Astronomy 330

Lecture 8

29 Sep 2010

Outline

- Review
  - Milky Way kinematics
    - Rotation and Oort's constants
      - Tangent points
      - $\omega_0 = V_0/R_0 = A-B$
      - $(dV/dR)_{R0} = -(A+B)$
    - Solar motion:
      - LSR
      - u,v,w
- Finish up MW kinematics
  - Disk
  - Halo
  - Measuring Galactic rotation: An example
- Start galactic dynamics

Solar Motion

- LSR $\equiv$ velocity of something moving in a perfectly circular orbit at R_0 and always residing exactly in the mid-plane (z=0).
- Define cylindrical coordinate system:
  - R (radial)
  - z (perpendicular to plane)
  - ϕ (azimuthal)
- *Residual* motion from the LSR:
  - u = radial, v = azimuthal, w = perpendicular
- *Observed* velocity of star *w.r.t. Sun*:
  - $U_* = u_* - u_\odot$, etc. for v, w
- Define means:
  - $\langle u_* \rangle = (1/N) \Sigma u_*$, summing over i=1 to N stars, etc. for v,w
  - $\langle U_* \rangle = (1/N) \Sigma U_*$, etc for V,W

Solar Motion

- Assumptions you can make
  - Overall stellar density isn't changing
    - there is no net flow in either u (radial) or w (perpendicular):
    - $\langle u_* \rangle = \langle w_* \rangle = 0$.
  - If you do detect a non-zero $\langle U_* \rangle$ or $\langle W_* \rangle$, this is the reflection of the Sun's motion:
  - $u_\odot = -\langle U_* \rangle$, $w_\odot = -\langle W_* \rangle$, $v_\odot = -\langle V_* \rangle + \langle v_* \rangle$

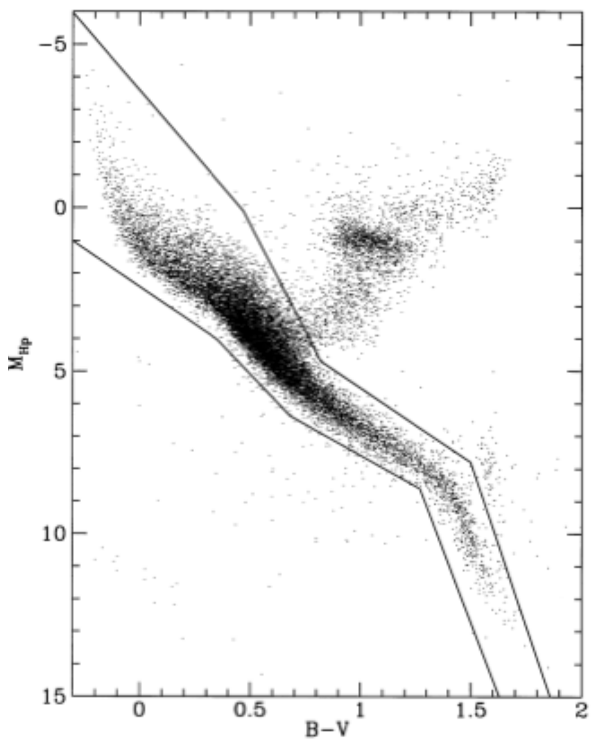
- Dehnen & Binney 1998 MNRAS 298 387 (DB88)
  - Parallaxes, proper motions, etc for solar neighborhood (disk pop only)
  - $u_\odot = -10.00 \pm 0.36$ km s^{-1} (inward; DB88 call this U_0)
  - $v_\odot = 5.25 \pm 0.62$ km s^{-1} (in the direction of rotation; DB88 call V_0)
  - $w_\odot = 7.17 \pm 0.38$ km s^{-1} (upward; DB88 call this W_0)
  - No color dependency for u and w, but for v….

Solar Motion

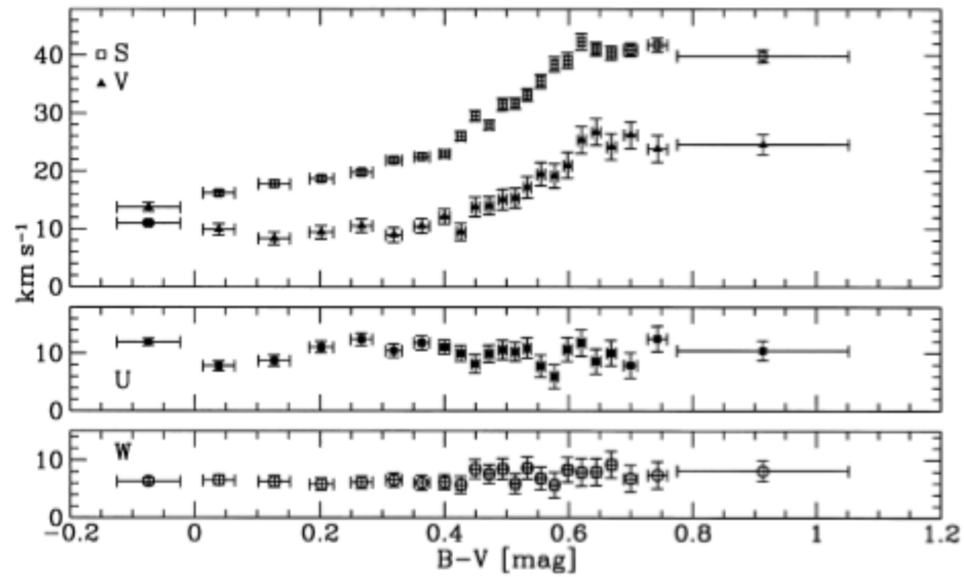
- Leading & Lagging
  - Stars on perfectly circular orbits with $R=R_0$ will have $<V> = 0$.
  - Stars on elliptical orbits with $R>R_0$ will have higher than expected velocities at R_0 and will "lead" the Sun
  - Stars on elliptical orbits with $R<R_0$ will have lower than expected velocities at R_0 and will "lag" the Sun
- Clear variation in $v_\odot$ with (B-V)!
  - Why?
  - Why only v and not u or w?
- We can also measure the random velocity, S^2, and relate this to $v_\odot$. This correlation is actually predicted by theory (as we shall see)!
  - $S = [<u^2> + <v^2> + <w^2>]^{1/2}$

