Astronomy 330

Lecture 13

15 Oct 2010

Outline

- Review:
  - Spiral arms
    - Winding problem
    - Density waves,
      - Epicycles
      - Linblad resonances
      - Co-rotation
  - Star-formation
    - Multiple tracers
    - Discontinuities in redshift
    - Insensitivity to low-mass stars
- Scaling relations
  - Tully-Fisher relation: a scaling law
- Dynamics of collisionless systems:
  - Measuring disk mass
  - Collisionless Boltzmann equation

Star formation

- Basic raw materials:
  - Molecular mass at some critical density
    - recall instabilities to gravitational collapse: the Jean's length
  - Spiral arms collect gas into shocks, accelerating collapse
- On a large enough scale, clouds should make stars according to the initial mass function (IMF) and do so largely in clusters
  - there are interesting deviations from this

- Whatever factors give rise to spiral structure, this is where most of the star-formation occurs.
- What we do see: Massive stars and the effect of their radiation
- What we don't see (directly): low-mass stars
  - How can we detect them?

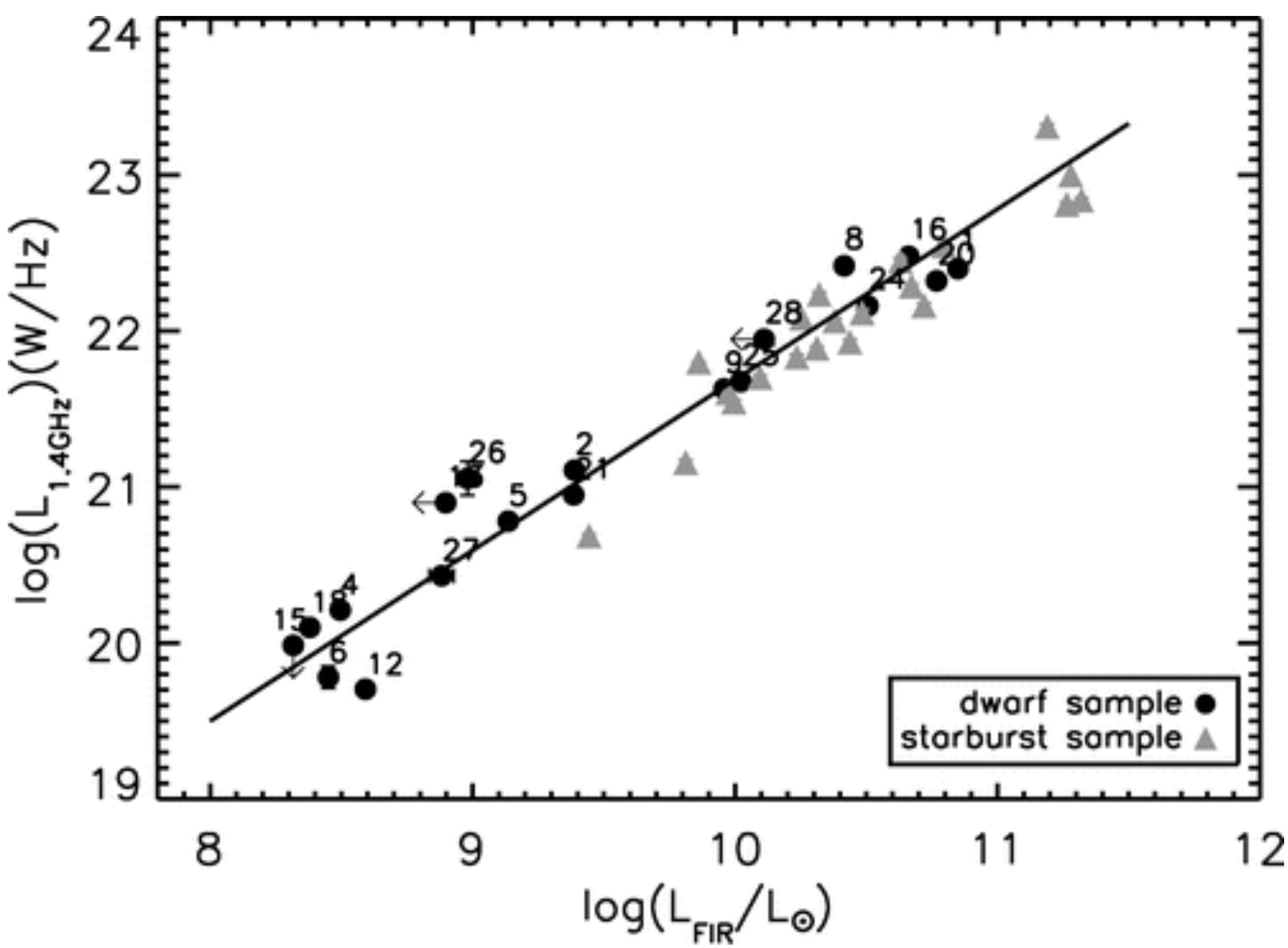
Tracers of massive stars: UV to Mid-IR

- UV luminosity
  - Directly traces the massive stars and their supply of ionizing photons
  - Susceptible to extinction!
  - Requires UV telescope (GALEX)
- Emission lines arising from ionized gas
  - Not as susceptible to extinction (at least Hα)
  - Measures the number of recombinations = number of ionizing photons = number of massive stars
- Warm Dust
  - Photons warm surrounding dust ➔dust reradiates in sub-mm and far-IR (based on properties of dust: T and composition)
  - Unaffected by extinction (it *is* the extincting material!)

