A500 / Problem Set #1 / Solutions

1. From Table 2.1 (Lecture 2) we have the flux zeropoint in the J band is 1600 Jy in the Johnson system (J), and by definition the zeropoint is 3631 Jy in the AB system ($J_{\rm AB}$). So

$$J - J_{\rm AB} = -2.5 \log(3631/1600) = -0.89 \text{ mag.}$$

Generically,

$$m_{\rm AB} = m_{\rm Johnson} + m_{AB} \text{ [Vega].}$$

2. From Lectures 2 we have

$$m_{AB} = -2.5 \log f_\nu - 48.60,$$

so

$$(g' - J)_{\rm AB} = -2.5 \log(f_\nu(g')/f_\nu(J)).$$

We have f_λ is constant, but we need to know the conversion from f_ν to f_λ. Since the integral of the flux over a given band-pass is the same for frequency and wavelength, it follows: $f_\nu \; d\nu = f_\lambda \; d\lambda$. This yields $\nu f_\nu = \lambda f_\lambda$, or $f_\nu \propto \nu^{-2} \propto \lambda^2$ for constant f_λ, and hence

$$(g' - J)_{\rm AB} = -5 \;\; \log(0.48/1.26) = 2.096 \text{ mag,}$$

i.e., the source is red.

3. (Lecture 2) Since $(B - V)_0 = 0$ in the Vega system by definition we have

$$(B - V) = -2.5 \log\{ \; [f_\nu(B)/f_{\nu,0}(B)] \; / \; [f_\nu(V)/f_{\nu,0}(V)] \; \},$$

where the "0" subscript refers to the magnitude system reference standard. With $f_\nu \propto \nu^\alpha$ and a little manipulation,

$$\alpha = \{ \; \log \; [f_{\nu,0}(B)/f_{\nu,0}(V)] - 0.4(B - V) \; \} \; / \; \log(\lambda_B/\lambda_V).$$

Plugging in B and V flux zeropoints and $B - V = 0.43$ in Vega system for an F6 dwarf we have $\alpha \sim -1$. This means $\nu f_\nu = \lambda f_\lambda$ is roughly constant, which is precisely the motivation for using F6 stars as flux standards. From the generic result in problem (1) we have

$$(B - V)_{\rm AB} = B - V + (B - V)_{\rm AB}\text{[Vega],}$$

where the AB color of Vega comes from the flux zeropoint definitions,

$$(B - V)_{\rm AB}\text{[Vega]} = -2.5 \log(4260/3640).$$

Alternatively, we can work from the definition of AB magnitude to find

$$(B - V)_{\rm AB} = -2.5 \log[f_\nu(B)/f_\nu(V)].$$

Either yields $(B - V)_{\rm AB} = 0.26$ for BD+17°4708.

4. Since we are calculating photons per arcsec^2 and we are given a surface-brightness in mag arcsec^{-2}, we can ignore solid angle in the calculation. $V = 29$ mag corresponds to

$$f_\nu = 3640 \times {\rm dexp}(-0.4 \times 29) \ {\rm Jy} \ = \ 9.14 \ {\rm nanoJy},$$

while 1 μJy = 15.1 photons sec^{-1}m^{-2} neper^{-1}. The problem didn't state the magnitude system; here we adopt Johnson, but note that Johnson and AB area nearly identical in V band by construction. Multiplying by the band-width in nepers of 0.16 and the collecting area of 0.0182 m^2 yields $f_\gamma = 4 \times 10^{-4}$ photons s^{-1} arcsec^{-2} at the top of the atmosphere. This is equivalent to the faintest surface-brightness currently detectable. (Note corrected statement in problem.)

5. (Lecture 2) At 550 nm (V band), $L_{\nu,\odot} = 5.30 \times 10^{18}$ erg sec^{-1} Hz^{-1}. Since 1 pc = 3.086×10^{18} cm, then 1 Jy = 9.52×10^{13} erg sec^{-1} pc^{-2} Hz^{-1}, so that $L_{\nu,\odot}$ pc^{-2} = 55672 Jy. Converting this to a surface-brightness over 4π sterad gives $L_{\nu,\odot}$ pc^{-2} sterad^{-1} = 4430 Jy sterad^{-1} = 0.104 μJy arcsec^{-2} (1 arcsec = 206265^{-1} radians). Using the result from problem 4 we have: $\mu_V = 29$ mag arcsec^{-2} = 9.14 nano-Jy arcsec^{-2} = 0.087 $L_\odot$ pc^{-2}. A more intuitive solution is to consider placing the Sun at a distance such that 1pc subtends an arcsec on the sky. This corresponds to d = 206265 arcsec. The same result follows using the distace modulus and the absolute V-band magnitude of the Sun (4.83, Vega mag).