Parenago’s Discontinuity

Clues to disk evolution:



Hipparcos catalogue: geometric parallax and proper motions



τ_{ms} ➔ mean age

Binney et al. (2000, MNRAS, 318, 658)

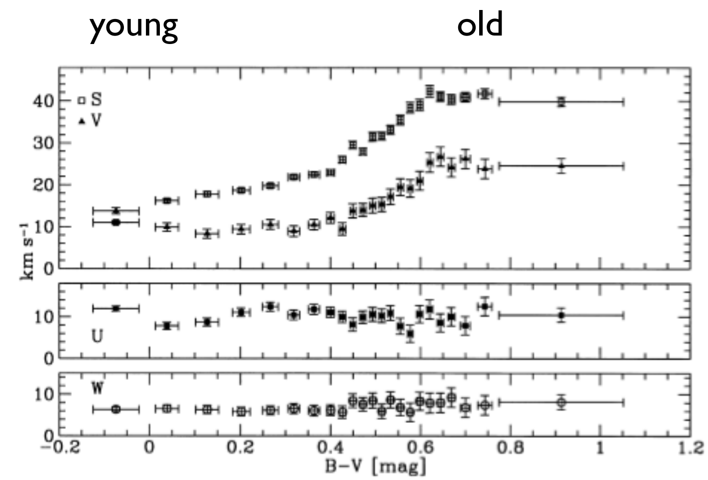
$S = S_0\,[1+(t/\mathrm{Gyr})^{0.33}]$ ← random grav. encounters

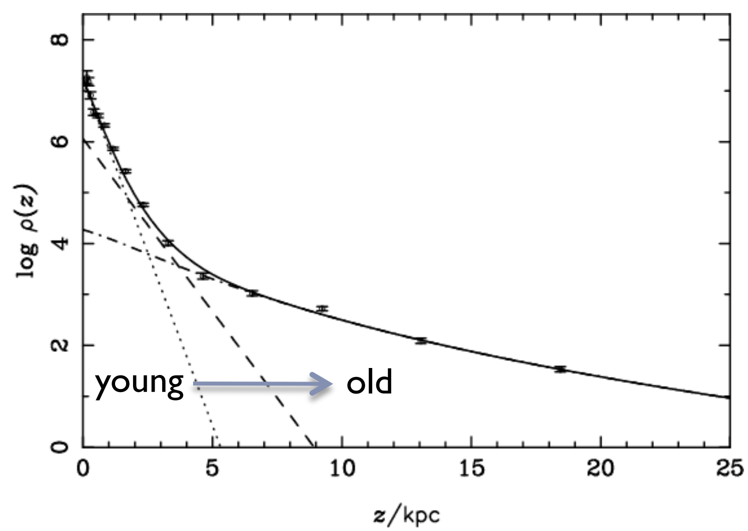
$S_0 = 8\ \mathrm{km\ s^{-1}}$ ← why might this be?

See also Wielen 1977, A&A, 60, 263

Parenago's Discontinuity: the disk

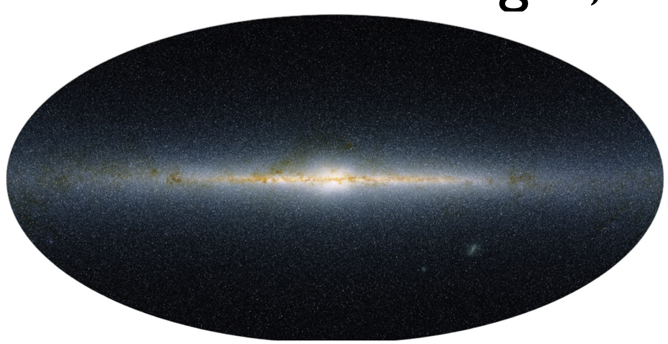
- The disk is observed to be well described by a double exponential in radius (R) and vertical height (z)
- Revisit nomenclature from lecture 6 to be consistent with S&G:
  - $\rho(R,z) = \rho_0 \exp(-z/h_z) \exp(-R/h_R)$
    - ρ is matter density, e.g., in stars $\rho_* = n_* \times m_*$
  - Integrate $\rho(R,z)$ in z to get $\Sigma(R)$, e.g. $M_\odot\ pc^{-2}$
    - $\Sigma(R) = \int \rho(R,z)\, dz$
  - Multiply by the mass-to-light ratio ($M/L = \Upsilon$) to get $I(R)$, the surface-brightness : $I(R) = \Upsilon^{-1} \times \Sigma(R)$
    - $\mu(R)$ often is used to denote surface-brightness in magnitudes arcsec^{-2}.
    - $\mu(R,\theta)$ would be surface-brightness at location R,θ in the disk (cyclindrical coordinates)
  - Integrate $\Sigma(R)$ in R to get total mass within a given radius $M(R)$, … or $I(R)$ to get total light
    - $M(R) = 2\pi \int \Sigma(R)\, r\, dr$
- Why is the distribution exponential in radius?
  - This is hard to answer definitively, but it is an observed fact.
- Why is the distribution exponential in height?
  - Here we will attempt to get a better physical standing in coming lectures.





Solar Motion

- Stellar motion in the disk is basically circular with some modest variations.
- There is an increase in the velocity dispersion of disk stars with color → age
  - Seen in vertical, radial, and azimuthal dimensions
  - Results in $v_{\odot}$ correlation with (B-V)
  - What about the thickness of the disk?
- Disk stars come in all different ages, but tend to be metal rich…



The Halo: Clues to formation scenario?