Tracers of massive stars: radio continuum

- Continuous radiation from free electrons (free-free emission)
  - Depends on electron density ρ_e
  - ρ_e depends on number of ionizing photons and number of massive stars
    - Unaffected by extinction
- Radio synchrotron emission
  - Massive stars explode ➔ expanding shocks accelerate particles to relativistic velocities ➔ combine with magnetic field ➔ synchrotron emission
    - Indirect measure of number of Sne
- The Far-IR/Radio continuum correlation:
  - Massive stars warm dust ➔ Far-IR
  - SNe accelerate cosmic rays ➔ radio continuum
    - But this hasn't really been demonstrated and
    - Implies a fixed fraction of SNe energy is converted into cosmic rays

Far-Infrared – Radio-Continuum correlation



Wu et al. 2008, AJ 676, 970

Tracers of star formation

- Is any one better than the others?
  - Depends on redshift
  - UV hard to measure at low redshift
    - not visible from ground
  - H α hard to measure at high redshift
    - moves into NIR where sky is bright
  - Sub-mm sensitivity and radio continuum sensitivity isn't what it needs to be for high-redshift measurements

- Primary limitations are two-fold:
  - Only measures the number of massive stars (those massive enough to emit lots of ionizing photons)
  - No single tracer can be used well over broad range in redshift

Scaling relations

- V, L, size correlate (the physical *scale* of disk systems)
  - "Larger" systems tend to have higher disk surface-brightness, older stellar populations, less gas, higher metallicity (i.e., the Hubble Sequence)
- Important 2[nd]-order effect:
  - matter-density increases with V, L, size
    - concentration, surface-brightness

    ➔ dynamical time-scales decrease

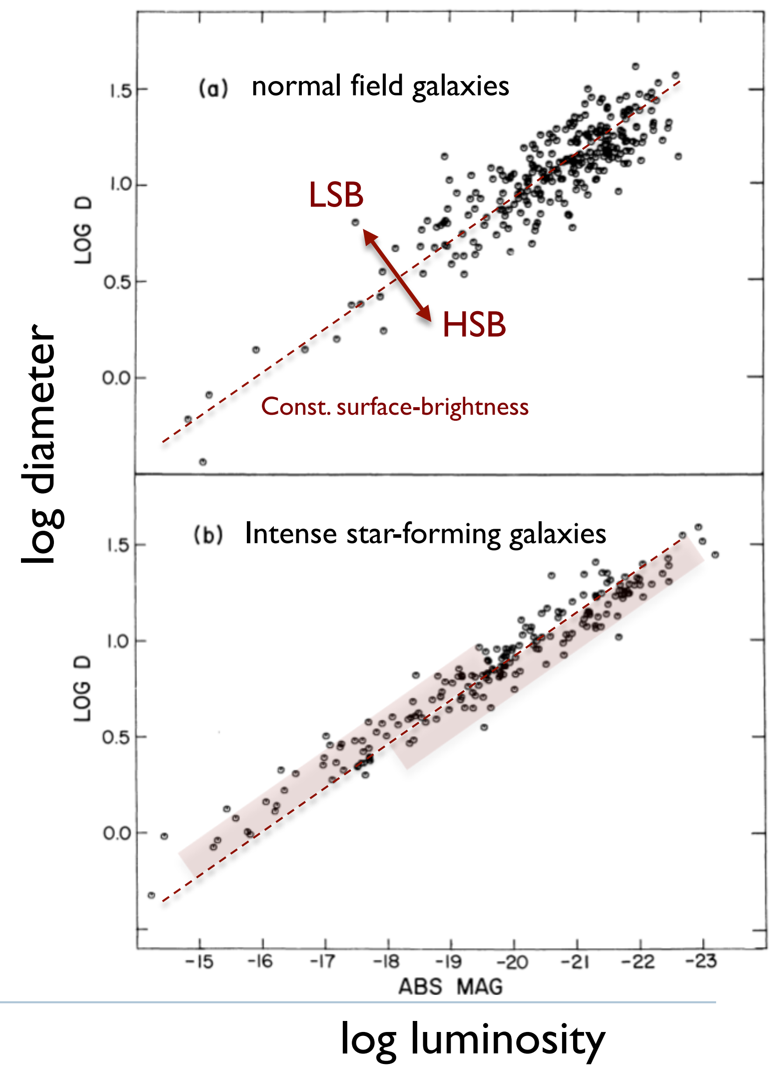
    $t_{dyn} \sim \sqrt{(1/G\rho)}$

    ➔ SFR, gas consumption and enrichment more rapid

    ➔ drives Hubble Sequence ???

    At some level it must.

