width $\Delta\lambda/\lambda = 1/R$:

$$N_{sky} \;=\; 15.1\; R^{-1}\; f_0 \;\mathrm{dexp}[-0.4(m_{sky} - m_0)] \quad [\mathrm{photons\ sec^{-1} m^{-2} arcsec^{-2}}]$$

From this it follows:

$$N_e \;=\; N_{sky}\; A_T\; \epsilon_T\; \epsilon_i\; \epsilon_d \quad [\mathrm{electrons\ sec^{-1}\ arcsec^{-2}}]$$

The total number of photo-electrons collected in some time, t over some solid-angle Ω is $N_e\ t\ \Omega$.

4c. The question was intended to ask how many detected photo-electrons are there per arcsec per second in the RSS detector from the sky. If you interpreted these as simply the number of photons per arcsec per second onto the detector, that's fine; your answer will differ by the factor of the detector QE.

In the V band (centered at 550 nm), $\mu_{\rm sky} = 21.5$ mag arcsec^{-2}, with a band-pass zeropoint of 3640 Jy in the Vega system and a band-width of 0.16 nepers (Lecture 2). For calculating photo-electrons from the sky, all factors in Problem 2 are relevant. Therefore we can equate the product of $A_T\ \epsilon_T\ \epsilon_i\ \epsilon_d$ with the product of the telescope obscuration, telescope efficiency, and instrument efficiency. The effective aperture diameter is 2.3m in this case, so $A_T\ \epsilon_T\ \epsilon_i\ \epsilon_d\ =\ \pi(2.3/2) = 4.15{\rm m}^2$. Next, we have $N_{sky} = 15.1\ R^{-1}\ 3640{\times}10^6\ {\rm dexp}(-0.4{\times}21.5) = 138.06\ R^{-1}$ [photons sec^{-1}m^{-2}arcsec^{-2}]. Hence $N_e\ =\ 625\ R^{-1}$ [electrons sec^{-1} arcsec^{-2}]. The number decreases to 573 for two etalon pairs.

4d. Define N'_e in units of electrons sec^{-1} pix, where it is understood that the pixel unit corresponds to the binned value in terms of solid angle. We can take the result for N_e from (4c) and factor in the solid-angle per un-binned pixel determined in (2b) to find $N'_e\ =\ 13.7\ n_x n_y\ R^{-1}$ [electrons sec^{-1} pixel] in all generality, where n_x and n_y are the integer-pixel binning factors in x and y dimensions on the detector (e.g., RA and DEC on sky). For two etalon pairs 13.7 goes to 12.6.

If you interpret the 10%-rule as given in the problem, this implies $\sqrt{N'_e\ t} = 10\ RN$, where RN is the rms read-noise in electrons. If you used the class notes, then $\sqrt{N'_e\ t} = 3\ RN$ – we'll call this 3:1 rule. Either is fine (but quite different); in practice, the latter is sufficient. For $RN = 2.5e^-$ solving for t we tabulate the results below, using 2×2 binning.

Background-limited Exposure Times for SALT FP Imaging

binning ($n_x n_y$)	R	t (sec) $9e^-/N_e$	$100e^-/N_e$
4	500	1314	14599
4	2500	6569	72992
4	12500	35714	396825

What you see is that in a single track of 45 min (2700 sec) it is impossible to become background limited other than for the lowest resolution FP mode with the more conservative (3:1) estimate. With higher efficiencies this would change.

4e. For two resolution elements sampling a FWHM of 2.5 arcsec, we want the binned pixels to be roughly 1.25 arcsec. Since each unbinned pixel is rouhgly 0.15 arcsec, we want close to 8×8 binning (if you chose 9×9 that's fine). Exposure times for these binnings simply scale by factors of 4/64 or 4/81 for the 8×8 or 9×9 binning, respectively. This brings the 3:1 estimates within reach for a single track for all resolutions.