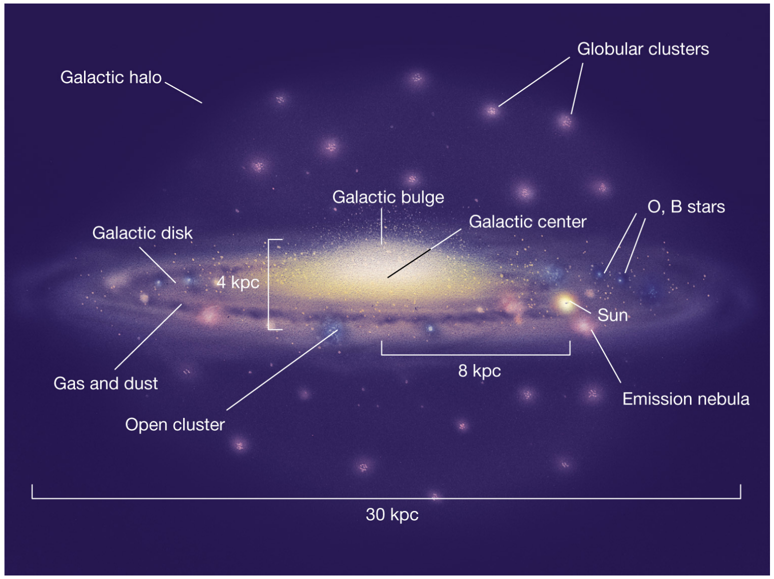
- First, a word from our dynamical sponsors, V and σ
- Velocity dispersion defined:
  - $\sigma^2_{los} = \int (v_{los} - \underline{v})^2 F(v_{los})\, dv_{los}$
  - or, $\sigma_{los} = ((v - \underline{v})2)^{1/2}$
  - where $F(v_{los})$ = velocity distribution function
- "Dynamical temperature:" a concept
  - V/σ
    - Measures the degree of coherent rotation to random motions
    - All stars move on nearly elliptical orbits in a gravitational potential.
    - If they are coherent in orientation and direction (i.e., planar) then the system is dynamically "cold" (i.e., the disk).
    - Incoherency of orbits is dynamically "hot"

The Halo: Clues to formation scenario?

- Layden 1995 AJ 110 2288
  - Age of halo RR Lyrae stars > 10 Gyr
  - $-2.0 < [Fe/H] < -1.5$; $V_{rot}/\sigma_{los} \sim 0$; $\sigma_{los} \sim 100\text{-}200$ km s^{-1}
  - $-1.0 < [Fe/H] < 0$; $V_{rot}/\sigma_{los} \sim 4$; $\sigma_{los} \sim 50$ km s^{-1}
- Relative to LSR
  - Radial: $\langle U \rangle = -13$ km s^{-1}
  - Vertical: $\langle W \rangle = -5$ km s^{-1}
  - Tangential: $\langle V \rangle_{[Fe/H]>-1.0} = 40$ km s^{-1}
  - $\langle V \rangle_{[Fe/H]<-1.0} = 200$ km s^{-1}
- Conclusion: there is an extended old, metal poor stellar halo dominated by random motions with very little, if any, net rotation ($0 < V < 50$ km/s)

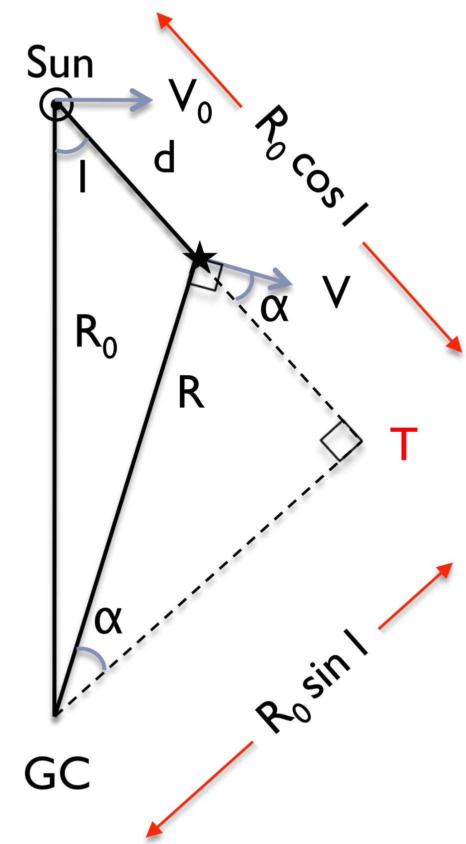
Globular Cluster Population

- Harris, W.E. 2001 "Star Clusters"
  - ~150 globular clusters in MWG
  - Distribution is spherically symmetric, density falls off as $R_{GC}^{-3.5}$
  - Bimodal metallicity distribution
    - [Fe/H] ~ -1.7 (metal-poor) ➔ found in halo
    - [Fe/H] ~ -0.2 (metal rich) ➔ found in bulge



Measuring Galactic Rotation

- Gas:
  - Good because the MW is optically thin at CO (mm) and HI (21cm) wavelengths
  - Bad because you have to use the tangent method –
    - essentially impossible to measure distances
- Stars:
  - Good because you can measure distances directly
  - Bad because it is difficult to measure distances for distant or faint stars
  - Bad because traditional studies are done in optical, which can't penetrate mid-plane dust
  - … enter the Sloan Digital Sky Survey (SDSS):



Measuring Galactic Rotation: Example

- Select stars of a single spectral type….A stars

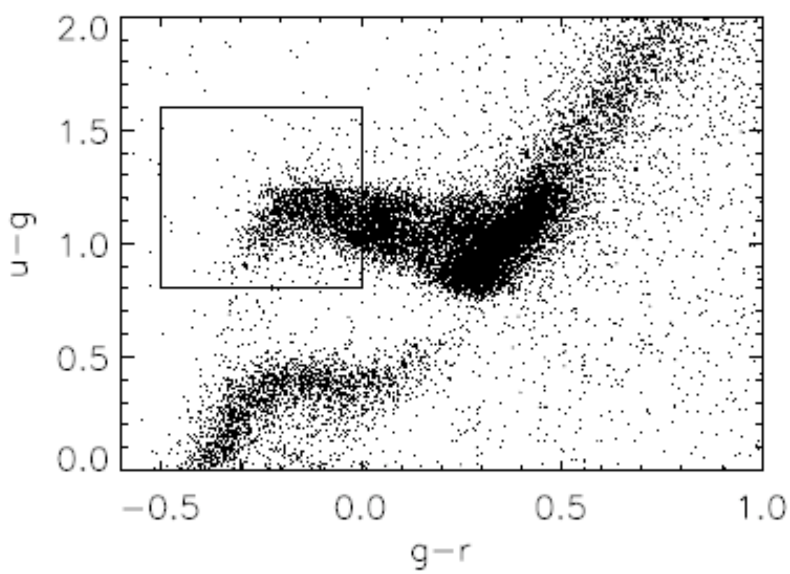
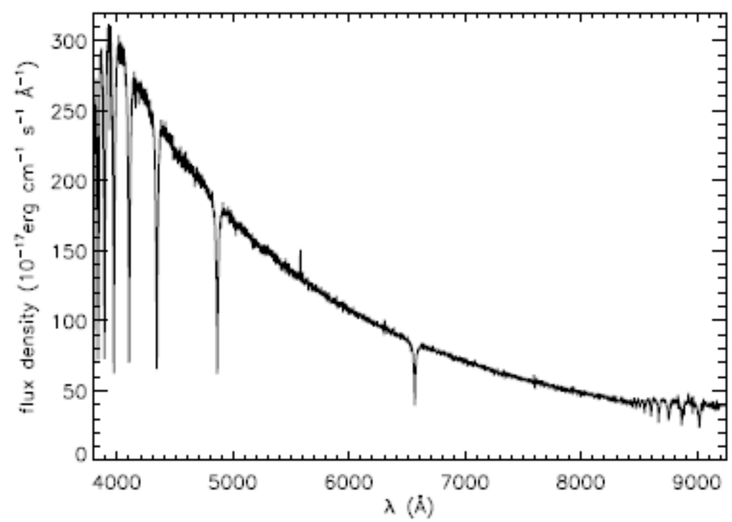