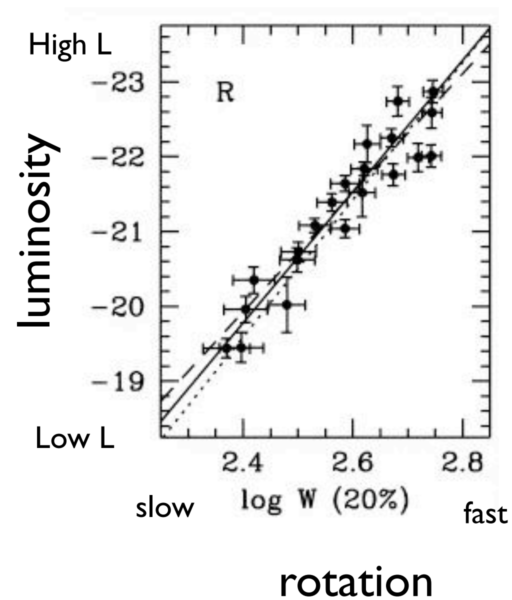
Huchra 1977, ApJ



Scaling relations *continued*

- What about mass?
  - The tightest correlation for disk galaxies is between V and L. This is called the **Tully-Fisher (TF) relation**

R-band (red light) TF:



Tully-Fisher relation: Measurement

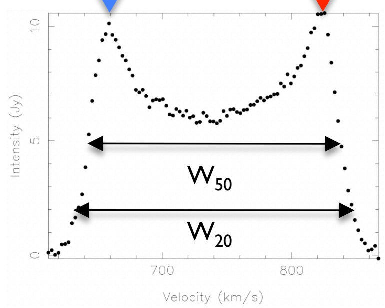
- Details of the measurement
- Velocity:
  - Measure of circular rotation
    - line-width or rotation curve
  - Corrections:
    - *inclination* (1/sin i)
    - turbulent broadening (if line width)
- Luminosity:
  - Corrections:
    - total flux
    - Galactic extinction
    - internal extinction (which depends on *inclination*)
    - distance
      - distance modulous
      - redshifting of band-pass, the so-called "k" correction

- Inclination:
  - Axial ratios of light profile (photometric ellipticity)
    - Correct for disk oblateness
  - Shape of iso-velocity contours (if 2D kinematics are available)

Surrogates measures of rotation

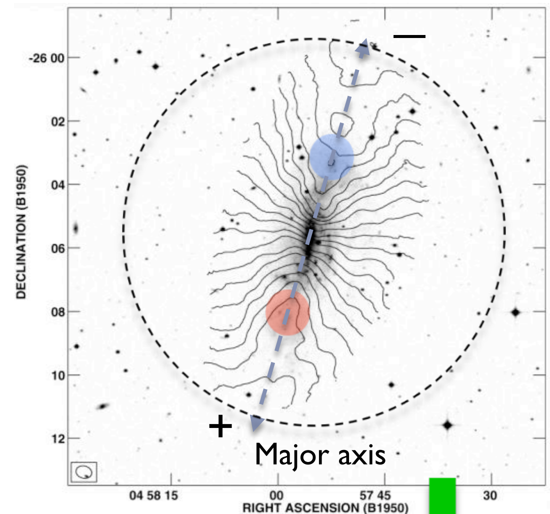
- Spatial information vs sensitivity:

4. Single dish (fiber):
Line width $W \sim 2V_c$



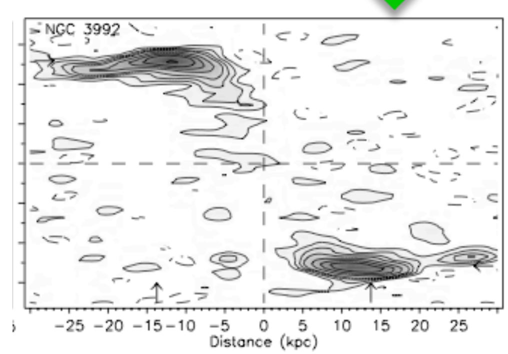
1. Interferometer/IFU:

➔ Velocity field
2D map of velocities,
or data cube

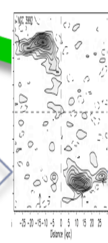


Slice down the major axis

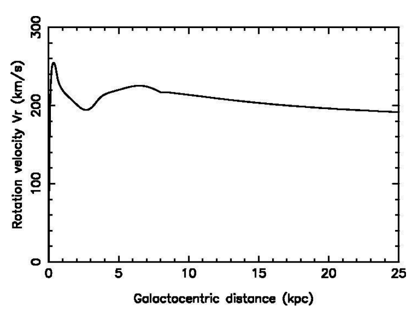
2. Position-velocity diagram (PVD):
Equivalent to long-slit spectrum



Integrate in x (and y)



3. Rotation curve



Flip (in V) and fold (in x)

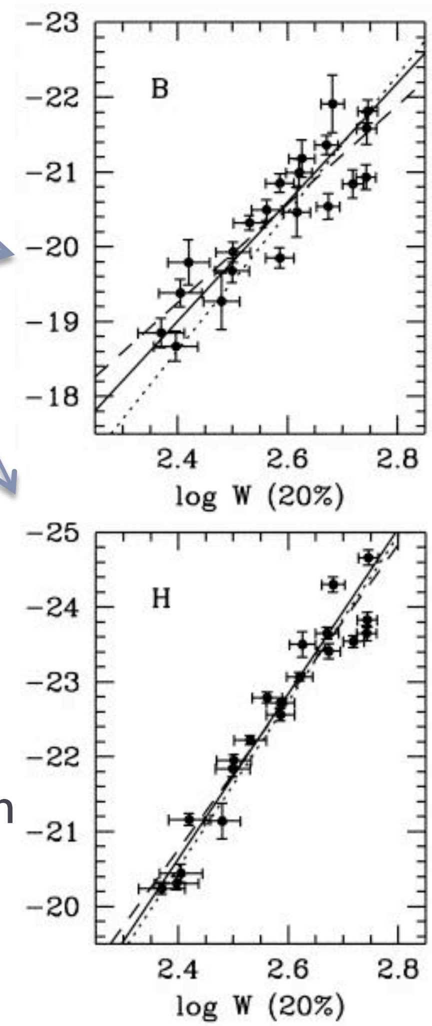
Tully-Fisher relationship: Scatter

- *Small!*
  - 0.5-0.3 mag in blue (B, 0.44 μ m)
  - 0.1 mag in near-IR (H, 1.6 μ m)
  - 0 mag (!) *intrinsic:* K-band for subset of galaxies with rotation curves and flat V(R) (Verheijen 2001)
    - *Too small?*

Why this trend?

gasp!

- Source of dispersion
  - Measurement errors (random)
  - Measurement errors (systematic)
    - Extinction
    - Shape of light-distribution (oblateness)➔inclination
    - Shape of rotation curve➔V_c
  - Cosmic variance
    - Variations in M/L with galaxy type



Tully-Fisher relation: Implications

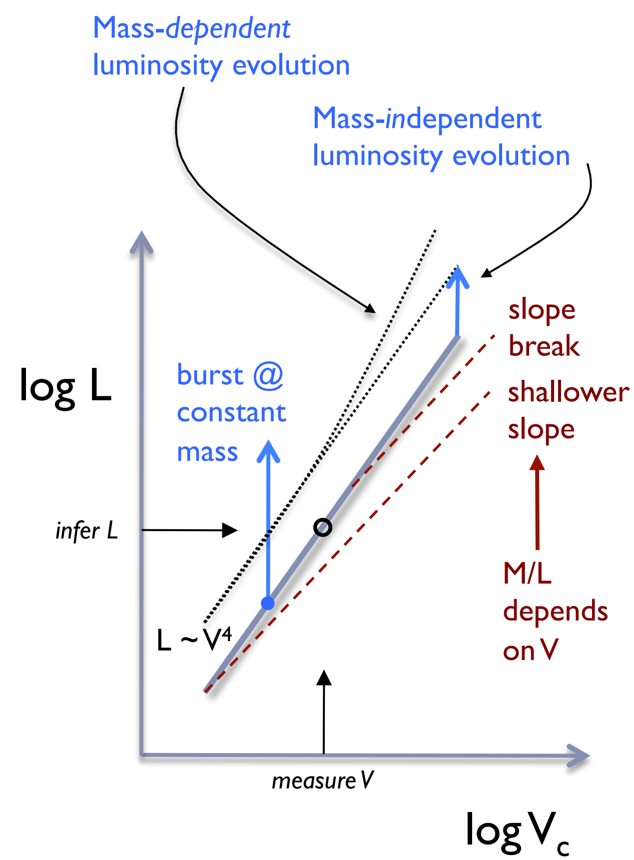
- Why is M/L so constant from galaxy to galaxy?
  - Here we're talking about M/L of the entire galaxy:
    - Mass is dominated by dark halo
    - Luminosity is dominated by disk
  - Total mass: M proportional to $[V^2_{max} h_R]$
  - Total luminosity: L proportional to $[I_0 h_R]$ (ignoring bulge)
  - ➔ L proportional to $[V^4_{max} (M/L)^2 I_0]$
  - A universal M/L implies remarkable constancy of the ratio of dark to luminous matter
    - Or worse, a fine-tuning of the dark-to-luminous mass ratio as the stellar M/L varies.
- What does this tell us about galaxy formation and feedback?

Tully-Fisher relation: diagnostic tool

- Standard candle: V is distance-independent

- Structural probe: slope and scatter
  - Since L is proportional to $[V^4_{max} (M/L)^2 I_0]$
  - ➔ *M* vs log(V) should have slope of 10
  - and should depend on surface-brightness
    - *Slope is < 10, varies with wavelength*
    - *No dependence on surface-brightness*

- Evolutionary probe
  - Changes in M/L with *time*
    - Assume M roughly constant
      - ☐ Secular changes in L: star-formation history
      - ☐ Stochastic changes in L (star-formation bursts)
        - ☐ Scatter increases with burst duty-cycle



Dynamics of collisionless systems

- Motivation:
  - Circular rotation is too simple and v_c gives us too little information to constrain Φ and hence ρ (e.g., rotation curves)
  - Without Φ and hence ρ we can't understand how mass has assembled and stars have formed
    - We can't even predict how the Tully-Fisher relation should evolve
  - Gas is messy because it requires understanding hydrodynamics, and likely magneto-hydrodynamics.
  - At are disposal are stars, nearly collisionless tracers of Φ!