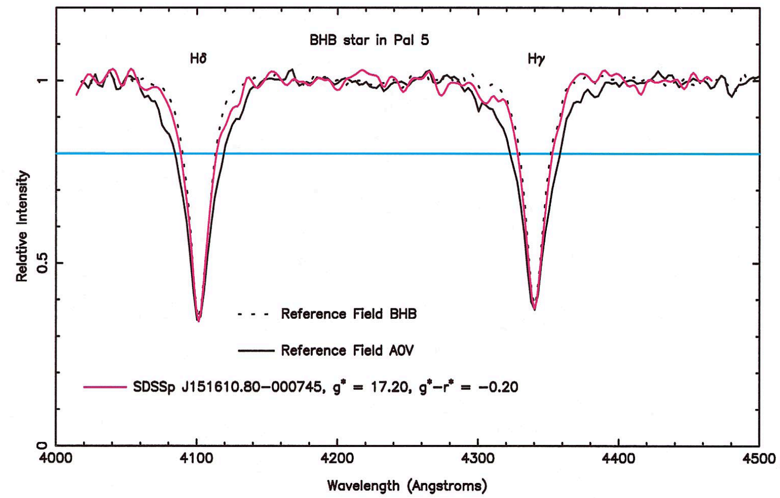
FIG. 1.—SDSS color-color diagram showing all spectroscopically targeted objects that were subsequently confirmed as stars. The large Balmer jump of A-type stars places them in the region where our "color-cut" selection box is drawn. This color selection approach follows Yanny et al. (2000).