Dynamics of collisionless systems

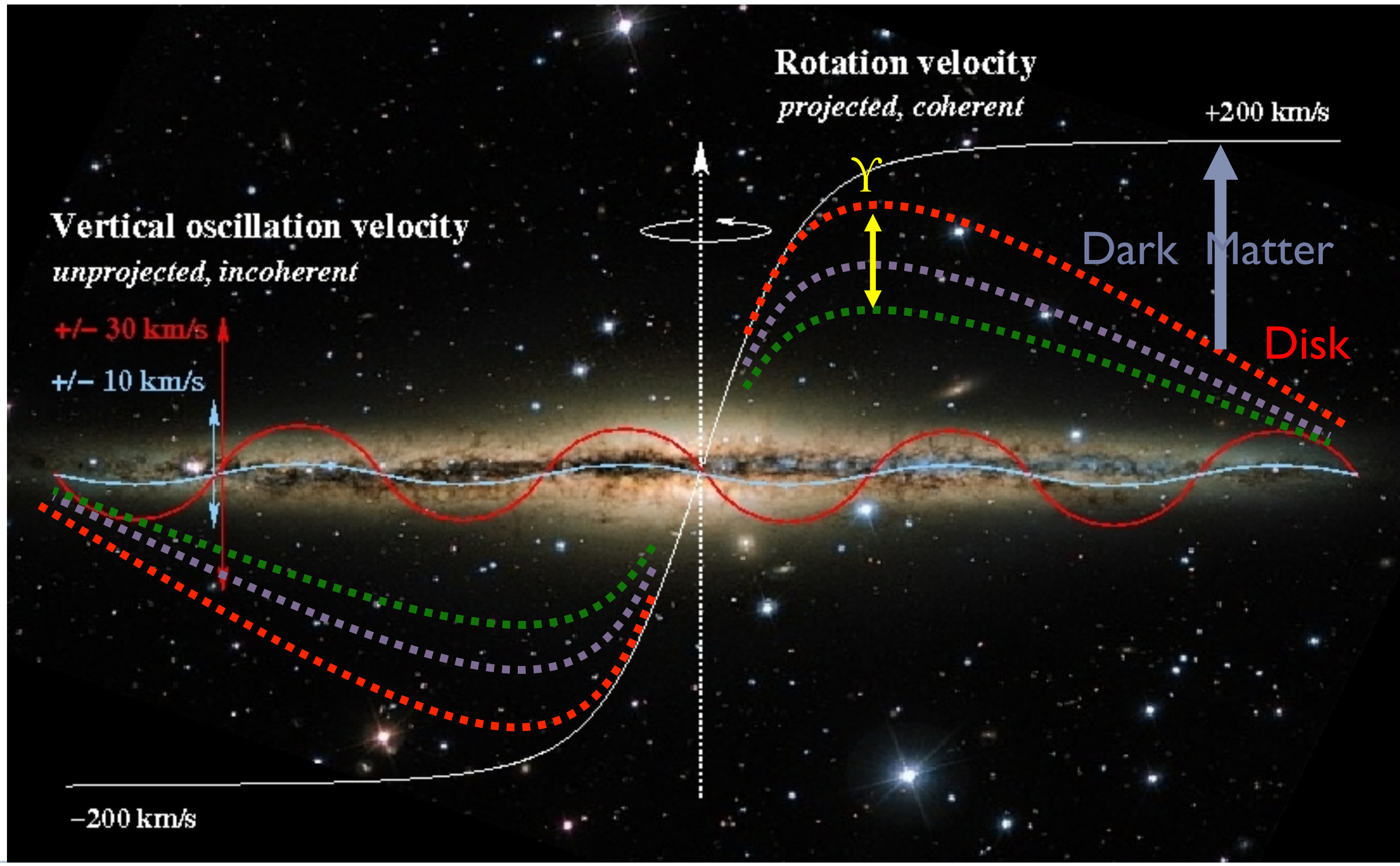
- How we'll proceed:
  - Start with the Continuity Equation (CE)
  - Use CE to motivate the Collissionless Bolztmann Equation (CBE), like CE but with a force term (remember $\nabla\Phi(\mathbf{x})$!)
  - Develop moments of CBE to relate v and σ and higher-order moments of velocity to Φ and ρ.
- Applications to realistic systems and real problems
  - Velocity ellipsoid
  - Asymmetric drift

Don't be intimidated by moment-integrals of differential equations in cylindrical coordinates: follow the terms, and look for physical intuition.

Disk heating
Disk mass
Disk stability

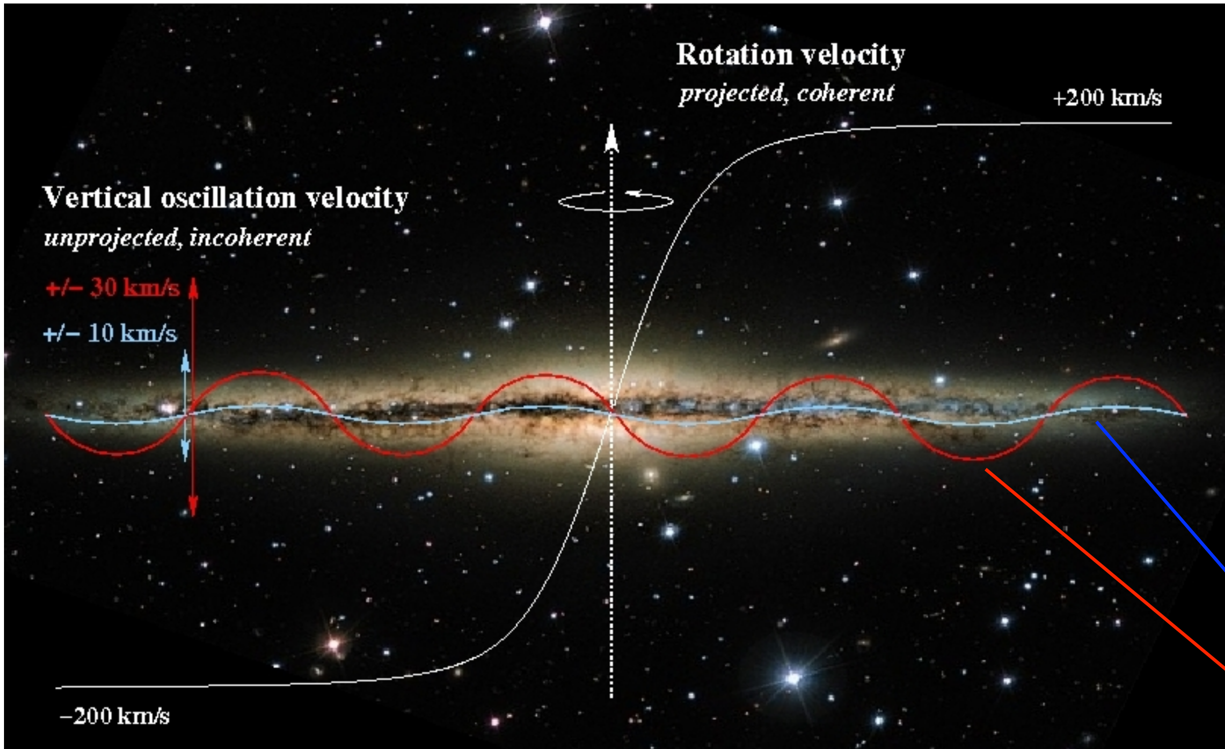
Example: Breaking the Disk-Halo Degeneracy

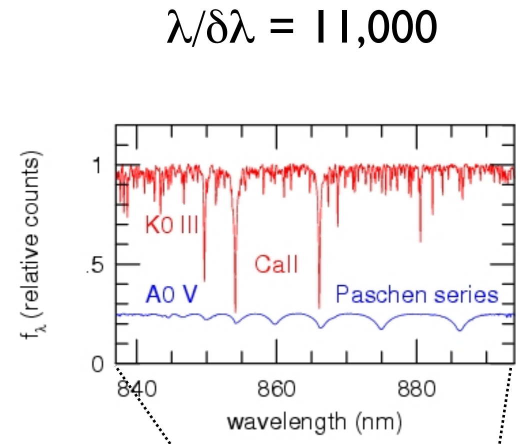
- Rotation provides the *total* mass within a given radius.
- Vertical oscillations of disk stars provides *disk* mass within given height



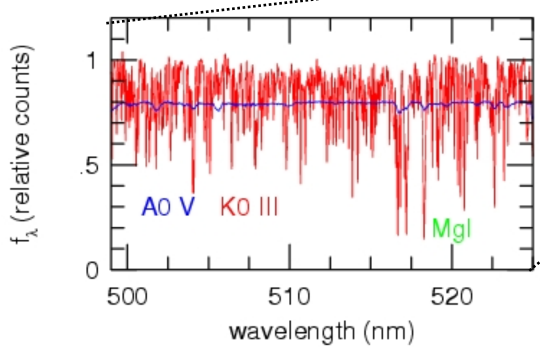
Vertical oscillations: a direct, dynamical approach

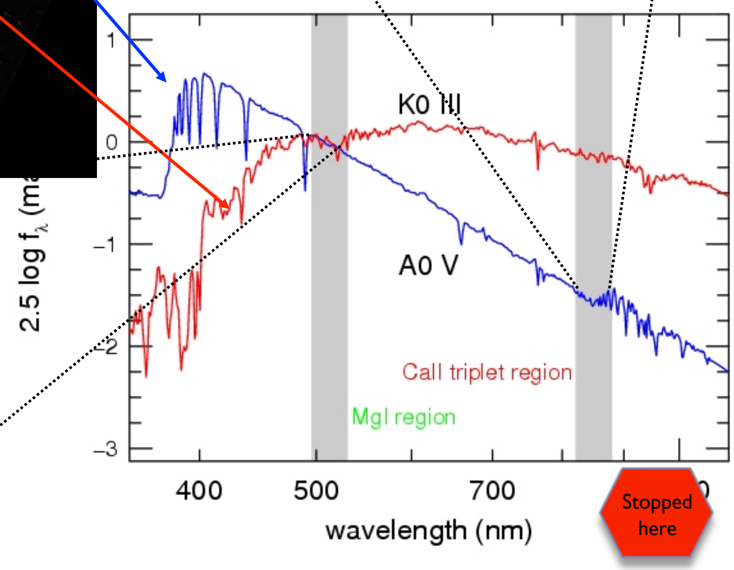
The kinematic signal





- Young stars
  - Hot: weak or intrinsically broad lines
  - Dyamically cold, thin layer (extinction)
- Old stars
  - Cool: many strong, narrow lines
- Dynamically warm, thick layer