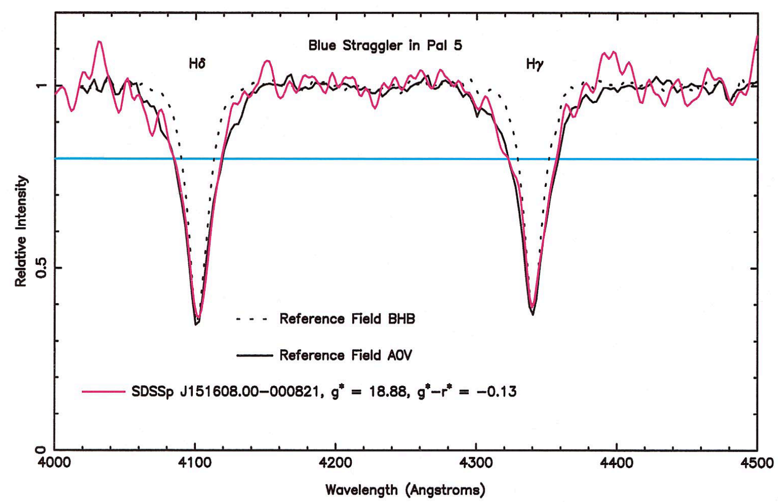


Xue et al. 2008

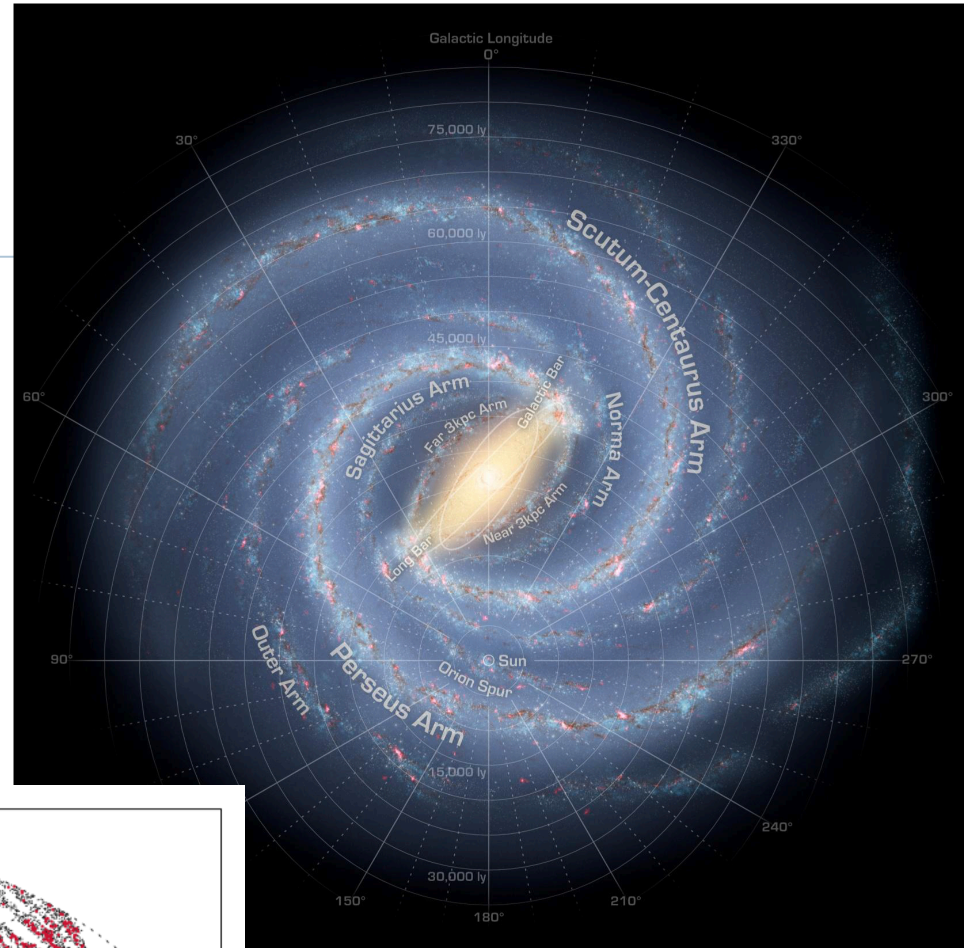
Measuring Galactic Rotation: Example

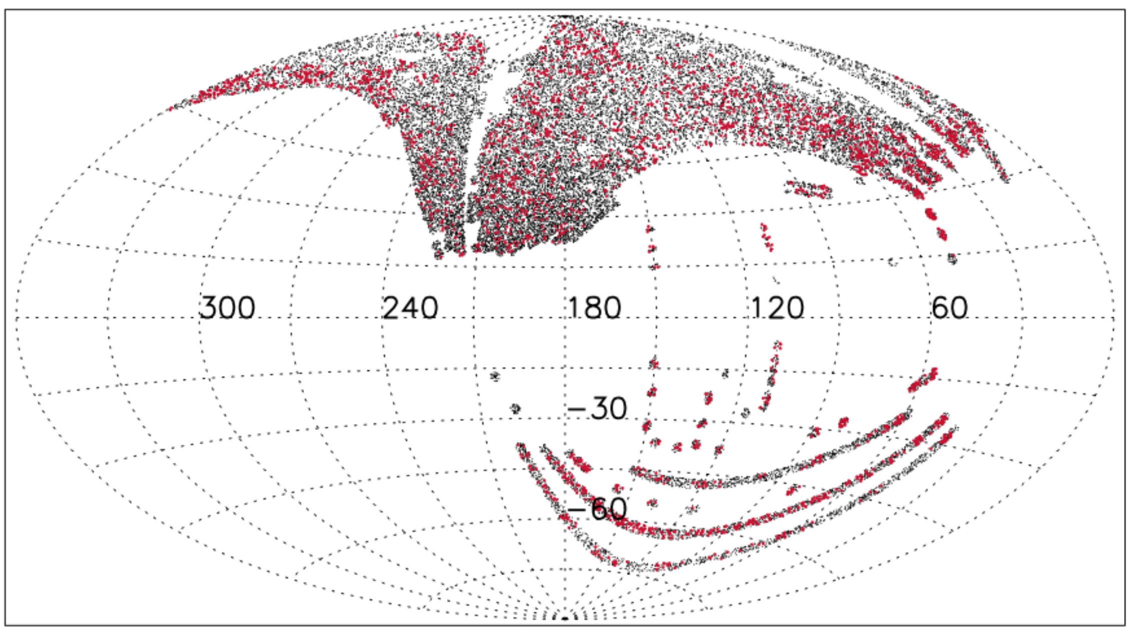
- Distinguish between blue horizontal branch stars and blue stragglers (MS) so the luminosity is known
- Infer distances





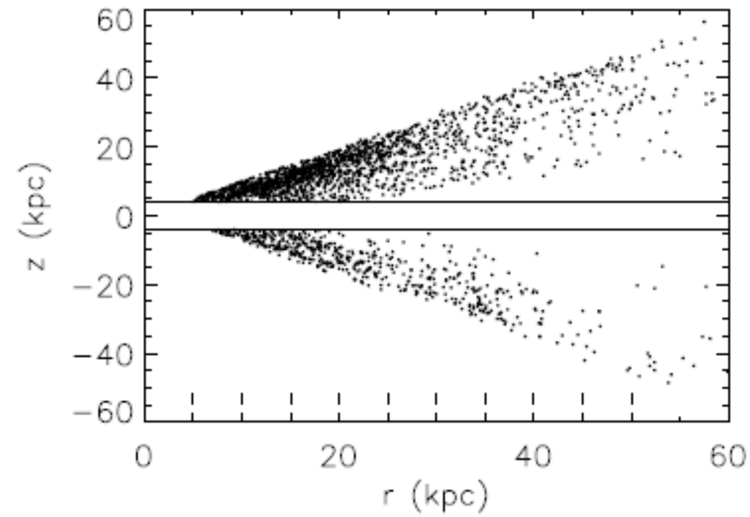
Sight Lines

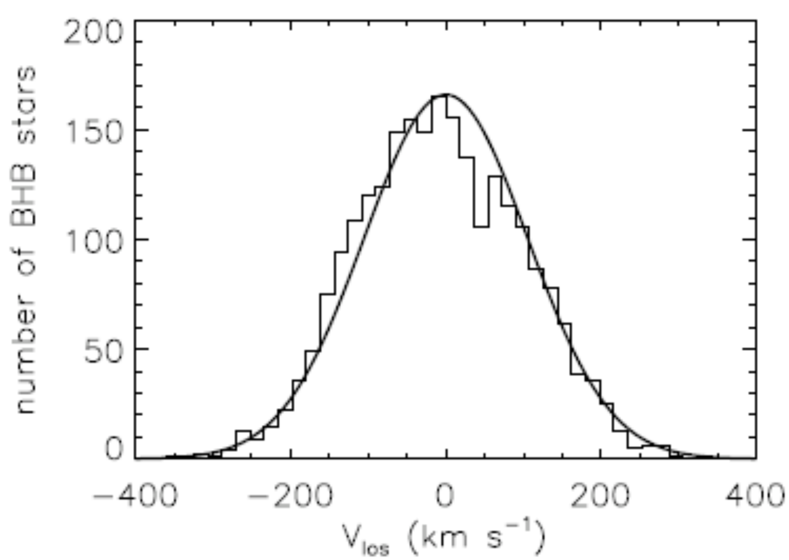

Galactic Longitude
0°
30°
330°
60°
300°
90°
270°
240°
210°
180°
150°
75,000 ly
60,000 ly
45,000 ly
15,000 ly
30,000 ly
Scutum-Centaurus Arm
Sagittarius Arm
Far 3kpc Arm
Galactic Bar
Norma Arm
Near 3kpc Arm
Long Bar
Outer Arm
Perseus Arm
Orion Spur
Sun