Disk Mass formula

Use *statistical* measure of *disk thickness* from edge-on galaxies …

vertical distribution*

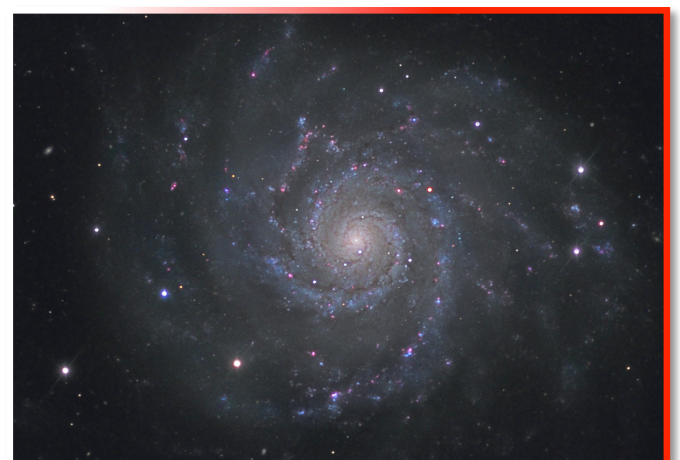
thickness

vertical oscillations

$$\Sigma = 100\left(\frac{k}{3/2}\right)^{-1}\left(\frac{h_z}{444\ \mathrm{pc}}\right)^{-1}\left(\frac{\sigma_z}{30\,\mathrm{km/s}}\right)^{2} \mathrm{M_{sol}\ pc^{-2}}$$

Disk mass surface density

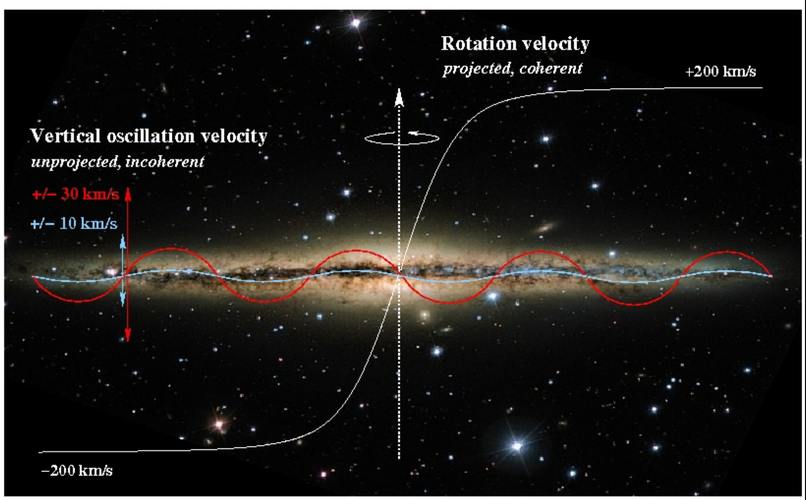
….and apply relation to face-on galaxies where the *vertical oscillations* of stars can be measured.



* 1.5<k<2 for exp, sech, sech^2

Edge-on or Nearly Face-on ?

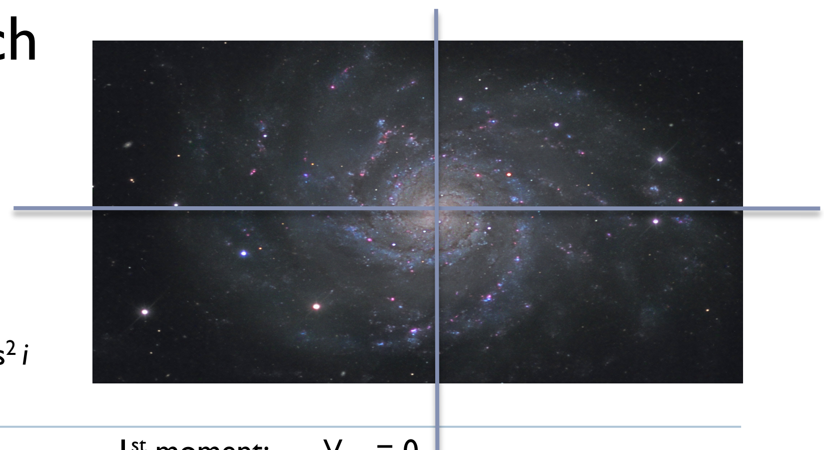
- ✓ Rotation projected
- × Vertical dispersion *inaccessible* except via statistical *kinematic* correlations
- ✓ Vertical height projected



- ✓ Rotation accessible at high spectral resolution
- ✓ Vertical dispersion projected
- × Vertical height *inaccessible* except via statistical *photometric* correlations

The problem

- If you look at completely face-on galaxies you can't measure rotation ➔ can't estimate total mass (total potential)
- Even if you look at *moderate inclination* (i~30°) galaxies, you get components of the stellar velocity dispersion (σ) which are not vertical (σ_z) but radial (σ_R) or tangential (σ_ϕ).
- In other words, $\boldsymbol{\sigma}$ is a vector – the velocity ellipsoid
- From the solar neighborhood we expect: $\sigma_R > \sigma_\phi > \sigma_z$
- But we can only observe 2 spatial dimensions:
  - How do we solve for σ_z ?
- And how do we solve for σ_R, which turns out to be interesting for understanding disk heating?



1st moment: $V_{los} = V \sin i$
2nd moment: $\sigma_{los}^2 = \sigma_\phi^2 \sin^2 i + \sigma_z^2 \cos^2 i$

1st moment: $V_{los} = 0$
2nd moment: $\sigma_{los}^2 = \sigma_R^2 \sin^2 i + \sigma_z^2 \cos^2 i$

los = line of sight

Continuity Equation

- The mass of fluid in closed volume V, fixed in position and shape, bounded by surface S at time t
  - $M(t) = \int \rho(\mathbf{x},t) d^3\mathbf{x}$
- Mass changes with time as
  - $dM/dt = \int (\partial \rho / \partial t)\, d^3\mathbf{x} = -\int \rho\, \mathbf{v} \cdot d^2\mathbf{S}$
  - mass flowing out area-element d^2S per unit time is $\rho\, \mathbf{v} \cdot d^2\mathbf{S}$

NB: ∂ = partial derivative

- The above equality allows us to write
  - $\int (\partial \rho / \partial t) d^3\mathbf{x} + \int \rho\, \mathbf{v} \cdot d^2\mathbf{S} = 0$
  - $\int [\, \partial \rho / \partial t + \nabla \cdot (\rho\, \mathbf{v})\,]\, d^3\mathbf{x} = 0$

Divergence theorem

- Since true for any volume
  - $\partial \rho / \partial t + \nabla \cdot (\rho\, \mathbf{v}) = 0$

This is CE

In words: the change in density over time (1st term) is a result of a net divergence in the flow of fluid (2nd term). Stars are a collisionless fluid.

Collisionless Boltzmann Equation

- Generalize concept of spatial density ρ to phase-space density $f(\mathbf{x},\mathbf{v},t)\, d^3\mathbf{x}\, d^3\mathbf{v}$, **where** $f(\mathbf{x},\mathbf{v},t)$ is the distribution function (DF)
  - $f(\mathbf{x},\mathbf{v},t)\, d^3\mathbf{x}\, d^3\mathbf{v}$ gives the number of stars at a given time in a small volume $d^3\mathbf{x}$ and velocities in the range $d^3\mathbf{v}$
  - The number-density of stars at location **x** is the integral of $f(\mathbf{x},\mathbf{v},t)$ over velocities:
    - $n(\mathbf{x},t) = \int f(\mathbf{x},\mathbf{v},t)\, d^3\mathbf{v}$

$$\nu(\mathbf{x}) \equiv \int f d^3\mathbf{v}$$

  - The mean velocity of stars at location **x** is then given by
    - $\langle \mathbf{v}(\mathbf{x},t) \rangle = \int \mathbf{v}\, f(\mathbf{x},\mathbf{v},t)\, d^3\mathbf{v} / \int f(\mathbf{x},\mathbf{v},t)\, d^3\mathbf{v}$

$$\overline{v}_i \equiv \frac{1}{\nu} \int f v_i d^3\mathbf{v}$$

quantities you can measure

S&G notation

Notation we'll adopt

CBE *continued*

- **Goal**: Find equation such that given $f(\mathbf{x},\mathbf{v},t_0)$ we can calculate $f(\mathbf{x},\mathbf{v},t)$ at any t, …

  …. and hence our observable quantities $n(\mathbf{x},t)$, $\langle v(\mathbf{x},t)\rangle$, etc.

  - $f(\mathbf{x},\mathbf{v},t_0)$ is our initial condition
  - The gravitational potential does work on $f(\mathbf{x},\mathbf{v},t)$

- Introduce some useful notation and relate to the potential
  - Let $\mathbf{w} \equiv (\mathbf{x},\mathbf{v}) = (w_1 \ldots w_6)$
  - $\mathbf{w}' \equiv dw / dt = (\mathbf{x}',\mathbf{v}') = (\mathbf{v}, -\nabla\Phi) = (w_1 \ldots w_3, -\nabla\Phi)$

CBE *continued*

- Recall CE gives: $d\rho/dt + \nabla\cdot(\rho\,\mathbf{v}) = 0$
- Replace $\rho(\mathbf{x},t) \rightarrow f(\mathbf{x},\mathbf{v},t)$
- CE gives:
  - $df/dt + \Sigma_{i=1,6}\, d(fw_i')/dw_i' = 0$ ….but:
    - $dv_i/dx_i = 0$ x_i, v_i independent elements of phase-space
    - $dv_i'/dv_i = 0$ $v' = -\nabla\Phi$, and the gradient in the potential does not depend on velocity.
  - $df/dt + \Sigma_{i=1,6}\, w_i'(df/dw_i) = 0$
  - $df/dt + \Sigma_{i=1,3}\, [v_i(df/dx_i) - (d\Phi/dx_i)(df/dx_i)] = 0$
- $df/dt + \mathbf{v}\cdot\nabla f - \nabla\Phi\cdot df/d\mathbf{v} = 0$

CBE

Vector notation