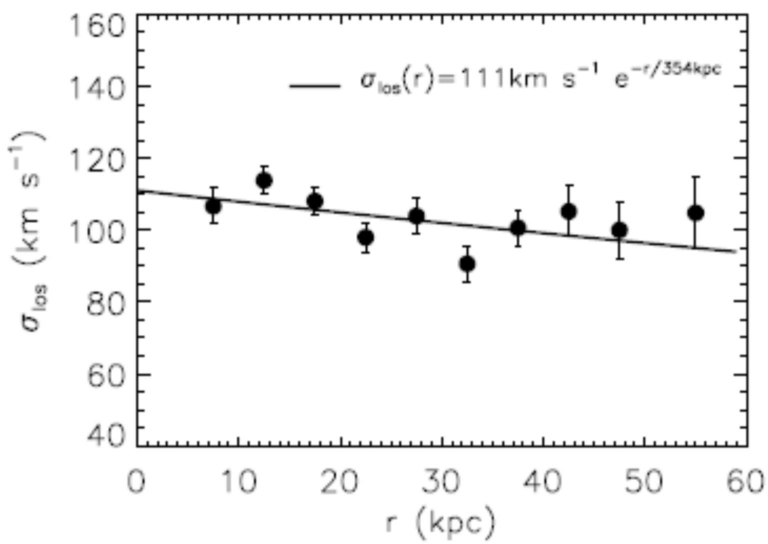

300
240
180
120
60
−30
−60

Measuring Galactic Rotation: Example

- Determine the spatial distribution w.r.t. the GC ➔
- Measure the observed distribution of line-of-sight velocities (🡇 V_{los}), and the dispersion of these velocities, σ_{los}, as a function of Galactic radius ➘







Measuring Galactic Rotation: Example

- And now the trick: Estimate circular velocity (the rotation curve) from the velocity-dispersion data.
- Why not just use the estimated distances to measure V_c directly?

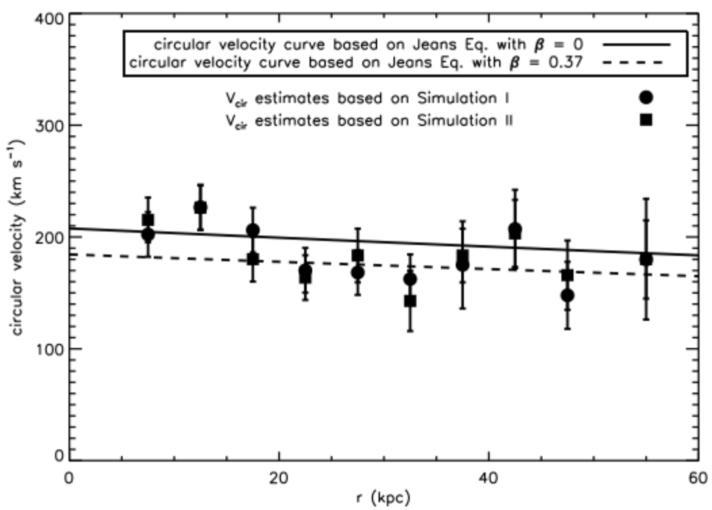


FIG. 15.—Distribution of circular velocity estimates, $V_{\rm cir}$, for two different simulated galaxies. The circles represent the $V_{\rm cir}$ estimates for the observed halo BHB stars based on simulation I, and the squares represent the $V_{\rm cir}$ estimates based on simulation II. The two lines show the circular velocity curve estimates derived from the velocity dispersion profile (Fig. 10) and the Jeans equation with $\beta = 0.37$ and $\beta = 0$.

For reference, we show how these estimates of $V_{\rm cir}(r)$ compare to those derived from the Jeans equation and the fit to $\sigma_{\rm los}(r)$ shown in Figure 10. From the Jeans equation, $V_{\rm cir}(r)$ can be estimated from the velocity dispersion, σ_r (Binney & Tremaine 1987), as follows:

$$-\frac{r}{\rho}\frac{d(\sigma_r^2\rho)}{dr} - 2\beta\sigma_r^2 = V_{\rm cir}^2(r), \qquad (8)$$

with

$$\beta = 1 - \frac{\sigma_t^2}{\sigma_r^2}, \qquad (9)$$

where $\sigma_r(r)$ and $\sigma_t(r)$ are the radial and tangential velocity dispersions, respectively, in spherical coordinates and $\rho(r)$ is the stellar density.

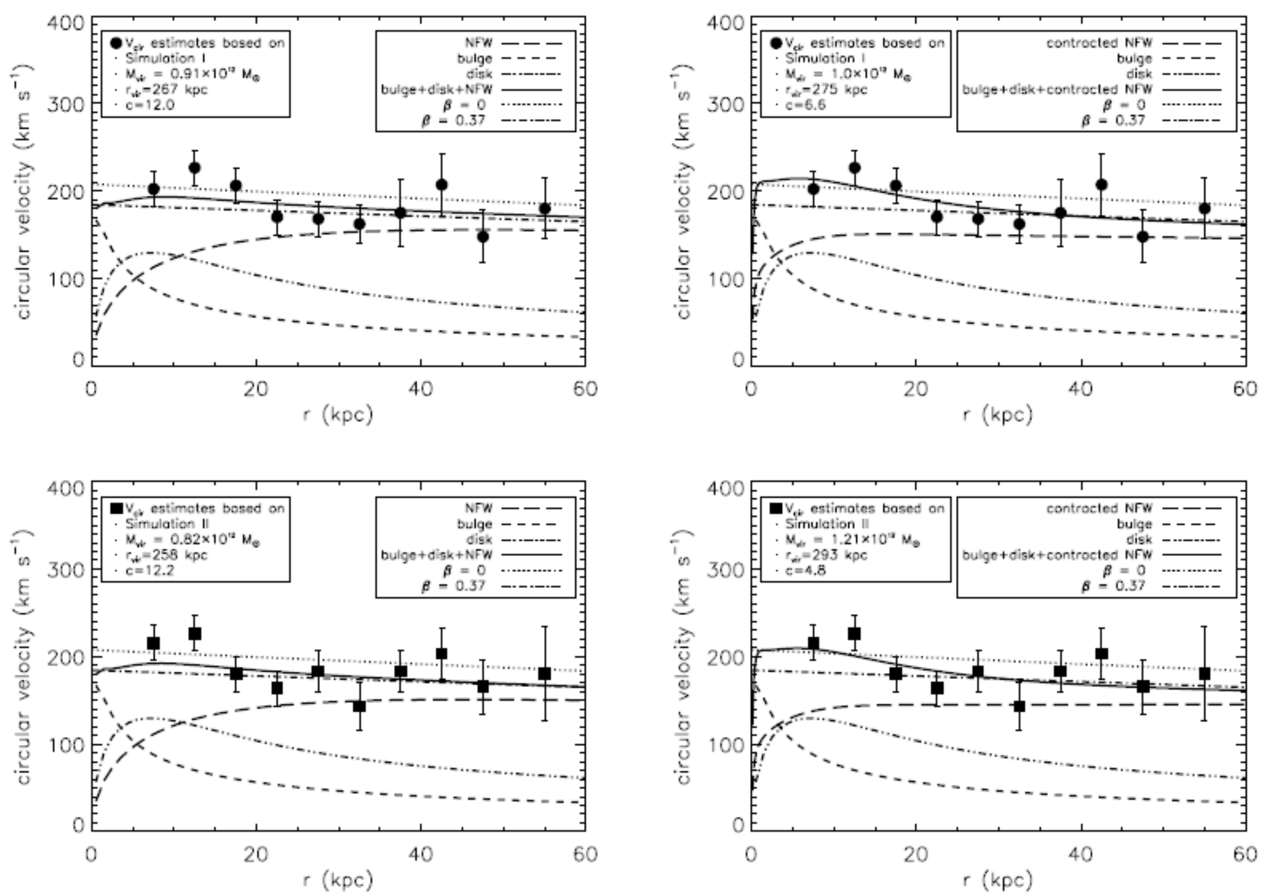
- So we need to learn some dynamics….

Why Dynamics?

- We can then also interpret the data in terms of a physical model:

Mass decomposition of the rotation curve into bulge, disk and halo components :

➔ Dark Matter
➔ Stellar M/L ≡ Υ_*
➔ The IMF
➔ Missing physics



Stopped here

Galactic Dynamics

- Basic morphology of galaxies (and parts of galaxies) is determined by the orbits of stars
  - disk galaxies are disk-like because most of the stars orbit in nearly circular orbits in a flattened plane.
- What determines the stellar orbits? The gravitational potential: $\Phi(r,\theta,z)$.
- What determines the gravitational potential? The distribution of mass, $\rho\,(r,\theta,z)$.

Fundamentals: Gravitational Potentials

- Newton's gravitational force law for a point-mass M
  - $d(mv)/dt = -GmMr/r^3$
    $= -m\nabla\Phi(r)$
    - v,r vectors, r scalar, ∇ is the gradient
    - Φ is the gravitational potential, $\Phi = -GM/r$
  - Thus, $F(x) = -\nabla\Phi$
    - the force is determined by the gradient of the potential.

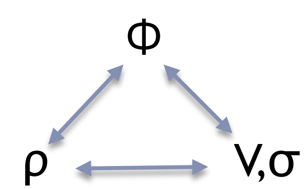
- Gravitational potential generalized:
  - $\Phi(x) \equiv -G \int (\rho(x')/|x'-x|)\, d^3x'$
  - $F(x) = G \int [(x'-x)/|x'-x|^3]\ \rho(x')d^3x'$
    - Force on a unit mass at position, x, from a distribution of mass $\rho(x)$.
- Take the divergence of F(x) [$\nabla\cdot F(x)$] to get Poisson's equation:

$$\nabla^2\Phi(x) = 4\pi G\rho(x)$$

See S&G for derivation, and we will review next time

Application to galaxies

- Here's the process:
  - We start by looking at some very simple geometric cases
  - Define a few terms that help us think about and characterize the potentials
  - Become more sophisticated in the form of the potential to be more realistic in matching galaxies
- Concepts:
  - circular and escape velocities
  - Time scales: dynamical, free-fall
  - Potential (W or PE) and kinematic energy (K or KE)
  - Energy Conservation and Virial Theorem
  - Angular momentum
- Application: rotation curves of galaxies



Spherical mass distributions

- Start simple…. Newton showed:
- A body that is inside a spherical shell of matter experiences no net gravitational force from that shell.
  - Mass contained in solid-angle $\delta\Omega$ of shell as seen by body depends on distance to shell:
    - $\delta m = \Sigma\,\delta\Omega \times r^2$, where Σ is the mass–surface–density of the shell.
    - Hence in any two directions:
      - $\delta m_1 / \delta m_2 = (r_1/r_2)^2 \rightarrow \delta F_1 = -\delta F_2$
      - particle is attracted equally in opposite directions
    - $\nabla\Phi = -F = 0$
- The gravitational force on a body that lies outside a closed spherical shell of matter is the same as it would be if all the shell's matter were concentrated into a point at its center.
  - $\Phi = -GM/R$

Spherical distributions: characteristic v

- The gravitational attraction of a density distribution, ρ(r'), on a particle at distance, r, is:
  - $F(r) = -(d\Phi/dr) = -GM(r)/r^2$
  - $M(r) = 4\pi\int\rho(r')r'^2dr'$
- Circular speed: $v_c^2 = r(d\Phi/dr) = GM(r)/r$
  - In a spherical potential $d\Phi/dr$ is the radial acceleration
  - for a spherical galaxy, v_c goes as $r^{1/2}$
- Escape speed: $v_e(r) = (2|\Phi(r)|)^{1/2}$

  - We'll revisit these when we consider energy

Homogeneous Sphere: v_c and t_{dyn}

- $M(r) = (4/3)\pi r^3 \rho$
  - ρ is constant
- For particle on circular orbit, $v_c = (4\pi G \rho /3)^{1/2} r$
  - rises linearly with r.
  - Check out the Galaxy's inner rotation curve.
  - What does this say about the bulge?
- Orbital period: $T = 2\pi r/v_c = (3\pi/G \rho)^{1/2}$
- Now release a point mass from rest at r:
  - $d^2r/dt^2 = -GM(r)/r^2 = -(4\pi G \rho /3)r$
  - Looks like the eqn of motion of a harmonic oscillator with frequency $= 2\pi/T$
  - Particle will reach r = 0 in 1/4 period (T/4), or
  - $t_{dyn} \equiv (3\pi /16G \rho)^{1/2}$

Isochrone Potential

- Since nothing is really homogeneous…
- $\Phi(r) = -GM/[b+(b^2+ r^2)^{1/2}]$
  - b is some constant to set the scale
  - $V_c^2(r) = GMr^2/[(b+a)^2a]$ ➔ $(GM/r)^{1/2}$ at large r
  - $a \equiv (b^2+r^2)^{1/2}$
- This simple potential has the advantage of having constant density at small r, falling to zero at large r
  - $\rho_0 = 3M / 16\pi Gb^3$
- Similar to the so-called Plummer model used by Plummer (1911) to fit the density distribution of globular clusters:
  - $\Phi(r) = -GM / (b^2+ r^2)^{1/2}$
  - $\rho(r) = (3M / 4\pi Gb^3)\ (1+r^2/b^2)^{-5/2}$

Singular Isothermal Sphere

- Hydrostatic equilibrium: pressure support balances gravitational potential
  - $dp/dr = (k_B\, T/m)\, d\rho/dr = -\rho\, GM(r)/r^2$
    - $\rho(r) = \sigma^2/2\pi G r^2$
      - where $\sigma^2 = k_B\, T/m$
    - $\Phi(r)$ is straight-forward to derive given our definitions
- A special class of power-law potentials for $\alpha=2$
  - $\rho(r) = \rho_0\, (r_0/r)^\alpha$
  - $M(r) = 4\pi\rho_0\, r_0^\alpha\, r^{(3-\alpha)} / (3-\alpha)$
  - $V_c^2(r) = 4\pi\rho_0\, r_0^\alpha\, r^{(2-\alpha)} / (3-\alpha)$

Look what happens to V(r) when $\alpha=2$

- Singular at origin so define characteristic values:
  - $\rho' = \rho/\rho_0$
  - $r' = r/r_0$
  - $r_0 \equiv (9\sigma^2 / 4\pi G\rho_0)^{1/2}$

Flat rotation curves: the disk

- Disk component
- $\Sigma(r) = \Upsilon \times \mu(r)$
  - Σ is the mass surface-density
  - Υ is the mass–to–light ratio (M/L)
  - μ is the surface–brightness
  - Surface mass density ($M_{\odot}\ pc^{-2}$) is just the mass to light ratio times the surface brightness ($L\ pc^{-2}$)
- Mass ➔ potential ➔ circular velocity
  - The trick here is to deal with the non-circular density distibution.

Flat rotation curves: the exponential disk

- $\Sigma(r) = \Sigma_0 \exp(-r/h_R)$
- Mass:
  - $M(r) = 2\pi\int \Sigma(r')r'\,dr' = 2\pi\,\Sigma_0 h_R^2[1-\exp(-r/h_R)(1+r/h_R)]$
- ➔ potential
  - $\Phi(r,z=0) = -\pi G\,\Sigma_0\, r\,[I_0(y)K_0(y) - I_1(y)K_1(y)]$
  - $y = r/2h_R$
  - I, K are modified Bessel functions of the 1st and 2nd kinds.

A bit of work; see Freeman (1970) and Toomre (1963)

- ➔ circular velocity
  - $V_c^2(r) = r\,d\Phi/dr = 4\pi G\,\Sigma_0\, h_R\, y^2\,[I_0(y)K_0(y) - I_1(y)K_1(y)]$

  - Note: This is for an infinitely-thin exponential disk. In reality, disks have a thickness with axis ratios h_R:h_z between 5:1 and 10:1

Rotation from an exponential disk

$q = h_z/h_r$
Disk oblateness

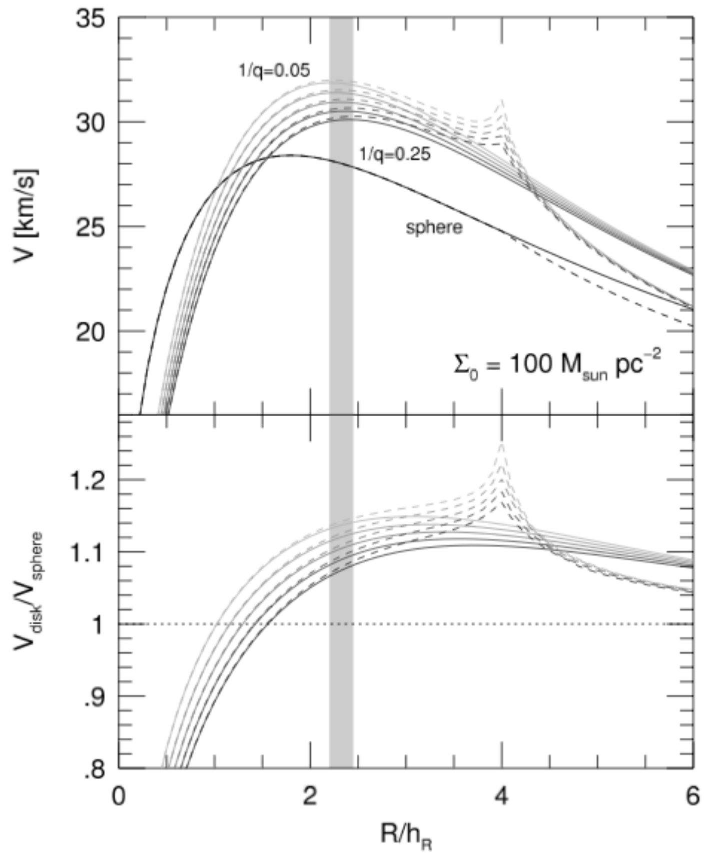


Fig. 17.— Rotation speed of an exponential disk with central mass surface density of 100 $\mathcal{M}_\odot$ pc^{-2} and oblateness $0.05 < q < 0.25$ versus radius normalized by scale-length, compared to a spherical density distribution with the same enclosed mass. Bottom panel shows the ratio of spherical to disk velocities. Dashed and solid lines show disks truncated at R/h_R=4 and 10, respectively. The radial range where these disks have peak velocities is shaded in gray.

It isn't flat